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Amplification through chaotic synchronization in spatially extended beam-plasma systems

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In this paper, we have studied the relationship between chaotic synchronization and microwave signal amplification in coupled beam-plasma systems. We have considered a 1D particle-in-cell numerical model of unidirectionally coupled beam-plasma oscillatory media being in the regime of electron pattern formation. We have shown the significant gain of microwave oscillation power in coupled beam-plasma media being in the different regimes of generation. The discovered effect has a close connection with the chaotic synchronization phenomenon, so we have observed that amplification appears after the onset of the complete time scale synchronization regime in the analyzed coupled spatially extended systems. We have also provided the numerical study of physical processes in the chain of beam-plasma systems leading to the chaotic synchronization and the amplification of microwave oscillations power, respectively. *Published by AIP Publishing.*

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Chaotic synchronization in spatially extended systems is one of the most interesting and intricate phenomena in nonlinear science. In this paper, we study the time scale synchronization in unidirectionally coupled beam-plasma systems being in the regime of electron pattern formation and its relationship with the phenomenon of amplification of chaotic signals. We show that the onset of a complete time scale synchronization regime is accompanied by a sharp increase of the amplification coefficient of the output microwave power. Such an effect is observed both in coupled chaotic systems and in the case of the influence of chaotic signal on the spatially extended electronic system demonstrating periodic dynamics.

I. INTRODUCTION

The study of chaotic synchronization in the coupled beam-plasma systems is one of the most interesting and intricate problems of modern physics of plasmas and microwave vacuum electronics.^{1–12} The regimes of chaotic synchronization in such systems have wide practical applications, for example, for creation of a powerful array of microwave oscillators, secure information transmission, and control of chaos in the microwave devices.^{13–23}

Several types of the synchronous behavior in coupled chaotic beam-plasma systems are known at present. They are the phase synchronization, complete synchronization, generalized synchronization, and time scale synchronization.^{7,10,11,24} One of the most important types from them is the regime of time scale synchronization.

Time scale synchronization²⁵ means the presence of the synchronous dynamics in a certain range $[s_1; s_2]$ of time scales s introduced by means of the continuous wavelet transform^{26,27}

$$W(s, t_0) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t) \psi^* \left(\frac{t - t_0}{s} \right) dt, \quad (1)$$

where $x(t)$ is a time series under study, $\psi(\eta) = (1/\sqrt[4]{\pi}) \exp(j\Omega_0\eta) \exp(-\eta^2/2)$ is the Morlet complex mother wavelet function, and $\Omega_0 = 2\pi$ is the parameter of the wavelet. The behavior of the system on different time scales can be characterized by means of the wavelet spectrum

$$W(s, t_0) = |W(s, t_0)| \exp i\phi(s, t_0), \quad (2)$$

where $|W(s, t_0)|$ and $\phi(s, t_0)$ are the amplitude and phase of the wavelet spectrum, respectively. It is also necessary to introduce into consideration the distribution of integral energy by time scales

$$\langle E(s) \rangle = \int |W(s, t_0)| dt_0. \quad (3)$$

For two coupled dynamical systems with time series $\mathbf{x}_{1,2}(t)$, time scale synchronization takes place, if there is the range of the synchronous time scales $s \in [s_1; s_2]$ where the phase locking condition

$$|\phi_1(s, t) - \phi_2(s, t)| < 2\pi \quad (4)$$

(where the phases $\phi_{1,2}$ should be considered as monotonically increasing or monotonically decreasing, $-\infty < \phi_{1,2} < +\infty$) is satisfied, and the part of the wavelet spectrum energy fallen in this range is positive,^{24,28,29} i.e.,

$$E_{snhr} = \int_{s_1}^{s_2} \langle E(s) \rangle ds > 0. \quad (5)$$

Time scale synchronization is known to combine different types of chaotic synchronization known at present. In particular, the regimes of phase, generalized and complete

synchronization can be considered as partial cases of the time scale synchronization regime. The difference between them is defined by a number of synchronous time scales that can be characterized by a measure of time scale synchronization. The measure of time scale synchronization first introduced in Ref. 25 is defined as a part of energy of the wavelet spectrum fallen in the synchronized time scales, i.e.

$$\gamma_{1,2} = \frac{1}{E_{1,2}} \int_{s_1}^{s_2} \langle E_{1,2}(s) \rangle ds, \quad (6)$$

where

$$E_{1,2} = \int_0^\infty \langle E_{1,2}(s) \rangle ds \quad (7)$$

indices 1 and 2 correspond to the first and second interacting systems, respectively. It is equal to zero ($\gamma_{1,2} = 0$) in the regime of asynchronous dynamics of interacting systems and it is equal to one ($\gamma_{1,2} = 1$) in the complete synchronization regime. The intermediate values of the synchronization measure ($0 < \gamma_{1,2} < 1$) correspond to the regimes of the phase or generalized synchronization, at that the quantitative value of $\gamma_{1,2}$ defines the degree of the phase or/and generalized synchronization.^{24,25} In the case of unidirectional coupling, it is necessary to calculate such a measure only for the response system, i.e., $\gamma = \gamma_2$.

It should be noted that the consideration of the system dynamics on the different time scales allows detecting the presence of the synchronous regime even in the case when the classical methods of the synchronous regime detection are misleading (for example, the detection of the phase synchronization in systems with an ill-defined phase).²⁸ Therefore, such a method allows analyzing the behavior of complex spatially extended beam-plasma systems and defining the degree of their synchronization.

In the present paper, we analyze the effects taking place in unidirectionally coupled beam-plasma systems at the time scale synchronization regime onset. As would be shown below, the onset of such a regime is accompanied by the amplification of the output microwave power of signals generated by interacting systems.

II. SYSTEM UNDER STUDY

The system under study is represented by two unidirectionally coupled low-voltage virtual cathode oscillators

(VCO) or vircators, the drive and response ones. Its schematic representation is shown in Fig. 1.

A low-voltage vircator is a plane diode gap penetrated by the electron beam with overcritical perveance $p_{cr} = IV^{3/2}$,^{30,31} where I is the beam current and V is the applied potential. To form the overcritical perveance, the output grid of the system is subjected to the retarding potential. The increase of the retarding potential results in the virtual cathode formation.^{23,32,33}

We have used a one-dimensional non-stationary model of beam dynamics to simulate the nonlinear non-stationary processes in the charged particle beam with a virtual cathode.³⁴ The simulation has been performed by the particle-in-cell (PIC) method. Due to such an approach, the electron beam has been considered as a set of large charged particles injected in the interaction space in equal moments of time with the constant velocity. For each particle, the non-relativistic equations of motion have been solved. In dimensionless form, such equations are given by³¹

$$\frac{d^2 x_i}{dt^2} = -E(x_i), \quad (8)$$

where x_i is the coordinate of the i^{th} charged particle, $E(x_i)$ is the space-charge field intensity in the coordinate x_i , and $i = 1 \dots N$, N is a full number of large charged particles used in numerical simulation.

The intensity and potential of the space charge electric field have been defined in the uniform space grid with step Δx . The potential of the space charge electric field in the quasi-static approximation has been defined by the Poisson equation which in one-dimensional approximation takes the following form:

$$\frac{d^2 \phi}{dx^2} = \alpha^2 \rho(x), \quad (9)$$

where $\alpha = \omega_p L/v_0$ is the Pierce parameter, where ω_p is a plasma frequency, L is a length of the drift gap, and v_0 is undisturbed velocity of the electron beam. The boundary condition for the Poisson equation is the requirement of the presence of the retarding potential difference between the grids of the system, i.e., $\phi(0) = 0$, $\phi(1) = \Delta\phi$. The intensity E of the space-charge field has been defined in such a case by the numerical differentiation of the obtained values of the potential $E = -\partial\phi/\partial x$.

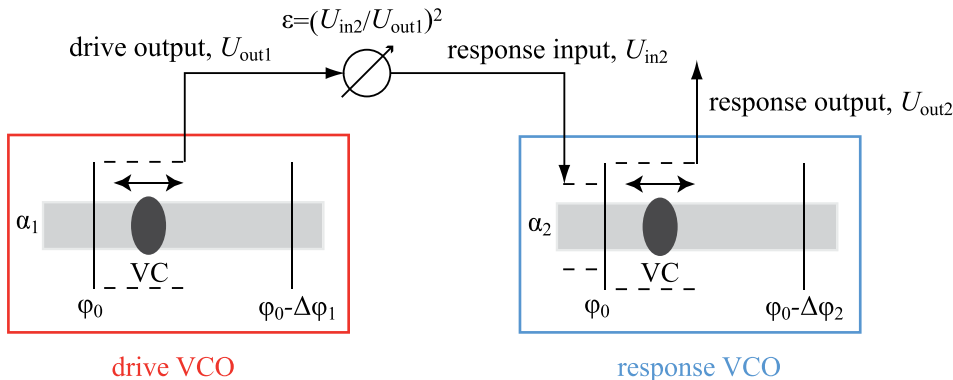


FIG. 1. The model of two unidirectionally coupled low-voltage vircators (VCO). Here, blue rectangles represent electron beam piercing the drift space bounded by two grid electrodes in the regime of VC formation and spatio-temporal oscillations. Dashed lines correspond to helical systems for the microwave signal input and output. Arrows reflect the direction of microwave energy transition from drive system to the response one.

Calculation of the space charge density has been performed by the particle-in-cell (PIC) method which consists of the finding of the space charge by means of the bilinear weighting of the particle charge on the grid.³⁴ In the PIC method, the space charge density in node j of the space grid ($x_j = j\Delta x$) is given by

$$\rho(x_j) = \frac{1}{n_0} \sum_{i=1}^N \Theta(x_i - x_j), \quad (10)$$

where N is the full number of charged particles, n_0 is a parameter of numerical scheme equal to the number of particles in cell being in non-perturbed state

$$\Theta(x) = \begin{cases} 1 - |x|/\Delta x, & |x| < \Delta x, \\ 0, & |x| > \Delta x \end{cases} \quad (11)$$

is the piece-linear function defining the procedure of “weighting” of the particle in the space grid with the step Δx .

For output of the power of microwave oscillations of the virtual cathode in the low-voltage vircator, the segment of the helix slow-wave structure has been used. It has been simulated in the framework of the equivalent circuit method.^{35,36} Due to such an approach, the segment of the helix slow-wave structure has been represented as a series of inductors $L[H/m]$ with shunt capacitors $C[F/m]$ and described by the telegraphy equations with additional terms describing the excitation of electromagnetic waves by the beam

$$\frac{\partial I}{\partial t} = -\frac{1}{L} \frac{\partial U_{out}}{\partial x}, \quad \frac{\partial U_{out}}{\partial t} = -\frac{1}{C} \frac{\partial I}{\partial x} + \frac{1}{C} \frac{\partial q}{\partial t}, \quad (12)$$

where $U_{out}(t)$ is the output signal of the low-voltage vircator (the integral value characterizing the system state). The telegraphy equations have been solved numerically under the assumption of the line conditioning on the left $x=0$ and right $x=l$ ends of the segment of the helix slow-wave structure. The distribution of the value of the beam charge $q(t, x)$ exciting electromagnetic waves in the transmission line has been obtained by the solution of task by the particle-in-cell method (see above). During the simulations, we have used the following values of capacity and inductance: $C = 10^4$ and $L = 1$.

To analyze time scale synchronization in two unidirectionally coupled low-voltage vircators, we have performed the numerical simulation of the dynamics of the drive (marked by index “1”) and response (marked by index “2”) generators in accordance with the Eqs. (8)–(12) mentioned above. Parameters of the used 1D PIC numerical scheme for simulation of each vircator have been selected in the following way: number of spatial cells $N_c = 400$, number of particles per cell in the undisturbed state $n_0 = 10$, spatial step $\Delta x = 1/N_c = 2.5 \times 10^{-3}$, and time step $\Delta t = \Delta x/n_0 = 2.5 \times 10^{-4}$. To improve the accuracy of calculations, we have fulfilled the high pass filtration procedure related to the electric field $E(x)$ and space-charge density $\rho(x)$ spatial distributions³⁴ that gives it possible to accurately calculate the derivative of the charge ρ in the telegraphy Eq. (12).

Mentioned parameters allow providing correct numerical PIC simulation of regular and chaotic regimes of oscillations in the considered systems.

The unidirectional coupling between drive and response low-voltage vircators has been realized by means of the microwave signal injection from the output of the drive generator to the input of the response one.³⁷ The input of signal in the response generator has been performed by means of the modulation of beam penetrating into the diode gap by the helical electrodynamic segment located between the electron gun and input grid of the interaction space subjected to the influence of the signal of the drive vircator. The attenuator has been located in the communication channel between generators that allows controlling the power of microwave signal affecting the response generator. Such peculiarity has been taken into account in the system under study by the addition of supplementary equations describing the modulating helix to the equations of the response generator. These equations are given by

$$\frac{\partial I_{2in}}{\partial t} = -\frac{1}{L} \frac{\partial U_{2in}}{\partial x}, \quad \frac{\partial U_{2in}}{\partial t} = -\frac{1}{C} \frac{\partial I_{2in}}{\partial x} \quad (13)$$

with the boundary condition

$$U_{2in}(0, t) = \sqrt{\varepsilon} U_{1out}(1, t - T), \quad (14)$$

where ε is a coupling coefficient calculated as a ratio of the power of drive signal to the output power of the response generator and $T=1$ is a time delay chosen arbitrary due to the unidirectional type of coupling between interacting systems.

The control parameters in the system of unidirectionally coupled low-voltage vircators are the retarding potential difference $\Delta\varphi_{1,2}$ between the grids of the drift gap, the Pierce parameters $\alpha_{1,2}$, and coupling parameter ε . The variation of the retarding potential of the output grid and Pierce parameters can result in the change of the dynamics of the electron beam in the generator and variation of the virtual cathode oscillation regime.

III. RESULTS

We have studied two different oscillation regimes of the interacting drive and response systems: (i) both the drive and response low-voltage vircators are in the regimes of chaotic generation ($\alpha_1 = 0.9$, $\Delta\varphi_1 = 0.53$, $\alpha_2 = 0.9$, $\Delta\varphi_2 = 0.55$); (ii) drive chaotic generator affects the response one demonstrating the regime of periodic oscillations ($\alpha_1 = 0.9$, $\Delta\varphi_1 = 0.53$, $\alpha_2 = 0.9$, $\Delta\varphi_2 = 0.6$). Fourier spectra, phase portraits, and time series of the output radiation for these regimes are shown in Figs. 2(a), 2(b), and 3(a), respectively. Figure 2(a) corresponds to the drive low-voltage vircator ($\alpha_1 = 0.9$, $\Delta\varphi_1 = 0.53$) and Figs. 2(b) and 3(a) refer to the response vircator in the chaotic ($\alpha_2 = 0.9$, $\Delta\varphi_2 = 0.55$) and periodic ($\alpha_2 = 0.9$, $\Delta\varphi_2 = 0.6$) regimes. It is easy to observe the presence of two well-pronounced spectral components f_1 and f_2 both in the regimes of chaotic and periodic generation.

To study the time scale synchronization regime onset in unidirectionally coupled low-voltage vircators, we have

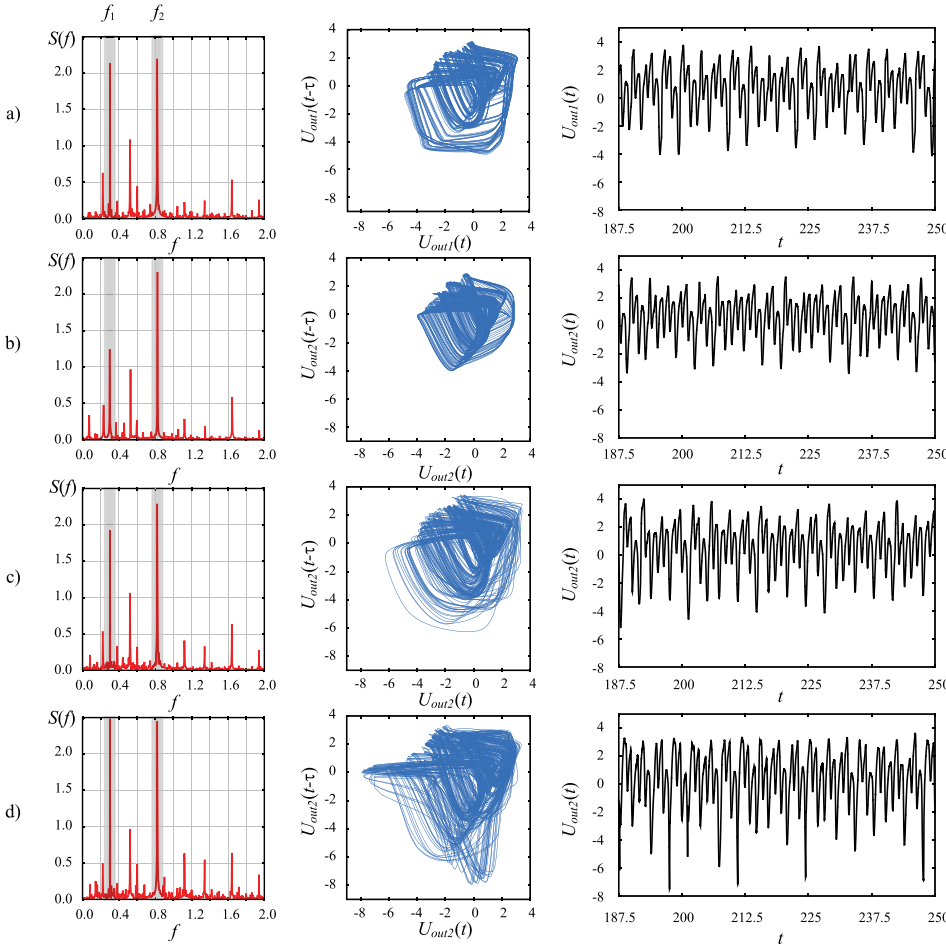


FIG. 2. (a) Fourier spectrum (left), phase portrait (middle), and time series (right) of the output radiation of the drive chaotic generator ($\alpha_1 = 0.9$, $\Delta\varphi_1 = 0.53$). (b)–(d) Evolution of the spectral compound, phase portraits, and time series of the output radiation of the response chaotic generator ($\alpha_2 = 0.9$, $\Delta\varphi_2 = 0.55$) with the coupling parameter value increasing: (b) $\varepsilon = 0.0$, (c) $\varepsilon = 0.3$, and (d) $\varepsilon = 0.8$. The time delay $\tau = 0.375$ for all considered cases.

analyzed the output signals $U_{out1}(t)$ and $U_{out2}(t)$ from the drive and response systems, respectively, for different values of the coupling parameter strength. Figure 4(a) shows the dependence of the measure of time scale synchronization γ (6) on the coupling parameter ε for the case when both low-voltage vircators are in the regimes of chaotic generation. The behavior of the dependence is a typical one for coupled chaotic systems, i.e., $\gamma = 0$ for the small values of the coupling parameter when the asynchronous dynamics is observed, then it starts increasing that corresponds to the time scale synchronization regime onset reaching the value of $\gamma = 1$ in the regime of complete synchronization. In the same Figure, the dependence of the coefficient of amplification P/P_0 on the coupling parameter is shown. The coefficient of amplification has been calculated as a ratio of the output power of the response generator for the fixed value of the coupling parameter strength to the same power of the response generator in the absence of coupling. It is clear that it is equal to one in the absence of coupling. The increase of the coupling strengths results, first of all, in the small variation of the coefficient with a further sharp increase of the amplification. Comparing the behavior of the synchronization measure and amplification coefficient [curves 1 and 2 in Fig. 4(a)] allows us to conclude that there are two important values of the coupling parameter: ε_1 corresponding to the case when the most part of the time scales is the synchronous one (γ is close to 1) and the output power of the response system starts increasing; ε_2 corresponding to the small

decrease of the measure of time scale synchronization and attainment of the output power on the level of saturation. Such a situation takes place for different values of the control parameters of the response generator and may be considered as a typical one. Figure 4(b) illustrates the dependencies of the quantitative values of ε_1 and ε_2 on the retarding potential difference $\Delta\varphi_2$ in the response system. It is clearly seen that for all considered values of the response parameters there are the fields of the power amplification and saturation. At that, the amplification of the power takes place for the relatively small values of the coupling strengths that allows achieving the power amplification due to the coupling of interacting generators.

To understand the physical mechanisms resulting in the power amplification in the regime of time scale synchronization, we have analyzed the transformation of the spectral compound of the response system with the increase of the coupling parameter strengths. Figures 2(b)–2(d) illustrate the Fourier spectra and phase portraits of the response system for different values of the coupling parameter (see caption). It is clearly seen the presence of two main spectral components f_1 and f_2 (marked by grey rectangles) in the Fourier spectra for all considered values of the coupling strength. They correspond to the different types of the electron bunching in the beam. As it has been mentioned above, two main spectral components are also presented in the Fourier spectrum of the drive system [Fig. 2(a)], at that, their intensities are comparable with each other. On the contrary, in the

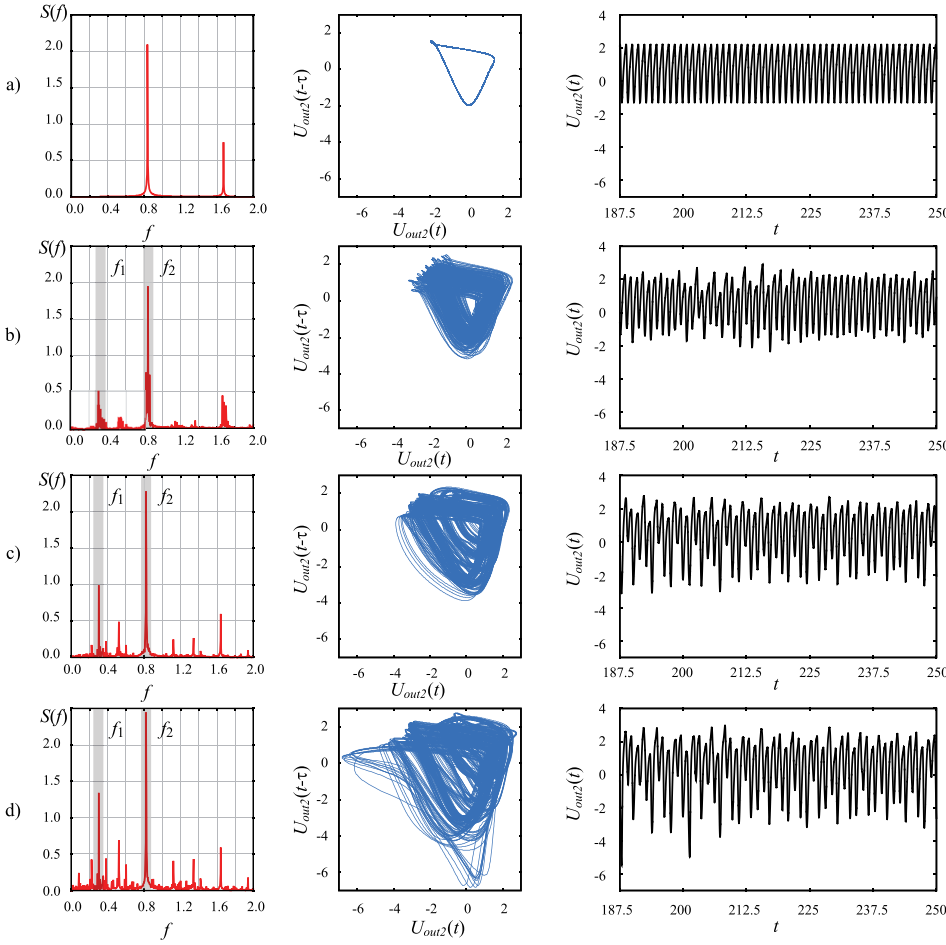


FIG. 3. Evolution of the spectral compound, phase portraits, and time series of the output radiation of the response periodic generator ($\alpha_2 = 0.9$, $\Delta\varphi_2 = 0.6$) with the coupling parameter value increasing: (a) $\varepsilon = 0.0$, (b) $\varepsilon = 0.2$, (c) $\varepsilon = 0.6$, and (d) $\varepsilon = 0.9$. The time delay $\tau = 0.375$ for all considered cases.

autonomous regime in the Fourier spectrum of the response system the spectral component on the frequency f_2 is more pronounced in comparison with the frequency f_1 [Fig. 2(b)]. The amplification of the output power with the coupling parameter value increasing is related to the increase of oscillations on the frequency f_1 due to the enhancement of the modulation bunching of the beam [Fig. 2(c)]. Saturation takes place when the intensities of the spectral components on the frequencies f_1 and f_2 become equal to each other [Fig. 2(d)]. It is also clearly seen the sequential deformation of the phase portrait and time series of the output radiation of the response system connected with the appearance and amplification of low-frequency oscillations on the frequency f_1 . At that, the type of the phase portrait of output oscillations shown in Fig. 2(d) is a typical one for the field of saturation. It should be noted that the coupling parameter value increasing results, first of all, in the complication of the response system dynamics that manifests itself in the growth of the value of the highest Lyapunov exponent, with its further decrease and transition in the field of the negative values. Such transition is connected with the generalized synchronization regime onset in coupled beam-plasma systems.^{10,12}

As it has been mentioned above, the amplification of the response vircator output power is related to the rising of the charged particle bunching efficiency while propagating through the drift space. The same amplification mechanism has been shown in the previous work³⁸ where we have investigated theoretically and experimentally the dynamics of the

low-voltage vircator driven by the external harmonic signal. It is well known that the velocity modulation of the electron beam leads to the formation of electron bunches in the drift space.³⁹ In such devices as virtual cathode oscillators, where the bunching of the electrons takes place under the interaction of charged particles with each other, velocity modulation of the electron beam at the frequencies close to the beam eigenfrequencies causes the growth of the space-charge density within the oscillating electron cloud in the region of the virtual cathode. Figure 5 shows the evolution of the beam space-charge density distribution $\rho(x)$ in the response vircator with the increase of the coupling parameter ε between coupled vircators. It is clearly seen that the coupling parameter growth causes the increase of the space-charge density distribution peak value. It is notable that the space-charge density distribution peak value corresponding to the saturation area ($\varepsilon \geq \varepsilon_2$) is approximately two times greater than one corresponding to the autonomous regime of the response vircator.

Similar effects are observed in the case when the periodic low-voltage vircator is subjected to the influence of the chaotic signal (see Figs. 3 and 6, where synchronization measure and coefficient of amplification as well as Fourier spectra and phase portraits of output radiation for different values of the coupling parameter have been shown). Indeed, the chaotic signal from the drive generator results in the chaoticization of the response system dynamics and even for very small values of the coupling parameter the dynamics of the

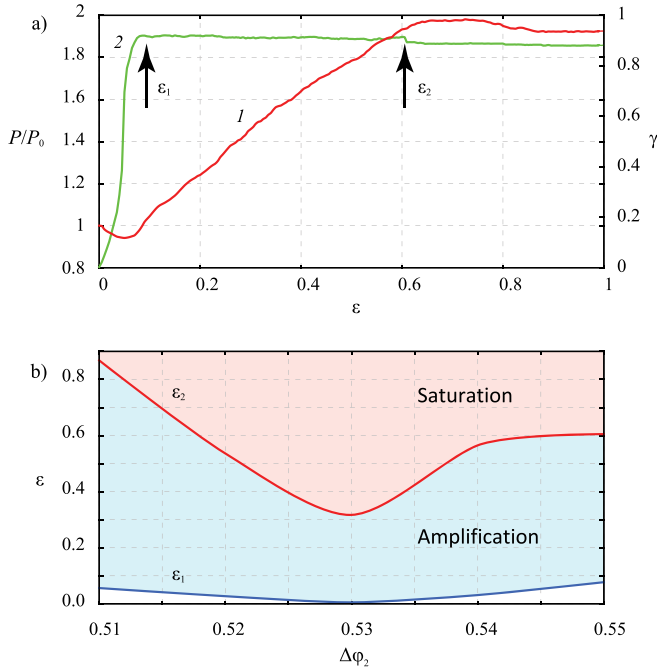


FIG. 4. (a) Dependence of the coefficient of amplification P/P_0 (curve 1, left axis) and the measure of time scale synchronization γ (curve 2, right axis) on the coupling parameter ϵ . Both low-voltage vircators are in the regimes of chaotic generation with the control parameters $\alpha_1 = 0.9$, $\Delta\phi_1 = 0.53$ (drive generator) and $\alpha_2 = 0.9$, $\Delta\phi_2 = 0.55$ (response generator). (b) Dependence of the quantitative values of ϵ_1 and ϵ_2 [marked by arrows in (a)] on the control parameter of the response generator $\Delta\phi_2$ in the field of chaotic generation for the same values of the control parameters α_1 , $\Delta\phi_1$, and α_2 . The fields of the power amplification and saturation are marked on the $(\epsilon, \Delta\phi_2)$ -plane.

response low-voltage vircator become chaotic [Fig. 3(b)]. At that, two main spectral components f_1 and f_2 are present in the Fourier spectrum of the output radiation of the response system as before. The spectral component on the frequency f_2 is more pronounced in comparison with the last one on the frequency f_1 . But with further increase of the coupling parameter the intensity of such spectral component grows up that changes the dynamics of the response system considerably [compare Figs. 3(c) and 3(d) with Fig. 3(b)]. The changes in the response system dynamics are accompanied by the growth of the output power of the response system.

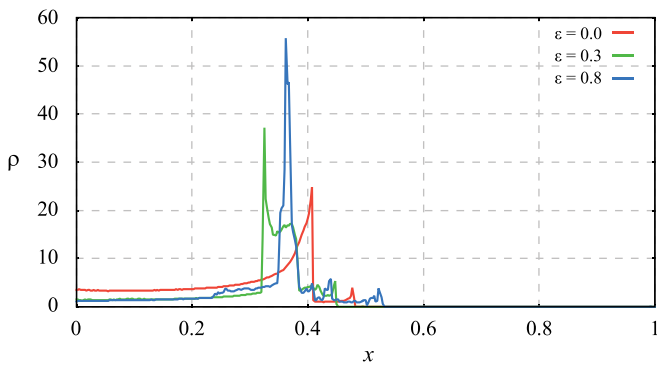


FIG. 5. Spatial distributions of the beam space-charge density $\rho(x)$ in the response low-voltage vircator demonstrating chaotic dynamics for different values of the coupling parameter ϵ . This figure illustrates the beam dynamics at the following control parameter values of coupled vircators: $\alpha_1 = 0.9$, $\Delta\phi_1 = 0.53$, $\alpha_2 = 0.9$, $\Delta\phi_2 = 0.55$.

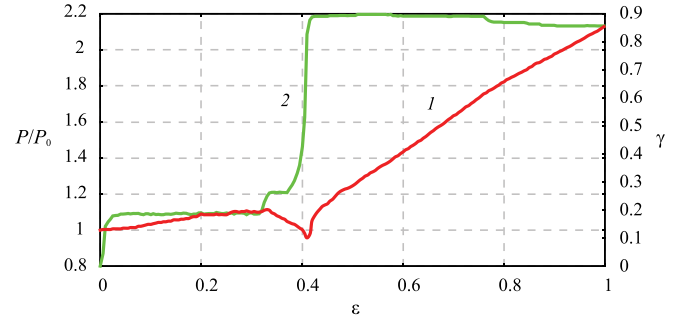


FIG. 6. Dependence of the coefficient of amplification P/P_0 (curve 1, left axis) and the measure of time scale synchronization γ (curve 2, right axis) on the coupling parameter ϵ . The drive low-voltage vircator is in the regimes of chaotic generation ($\alpha_1 = 0.9$, $\Delta\phi_1 = 0.53$), whereas the response one is in the periodical regime ($\alpha_2 = 0.9$, $\Delta\phi_2 = 0.6$).

Figure 6 shows the dependence of the coefficient of amplification P/P_0 of the response low-voltage vircator on the coupling parameter ϵ (curve 1). It is clearly seen that as in the case of unidirectionally coupled chaotic systems the sharp increase of the amplification coefficient for intermediate values of the coupling parameter is observed. Such amplification is also connected with the time scale synchronization regime onset. In Fig. 6, the dependence of synchronization measure γ on the coupling parameter ϵ is also shown (curve 2). It is easy to see that the achievement by the γ -value of the quantities close to one appears sharply, and such moment coincides with the increase of amplification coefficient. So, on the basis of made consideration one can conclude that the regimes of time scale synchronization and growth of the amplification coefficient in unidirectionally coupled low-voltage vircators are closely connected with each other. Independent of the regime realized in the response system the onset of complete time scale synchronization regime is accompanied by the sharp growth of the amplification coefficient that is connected with the considerable changes in the spectral compound of the output microwave radiation in the response generator.

IV. CONCLUSIONS

In conclusion, in the paper we have studied the relationship between the time scale synchronization regime onset and the amplification of the output microwave radiation power in two unidirectionally coupled low-voltage vircators simulated in the framework of the 1D particle-in-cell method. We have shown that the complete time scale synchronization regime onset is accompanied by the sharp increase of the amplification coefficient calculated for the response system. Such an effect does not depend on the characteristics of the response system, i.e., it is observed both in the case of the interaction between two chaotic vircators and in the case of the influence of the chaotic signal to low-voltage vircator being in the periodic regime. It worth noting that in the early work³⁸ the similar effect of the output microwave power gain under the external signal action has been confirmed experimentally. In that case, the external harmonic signal has also been introduced to the response system as the preliminary velocity modulation of the electron beam at the entrance of the drift space.

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- ¹P. C. Jagher de, F. W. Sluijter, and H. J. Hopman, "Relativistic electron beams and beam-plasma interaction," *Phys. Rep.* **167**, 177–239 (1988).
- ²R. A. York and R. C. Compton, "Experimental-observation and simulation of mode-locking phenomena in coupled-oscillator arrays," *J. Appl. Phys.* **71**, 2959–2965 (1992).
- ³M. G. Rosenblum, A. S. Pikovsky, and J. Kurths, "Phase synchronization of chaotic oscillators," *Phys. Rev. Lett.* **76**, 1804–1807 (1996).
- ⁴P. S. Landa, A. A. Zaikin, M. G. Rosenblum, and J. Kurths, "Control of noise-induced oscillations of a pendulum with a randomly vibrating suspension axis," *Phys. Rev. E* **56**, 1465–1470 (1997).
- ⁵M. S. Hur, H. J. Lee, and J. K. Lee, "Parametrization of nonlinear and chaotic oscillations in driven beam-plasma diodes," *Phys. Rev. E* **58**, 936–941 (1998).
- ⁶E. Rosa, W. B. Pardo, C. M. Ticos, J. A. Walkenstein, and M. Monti, "Phase synchronization of chaos in a plasma discharge tube," *Int. J. Bifurcation Chaos* **10**, 2551–2563 (2000).
- ⁷C. M. Ticos, E. Rosa, W. B. Pardo, J. A. Walkenstein, and M. Monti, "Experimental real-time phase synchronization of a paced chaotic plasma discharge," *Phys. Rev. Lett.* **85**, 2929 (2000).
- ⁸J. J. Lynch and R. A. York, "Synchronization of oscillators coupled through narrow-band networks," *IEEE Trans. Microwave Theory Tech.* **49**, 237–249 (2001).
- ⁹J. Buckwalter, T. H. Heath, and R. A. York, "Synchronization design of a coupled phase-locked loop," *IEEE Trans. Microwave Theory Tech.* **51**, 952–960 (2003).
- ¹⁰R. A. Filatov, A. E. Hramov, and A. A. Koronovskii, "Chaotic synchronization in coupled spatially extended beam-plasma systems," *Phys. Lett. A* **358**, 301–308 (2006).
- ¹¹T. Fukuyama, R. Kozakov, H. Testrich, and C. Wilke, "Spatiotemporal synchronization of coupled oscillators in a laboratory plasma," *Phys. Rev. Lett.* **96**, 024101 (2006).
- ¹²A. E. Hramov, A. A. Koronovskii, V. A. Maximenko, and O. I. Moskalenko, "Computation of the spectrum of spatial lyapunov exponents for the spatially extended beam-plasma systems and electron-wave devices," *Phys. Plasmas* **19**, 082302 (2012).
- ¹³W. Woo, J. Benford, D. Fittinghoff, B. Harteneck, D. Price, R. Smith, and H. Sze, "Phase locking of high-power microwave oscillator," *J. Appl. Phys.* **65**, 861 (1989).
- ¹⁴A. E. Dubinov and V. D. Selemir, "Electronic devices with virtual cathodes (review)," *J. Commun. Technol. Electron.* **47**, 575–600 (2002).
- ¹⁵A. A. Koronovskii, O. I. Moskalenko, and A. E. Hramov, "On the use of chaotic synchronization for secure communication," *Phys.-Usp.* **52**, 1213–1238 (2009).
- ¹⁶D. Su and C. Tang, "Plasma-based multistage virtual cathode radiation," *Phys. Plasmas* **18**, 123104 (2011).
- ¹⁷A. S. Shlapakovski, T. Queller, Y. Bliokh, and Y. Krasik, "Investigations of a double-gap vircator at sub-microsecond pulse durations," *IEEE Trans. Plasma Sci.* **40**, 1607–1617 (2012).
- ¹⁸W. Li, J. X. Ma, J. Li, and Z. Wei, "Experimental investigation of virtual cathode sheath in the presence of secondary electrons produced by streaming ions," *Phys. Plasmas* **19**, 113511 (2012).
- ¹⁹A. E. Hramov, A. A. Koronovskii, and I. S. Rempen, "Controlling chaos in spatially extended beam-plasma system by the continuous delayed feedback," *Chaos* **16**, 013123 (2006).
- ²⁰O. I. Moskalenko, N. S. Phrolov, A. A. Koronovskii, and A. E. Hramov, "Synchronization in the network of chaotic microwave oscillators," *Eur. Phys. J. Spec. Top.* **222**, 2571–2582 (2013).
- ²¹A. A. Koronovskii, O. I. Moskalenko, A. S. Pavlov, N. S. Frolov, and A. E. Hramov, "Generalized synchronization in the action of a chaotic signal on a periodic system," *Tech. Phys.* **59**, 629–636 (2014).
- ²²J. Ju, D. Cai, G. Du, Y. Wang, L. Liu, and J. Zhang, "Characterization of cesium iodide-coated carbon-fiber aluminum cathode for an s-band high-efficiency vircator," *IEEE Trans. Plasma Sci.* **43**, 3522–3526 (2015).
- ²³J. Benford, J. A. Swegle, and E. Schamiloglu, *High Power Microwaves*, Series in Plasma Physics, 3rd ed. (CRC Press, Taylor and Francis Group, 2016).
- ²⁴A. E. Hramov, A. A. Koronovskii, P. V. Popov, and I. S. Rempen, "Chaotic synchronization of coupled electron-wave systems with backward waves," *Chaos* **15**, 013705 (2005).
- ²⁵A. E. Hramov and A. A. Koronovskii, "An approach to chaotic synchronization," *Chaos* **14**, 603–610 (2004).
- ²⁶B. Torresani, *Continuous Wavelet Transform* (Savoire, Paris, 1995).
- ²⁷A. E. Hramov, A. A. Koronovskii, V. A. Makarov, A. N. Pavlov, and E. Sitnikova, *Wavelets in Neuroscience*, Springer Series in Synergetics (Springer, Heidelberg/New York/Dordrecht/London, 2015).
- ²⁸A. E. Hramov and A. A. Koronovskii, "Time scale synchronization of chaotic oscillators," *Physica D* **206**, 252–264 (2005).
- ²⁹A. E. Hramov, A. A. Koronovskii, and Y. Levin, "Synchronization of chaotic oscillator time scales," *JETP* **100**(4), 784–794 (2005).
- ³⁰Y. Kalinin, A. A. Koronovskii, A. E. Hramov, E. N. Egorov, and R. A. Filatov, "Experimental and theoretical investigations of stochastic oscillatory phenomena in a nonrelativistic electron beam with a virtual cathode," *Plasma Phys. Rep.* **31**, 938–952 (2005).
- ³¹R. A. Filatov, A. E. Hramov, Y. P. Bliokh, A. A. Koronovskii, and J. Felsteiner, "Influence of background gas ionization on oscillations in a virtual cathode with a retarding potential," *Phys. Plasmas* **16**, 033106 (2009).
- ³²R. A. Mahaffey, P. A. Sprangle, J. Golden, and C. A. Kapetanakis, "High-power microwaves from a non-isochronous reflecting electron system," *Phys. Rev. Lett.* **39**, 843 (1977).
- ³³A. N. Didenko, Y. Krasik, S. P. Perelugin, and G. P. Fomenko, "Generation of power microwave radiation of relativistic beam in triode system," *Tech. Phys. Lett.* **5**, 321 (1979).
- ³⁴C. K. Birdsall and A. B. Langdon, *Plasma Physics via Computer Simulation* (Taylor and Francis Group, 2005).
- ³⁵E. N. Egorov, Y. Kalinin, A. A. Koronovskii, A. E. Hramov, and M. Morozov, "Microwave generation power in a nonrelativistic electron beam with virtual cathode in a retarding electric field," *Tech. Phys. Lett.* **32**, 402–405 (2006).
- ³⁶I. J. Morey and C. K. Birdsall, "Travelling-wave-tube simulation: The IBC code," *IEEE Trans. Plasma Sci.* **18**, 482 (1990).
- ³⁷N. S. Frolov, A. A. Koronovskii, and A. E. Hramov, "Investigation of the generation characteristics in the chain of unidirectionally connected low-voltage vircators," *Bull. Russ. Acad. Sci.* **72**, 1592–1595 (2011).
- ³⁸N. S. Phrolov, A. A. Koronovskii, Y. Kalinin, S. A. Kurkin, and A. E. Hramov, "The effect of an external signal on output microwave power of a low-voltage vircator," *Phys. Lett. A* **378**, 2423–2428 (2014).
- ³⁹S. Tsimring, *Electron Beams and Microwave Vacuum Electronics* (John Wiley and Sons, Inc., Hoboken, New Jersey, 2007).