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Mathematical simulation of coherent resonance phenomenon in a network of Hodgkin-Huxley biological neurons

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ABSTRACT

In this paper we numerically simulate a two-layer network of coupled Hodgkin-Huxley neurons for modulating a processing visual perception by the human brain. We investigate the influence of the external stimulus amplitude on the dynamics of second layer neurons. We discover coherent resonance phenomenon in the system: there is an area of external stimulus amplitude when both SNR and characteristic correlation time are maximal. We also analyze the influence of internal noise amplitude on the system dynamics.

Keywords: Complex network, Hodgkin-Huxley neuron, neural network, coherence resonance

1. INTRODUCTION

Investigations of neuronal models subjected to different types of perturbations have received significant attention in the last years.[1–4] It is widely acknowledged that signal processing in neural systems takes place in a noisy environment. Hence, it is of interest to understand the statistical properties of stochastic neuronal systems. Investigation of the influence of noise on spike generation in the presence of some external forcing signals is particular important, because noise plays a significant role in the detection, transmission and encoding of such signals.

As all real systems, the neural systems are noisy. Noise can lead to increase or decrease of order in the dynamical systems under noise.[5–7] To be mentioned here are the effects of noise induced order in chaotic dynamics,[8,9] synchronization by external noise,[10,11] and stochastic resonance.[3,12–14] Also, noise has been shown to play a stabilizing role in ensembles of coupled oscillators and maps.[15,16] Especially interesting is the phenomenon of stochastic resonance, which appears when a nonlinear system is simultaneously driven by noise and a periodic signal.[17–20] At a certain noise amplitude the periodic response is maximal.

Coherence resonance is an important finding emerging in many fields of science, including complex neuronal systems.[17,21] The phenomenon of coherence resonance was first discussed in a simple autonomous system in the vicinity of the saddlenode bifurcation.[22,23] The nonuniform noise-induced limit cycle leads to a peak at a definite frequency in the power spectrum. The signal-to-noise ratio (SNR) increases first to a maximum and then decreases when the intensity of noise increases, showing the optimization of the coherent limit cycle to the noise.

In this paper, we numerically simulate a two-layer network of coupled Hodgkin-Huxley neurons[24] for modulating a processing visual perception by the human brain.[25–28] The first and the second layers of the network consist of 5 and 50 neurons and represent visual area of the thalamus and visual cortex respectively. As a model neuron, we chose the Hodgkin-Huxley neuron. We simulate visual stimulus by adding some external stimulus of constant amplitude to the neurons in the first layer connected to the neurons in the second one unidirectionally. All neurons in each layer are globally coupled to all other neurons inside the layer and each neuron has its own zero mean white Gaussian noise. We investigate the influence of the external stimulus amplitude on the dynamics of second layer neurons. We calculate power spectra of signal averaged over all neurons in the second layer and then we calculate signal-to-noise ratio and characteristic correlation time. As a result we discover

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coherent resonance phenomenon in the system: there is an area of external stimulus amplitude when both SNR and characteristic correlation time are maximal. It means that the network processes visual information better for some values of external stimulus amplitude.

2. NUMERICAL MODEL

The system under study represents the networks of $N^{ex} = 5$ and $N = 50$ Hodgkin-Huxley neurons (Fig. 1). Inside each network all elements are connected to each other, and there is a probability $p = 0.3$ of making a one-way connection between a neuron from the first network to a neuron from the second one. To all N^{ex} neurons from the first network we inject the external current I^{ex} of constant amplitude simulating the visual stimulus.

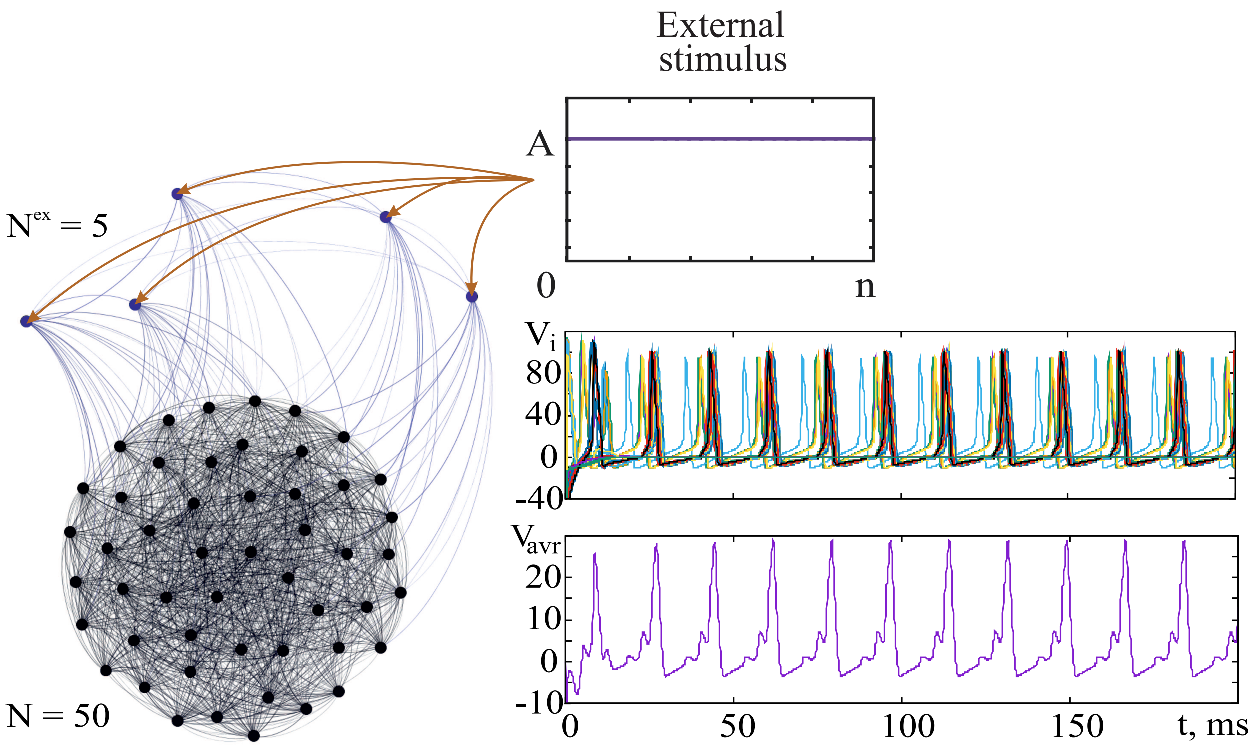

Figure 1. Research design. The external stimulus with amplitude A is applied to $N^{ex} = 5$ neurons in the first network. Each neuron in this network is connected to each neuron in the second network with a probability $p = 0.3$. From the system we take signal V_i from all neurons in the second network and signal V_{avr} averaged aver all these neurons. Each element has its own Gaussian noise.

The time evolution of the transmembrane potential of the HH neurons is given by:[24]

$$\begin{aligned} C_m \frac{dV_i}{dt} &= -g_{Na}^{max} m_i^3 h_i (V_i - V_{Na}) - g_K^{max} n_i^4 (V_i - V_K) - \\ &- g_L^{max} (V_i - V_L) + I_i^{ex} + I_i^{syn} \end{aligned} \tag{1}$$

where $C_m = 1\mu F/cm^3$ is the capacity of cell membrane, I_i^{ex} is an external bias current injected into a neuron in the network, V_i is the membrane potential of i-th neuron, $i = 1,...,N$, $g_{Na}^{max} = 120mS/cm^2$, $g_K^{max} = 36mS/cm^2$ and $g_L^{max} = 0.3mS/cm^2$ receptively denote the maximal sodium, potassium and leakage conductance when all ion channels are open. $V_{Na} = 50mV$, $V_K = -77mV$ and $V_L = -54.4mV$ are the reversal potentials for sodium, potassium and leak channels respectively. m, n and h represent the mean ratios of the open gates of the specific ion channels. n^4 and m^3h are the mean portions of the open potassium and sodium ion channels within a membrane patch. The dynamics of gating variables $(x = m, n, h)$ depending on rate functions $\alpha_x(V)$ and $\beta_x(V)$

are given:[29]

$$\frac{dx_i}{dt} = \alpha_{x_i}(V_i)(1-x_i) - \beta_{x_i}(V_i)x_i + \xi_{x_i}(t), \qquad x = m, n, h \tag{2}$$

$\xi_x(t)$ in Eq.2 is independent zero mean Gaussian white noise sources whose autocorrelation functions are given as below[30]

$$\langle \xi_{m_i}(t)\xi_{m_i}(t')\rangle = \frac{2\alpha_{m_i}\beta_{m_i}}{N_{Na}(\alpha_{m_i}+\beta_{m_i})}\delta(t-t') \tag{3}$$

$$\langle \xi_{h_i}(t)\xi_{h_i}(t')\rangle = \frac{2\alpha_{h_i}\beta_{h_i}}{N_{Na}(\alpha_{h_i}+\beta_{h_i})}\delta(t-t') \tag{4}$$

$$\langle \xi_{n_i}(t)\xi_{n_i}(t')\rangle = \frac{2\alpha_{n_i}\beta_{n_i}}{N_K(\alpha_{n_i}+\beta_{n_i})}\delta(t-t') \tag{5}$$

where N_{Na} and N_K represent the total number of sodium and potassium channels within a membrane patch, and are calculated as $N_{Na} = \rho_{Na}S$, $N_K = \rho_K S$ where $\rho_{Na} = 60\mu m^{-2}$ and $\rho_K = 18\mu m^{-2}$ are the sodium and potassium channel densities, respectively.[24] S is the membrane patch area of each neuron.

I_i^{syn} is the total synaptic current received by neuron i. We consider coupling via chemical synapses. The synaptic current takes the form[31]

$$I_i^{syn} = \sum_{j\in neigh(i)} g_c\alpha(t-t_0^j)(E_{rev}-V_i) \tag{6}$$

where the alpha function $\alpha(t)$ describes the temporal evolution of the synaptic conductance, g_c is the maximal conductance of the synaptic channel and t_0^j is the time at which presynaptic neuron j fires. We suppose $\alpha(t) = \mathrm{e}^{-t/\tau_{syn}}\Theta(t)$, there $\Theta(t)$ is the Heaviside step function and $\tau_{syn} = 3ms$.

3. RESULTS

We analyse the signal averaged over all N neurons from the second network $V_{avr} = \sum_{i=1}^{N} V_i/N$. The example of characteristic neuron dynamics one can on the Fig.1.

To investigate the dynamics os the system we analyse the coherence of a signal. For that we can calculate the signal-to-noise ratio (SNR) derived from the energy spectrum using the Fourier transform:[32]

$$E(f) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)\exp^{-i2\pi ft}\,dt \tag{7}$$

The maximum energy in the spectrum E_{max} appears at the average frequency of spiking neurons f_s. Therefore, this spectral component reflects the contribution of a regular behavior, while the noise contributes mainly to the background component E_N at the same frequency f_s.[33–35] The signal-to-noise ratio can be calculated from the power spectra as SNR $= E_{max}^2 - E_N^2$ (dB) at the dominant frequency f_s.[36]

Another way to measure of coherence of the system is the calculation of characteristic correlation time defined as[36]

$$\tau_c = \sum_{n_0}^{T} C(\tau)^2, \tag{8}$$

where t_0 is the transient time, T is the total time, $C(\tau)$ is the autocorrelation function given as

$$C(\tau) = \frac{\left\langle\, (x_{avr}(n) - \langle x_{avr}\rangle)\,(x_{avr}(n+\tau) - \langle x_{arv}\rangle)\,\right\rangle}{\left\langle\, (x_{avr}(n) - \langle x_{avr}\rangle)^2\,\right\rangle}, \tag{9}$$

where $\langle ... \rangle$ is the time average after transients. The larger the correlation time, the better the coherence.

In this work we calculate the dependencies of signal-to-noise ratio and characteristic correlation time from external stimulus amplitude (Fig.2). They both have the same dynamics: at low external stimulus amplitude

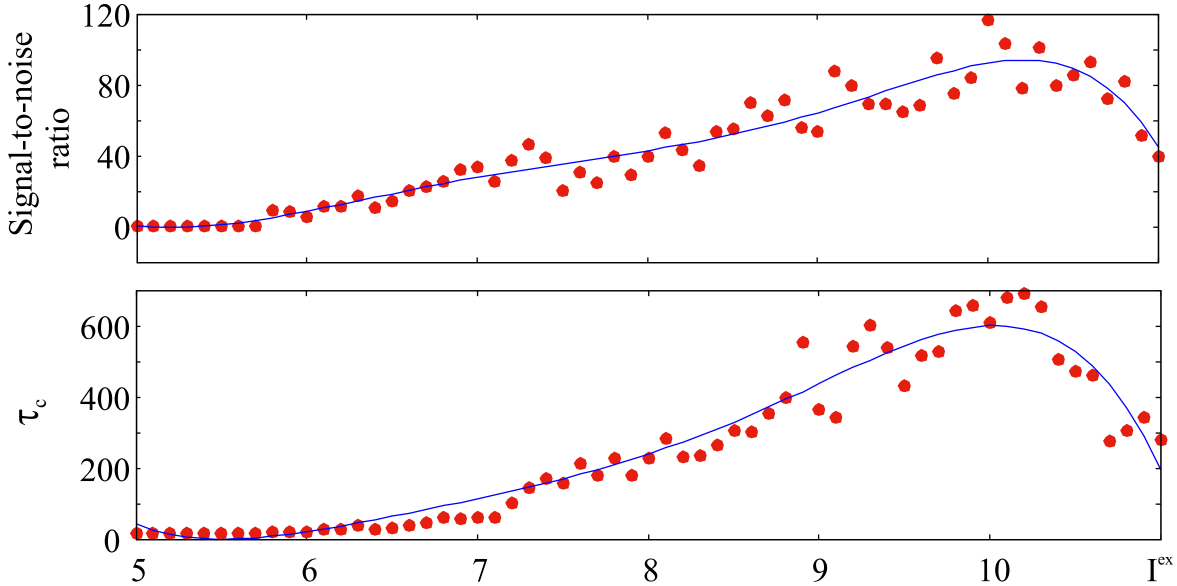

Figure 2. The dependencies of signal-to-noise ratio and characteristic correlation time from external stimulus amplitude (points) and the approximated dependencies by polynomial of order 6 (line) for $\xi = 0.1$.

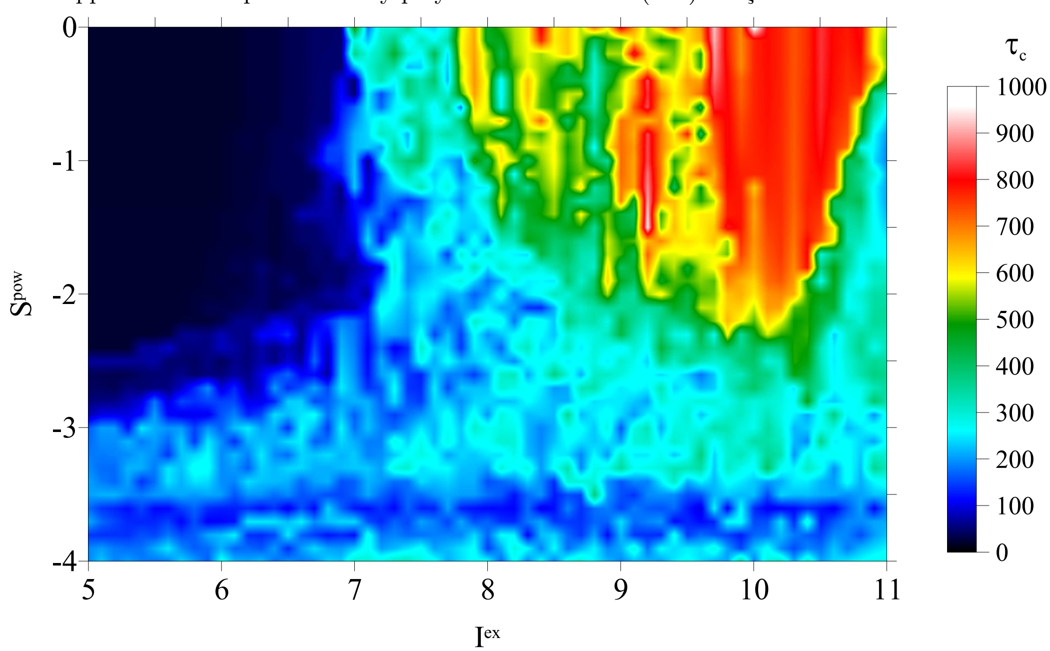

Figure 3. The dependence of characteristic correlation time τ_c from external stimulus amplitude I^{ex} and noise amplitude $\xi = 1/10^{Spow}$.

all neurons are in "silent" regime and there is no spikes generation. Increasing the stimulus amplitude leads to increasing the both signal-to-noise ratio and characteristic correlation time. After $I^{ex} = 10.0$ they start to decrease, so the dependencies have resonance at this amplitude value.

In the next step is the analysing of two-dimensional dependence of characteristic correlation time from amplitudes of external stimulus and internal noise. On the Fig.3 one can see the area of maximal coherence. Based on this figure we can make some conclusions: (1) We can see the black area of "silent" regime, when neurons cannot generate spikes due to the luck of external current, but with the presence of relatively big noise ($S_{pow} < 2.8$) they no need this current for spike generation; (2) For really big noise ($S_{pow} < 3.5$) there is no the dependence from external stimulus; (3) The bigger noise the narrower the maximal coherence area along external current amplitude.

4. CONCLUSION

We have numerically simulated the dynamics of the brain under visual perseption using 2 layer Hodgkin-Huxley neuron network. We have calculated characteristic correlation time and signal-to-noise ratio from power spectra of signal averaging over all neurons in the second layer to measure the coherence of the system. Analyzing the influence of amplitudes of internal noise and external stimulus on system dynamics we found that the coherence is maximal on the certain value of stimulus intensity. It means that the network processes visual information better for some values of external stimulus amplitude. We also analyze the influence of internal noise amplitude on the system dynamics. Calculating two-dimensional dependence of characteristic correlation time from external stimulus and internal noise amplitudes we find an area of maximal coherence of the network. It was found that for big values of noise the system dynamics doesn't depend from external stimulus amplitude.

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