ISSN 1062-8738, Bulletin of the Russian Academy of Sciences: Physics, 2017, Vol. 81, No. 1, pp. 110–113. © Allerton Press, Inc., 2017.
Original Russian Text © V.A. Maksimenko, M.V. Goremyko, V.V. Makarov, A.E. Hramov, D. Ghosh, B.K. Bera, S.K. Dana, 2017, published in Izvestiya Rossiiskoi Akademii Nauk, Seriya Fizicheskaya, 2017, Vol. 81, No. 1, pp. 121–124.

Excitation and Suppression of Chimeric States in the Multilayer Network of Oscillators with Nonlocal Coupling

V. A. Maksimenko[a, *], M. V. Goremyko[a, b], V. V. Makarov[a], A. E. Hramov[a], D. Ghosh[c], B. K. Bera[c], and S. K. Dana[d, e]

[a]*Saratov State Technical University, Saratov, 410028 Russia*
[b]*Government of Saratov Oblast, Saratov, 410042 Russia*
[c]*Indian Statistical Institute, Kolkata, 700108 India*
[d]*Indian Institute of Chemical Biology, Kolkata, 700032 India*
[e]*Center for Complex System Research, Kolkata, 700108 India*
**e-mail: maximenkovl@gmail.com*

Abstract—The interaction between ensembles of coupled nonlinear oscillators using the model of multilayer network is studied. It is found that the interaction between an ensemble with a chimera and an ensemble with both coherent and incoherent states of oscillators can lead to both suppression of the chimera and a transition to a coherent or incoherent state, or to the excitation of the chimeric state from the coherent or incoherent state, respectively.

DOI: 10.3103/S1062873817010166

INTRODUCTION

Research on the collective behavior of dynamic systems is now largely associated with the study of chimeric states [1–5]. Such states, characterized by coexistence in a network of subgroup of coherent and incoherent dynamic elements, were first described in 2002 in [1]. Chimeric states were then obtained in networks of nonlocal coupled nonlinear elements described by complex Ginzburg–Landau equations [6], and in a network of Kuramoto–Sakaguchi phase oscillators [7].

It has been shown that along with nonlocal coupled oscillators, symmetry breakdowns and transitions from a completely coherent state to the chimeric state can occur in networks of oscillators with global coupling [8, 9] or with local coupling [10–12]. In addition, it has been shown that chimeric states can emerge in ensembles of dynamic elements of different natures, including systems with periodic and chaotic dynamics [13], neural systems [14], discrete displays [15], and logical networks [16]. It should be noted that along with model systems, the occurrence of the chimeric state has been confirmed experimentally in chemical [17], electronic [18], electrochemical [19], optoelectronic [20], and mechanical [21] systems.

Despite the great interest in studying chimeric states, and thus the great many works denoted to this problem, consideration is generally limited to analyzing the behavior of individual networks and does consider the effects that can emerge as a result of their interaction. Such effects include the stability of the chimeric state in a network when it interacts with a network of coherent or incoherent elements.

To consider this problem, the chimeric states of interacting ensembles of nonlocal coupled nonlinear elements are investigated in this work using a model of a multilayer network. This model is often used both in analyzing experimental data and in the mathematical modeling of network dynamics caused by a multilayer model corresponding to a large number of real systems [22, 23].

One feature of a multilayer model is the presence of two types of couples on each element. The first type characterizes the interaction between the element and other network nodes located within a single layer. The second type determines the coupling of this element and elements associated with other layers of a network. Depending on the problem, the configurations of couplings between the elements of a multilayer network can be different. In this work, we consider the configuration described in [24].

MATHEMATICAL MODEL

According to [24], the investigated network consisting of $N \times M$ elements can be presented as a set of M layers (with N elements on each layer). The couplings between the elements inside the layer are distributed nonlocally [25] (each element is coupled with $2R$ adjacent elements); interaction between layers occurs via local couplings between two adjacent elements. A schematically of this model is shown in Fig. 1. Values φ_i^j correspond to dynamic variables characterizing the states of network node (in this case, the instantaneous value of phases of oscillators), and indices i and j correspond to the number of the element inside the layer and to the layer number, respectively. The couplings of element φ_i^j with the adjacent elements inside the layer are marked by the solid lines. Interlayer coupling, which occurs via the interaction between the element of first layer φ_i^1 and the adjacent element of second layer φ_i^2, is marked by the dashed line.

Kuramoto–Sakaguchi phase oscillator (1), which is often used as the basic model for the numerical and analytical study of chimeric states [26], is used in this work to simulate the dynamics of the network node [7]:

$$\frac{d\varphi_i^j}{dt} = \omega_i^j - \frac{\lambda_1}{2R+1}\sum_{k=i-2R}^{i+2R}\sin(\varphi_i^j - \varphi_k^j + \alpha) + \frac{\lambda_2}{M}\sum_{l\neq j}\sin(\varphi_i^j - \varphi_i^l). \quad (1)$$

In the formula (1), ω_i^j corresponds to the natural frequency of the oscillator; λ_1 is the coupling coefficient between the oscillators inside the layer; λ_2 is the coefficient of interlayer coupling; R is the coupling radius; M is the number of layers; and α is the constant phase shift. The network of identical oscillators $\omega_i^j = 1, \forall i, j$ is considered in this work.

Value S_I (see Eq. 2) characterizing the strength of incoherence is calculated in this work for quantitative diagnosis of the chimeric state. To calculate this quantity, the investigated ensemble of oscillators is divided into m groups of elements using n elements in the group. For this ensemble, S_I is defined as

$$S_\mathrm{I} = 1 - \frac{\sum_{r=1}^{m}\Theta(\delta - \sigma_r)}{m}, \quad (2)$$

where $\Theta(\cdot)$ is Heaviside function; σ describes standard deviation (3) characterizing the oscillators in the group with index r; and $\delta = 0.035$ is the threshold value. Quantity σ_r is calculated for each group using the relation

$$\sigma_r = \left\langle \sqrt{\frac{1}{n}\sum_{s=n(r-1)+1}^{rn}[\varphi_s - \Phi]^2} \right\rangle, \quad (3)$$

where $\langle\cdot\rangle$ denotes the averaging by time interval, Φ is the ensemble average phase.

Depending on the values of S_I, the network state can be interpreted as completely coherent ($S_\mathrm{I} = 0$), completely incoherent ($S_\mathrm{I} = 1$), or chimeric ($0 < S_\mathrm{I} < 1$), corresponding to the coexistence of coherent and incoherent clusters.

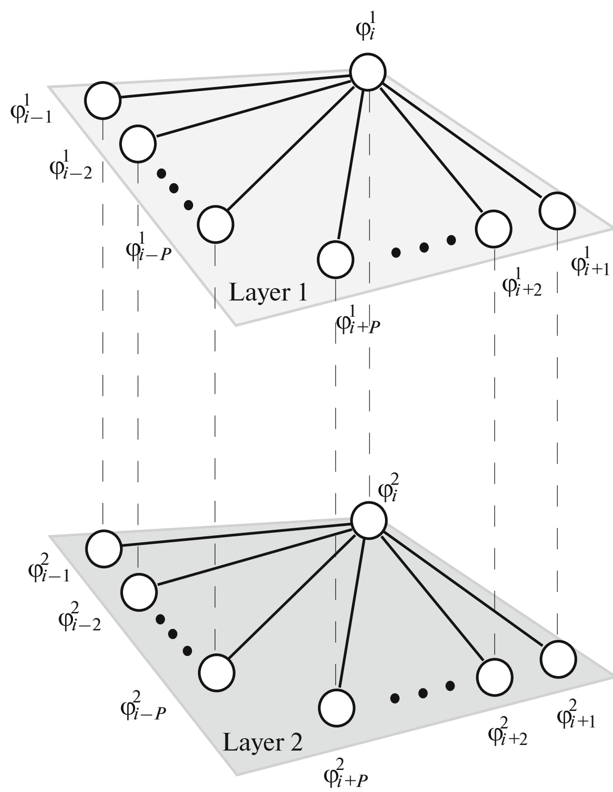

Fig. 1. Schematic image of a two-layer network of oscillators with nonlocal coupling inside the layers (nonlocal coupling was demonstrated using the example of the ith node). The dashed-and-dotted lines correspond to interlayer couplings.

RESULTS AND DISCUSSION

To study the excitation and suppression of the chimeric state, we investigated the behavior of a multilayer network numerically. The values of the parameters characterizing the nodes of network and its topology were chosen to be equal for each layer.

Quantity α defining the phase relation between interacting oscillators of this layer was considered the control parameter characterizing a network layer. Preliminary studies of the dynamics of a network of Kuramoto–Sakaguchi phase oscillators showed that a monotonous change in parameter α allows us to observe the transition from the coherent state ($\alpha < 1.45$) to the chimeric state ($\alpha \in [1.45–1.57]$), and then to the incoherent state ($\alpha > 1.57$).

In light of the relations for analyzing the interaction between chimeric and coherent states, the values of parameter α were set at $\alpha_1 = 1.45$, and $\alpha_2 = 1.2$. For interaction between chimeric and incoherent states, these values were set at $\alpha_1 = 1.45$, and $\alpha_2 = 1.7$.

The instantaneous distributions of the phases of oscillators for the specified values of parameters α_1 and α_2 when there is no interlayer coupling is shown in Fig. 2a. The left column corresponds to when the chi-

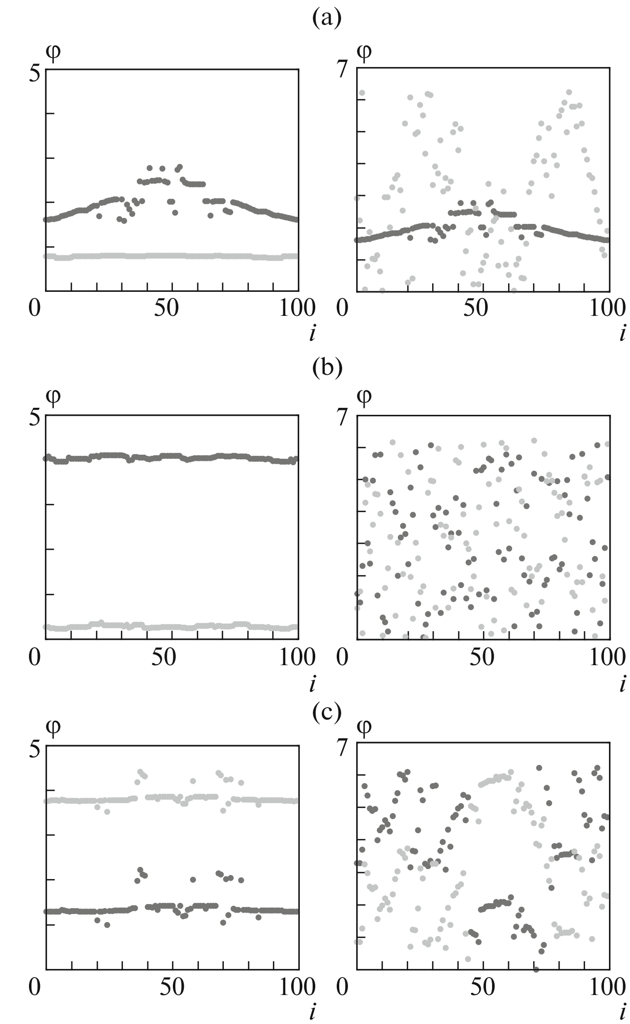

Fig. 2. Instantaneous distributions of the phases of oscillators on the layers for when there is no interlayer interaction, $\lambda_2 = 0$; when there is weak interaction, $\lambda_2 = 0.2$; and when there is strong interaction, $\lambda_2 = 0.5$. The left column corresponds to the chimeric state occurs on the first layer and the coherent state occurson the second layer. The right column corresponds to the chimeric and incoherent states, respectively.

meric state occurs on the first layer and the coherent state occurs on the second layer. The right column corresponds to the chimeric and incoherent states on the layers, respectively. The initial values of the phases of oscillators are in both cases distributed according to the laws

$$\varphi_i^1(0) = \begin{cases} \pi\left(\frac{4i}{N} - 1\right), \ i \in [0, \frac{N}{2}], \\ \pi\left(3 - \frac{4i}{N}\right), \ i \in [\frac{N}{2} + 1, N]. \end{cases}$$
$$\varphi_i^2(0) = \begin{cases} \pi\left(1 - \frac{4i}{N}\right), \ i \in [0, \frac{N}{2}], \\ \pi\left(\frac{4i}{N} - 3\right), \ i \in [\frac{N}{2} + 1, N]. \end{cases} \quad (4)$$

In both cases, noise is added to the initial distribution.

Figures 2b, 2c shows the instantaneous distributions of the phases of oscillators of the first and the second layers for weak ($\lambda_2 = 0.2$) and strong ($\lambda_2 = 0.5$) interlayer coupling, respectively. It can be seen that with weak interlayer coupling, the chimera in the first layer is suppressed and there is a transition to the coherent or incoherent state, depending on the initial state of the oscillators of the second layer (Fig. 2b). When the value of parameter λ_2 is increased, we observe interesting features in the dynamics of this system. It can be seen from Fig. 2c that the chimeric state on the first layer is stable. The excitation of the chimeric state in the ensemble of oscillators of the second layer with both initial coherent and incoherent dynamics is also observed.

It should be noted that as a result of strong interlayer interaction, the network transitions to the stable state characterized by synchronous behavior of its layers (identical distributions of the phases of oscillators are established on both layers). It is interesting that the multilayered structure increases the parameter space in which the chimeric state can occur. It is highly likely that this pattern remains true for a wide class of networks with multilayer structure in which chimeric states can occur.

CONCLUSIONS

The dynamics of a two-layer network of on nonlocal coupled Kuramoto–Sakaguchi phase oscillators was investigated. Cases where the ensemble of oscillators belonging to a single layer are in the chimeric state, while the oscillators of another layer are in coherent or incoherent states awerere considered. It was shown that at low values of interlayer coupling force, the chimeric state in the first layer is suppressed and all oscillators of the network transition to a coherent or incoherent state (depending on the state in the second layer of the network). With high values of interlayer interaction, we observe excitation of the chimeric state in the ensemble of oscillators of the second layer, whether it is in a completely coherent or incoherent state.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, project nos. 16-32-00334 and 15-52-45003; and by the RF Ministry of Education and Science, task nos. 3.23.2014K and 931. V.A. Maksimenko would like to thank the Dmitry Zimin Dynasty Foundation for their personal support.

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Translated by M. Kromin