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Diagnostics of the Regime of Hyperchaotic Dynamics from Sequences of Threshold-Crossing Time Intervals

A. N. Pavlov*, O. N. Pavlova, and J. H. Mohammad

Saratov State University, Saratov, 410012 Russia
**e-mail: pavlov.alexeyn@gmail.com*

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Abstract—We consider a method of hyperchaotic regime diagnostics in a system with self-sustained oscillations by monitoring point processes representing sequences of time intervals between the moments at which the system response signal crosses a threshold level. The possibility of determining two positive Lyapunov exponents from a single point process of short duration is demonstrated.

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The diagnostics of hyperchaos in dynamical systems is important in solving some technical problems related, e.g., to the development of communication systems with a high level of protection against unauthorized access [1–3], in which the use of hyperchaotic regimes of self-sustained oscillations as carrier signals provides effective protection of transmitted data. The task of hyperchaos diagnostics can be solved rather simply, provided that a mathematical model of the system with self-sustained oscillations is known and the spectrum of Lyapunov exponents is calculated by one of the well-known methods [4, 5]. The task becomes more complicated if the mathematical model is unknown and information on the system is restricted.

Previously, we have studied [6–8] the problem of determining Lyapunov exponents from point processes representing sequences of time intervals between moments at which the response signal of a system with self-sustained oscillations crosses a threshold level, by analogy with the interspike intervals of neurons and neuronal networks [9]. It has been established that the point process alone is insufficient for hyperchaos diagnostics and it is necessary to know at least two sequences of threshold-crossing interspike intervals for two variables of the system [8].

In the present Letter, we demonstrate that, provided that an appropriate signal is delivered to the input of a threshold device, the diagnostics of hyperchaos can be based upon a single sequence threshold-crossing interspike intervals and it is possible to use a sample sequence of relatively short duration.

The general idea of the proposed method is as follows. Consider a sequence of time moments T_i that correspond to the crossing of fixed threshold level Θ by signal $x(t)$ in one direction (e.g., downside up). Let us pass to values $I_i = T_{i+1} - T_i$ characterizing changes in the period of oscillations, and then to values of instantaneous frequency $\omega_i(T_i) = 2\pi/I_i$ averaged over the corresponding temporal interval. Finally, let us interpolate discrete readings of $\omega_i(T_i)$ by a smooth function (e.g., by a cubic spline) to pass to signal $S(t)$ set at a constant temporal step. This signal is used for reconstruction of the attractor by means of the time-delay method and for the subsequent calculation of two higher Lyapunov exponents [10]. The theoretical presuppositions of this method have been presented and tested on some model systems [6, 7]. It was established that, if the threshold level is set so that it is not crossed by signal $x(t)$ during several oscillations, a correct estimation of the maximum Lyapunov exponent can be obtained if the average interval I_i does not exceed the time of predictability of the dynamic regime under consideration [7].

In order to characterize hyperchaotic oscillations by point process I_i, let us consider a model of two coupled Rössler oscillators [11] described by the following set of equations:

$$\begin{aligned}
\frac{dx_1}{dt} &= -\omega_1 y_1 - z_1 + \gamma(x_2 - x_1),\\
\frac{dy_1}{dt} &= \omega_1 x_1 + a y_1,\\
\frac{dz_1}{dt} &= b + z_1(x_1 - \mu),\\
\frac{dx_2}{dt} &= -\omega_2 y_2 - z_2 + \gamma(x_1 - x_2),\\
\frac{dy_2}{dt} &= \omega_2 x_2 + a y_2,\\
\frac{dz_2}{dt} &= b + z_2(x_2 - \mu),
\end{aligned} \tag{1}$$

where a, b, and μ are control parameters that determine the dynamics of each system and γ is the cou-

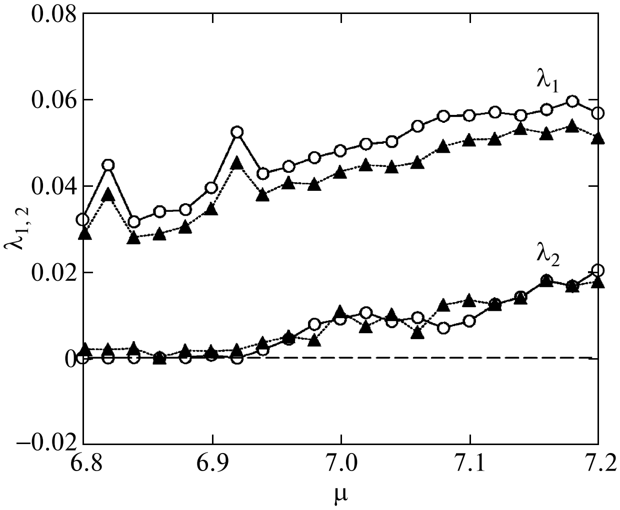

Fig. 1. Plots of two higher Lyapunov exponents $\lambda_{1,2}$ vs. parameter μ for model of coupled Rössler oscillators (1). Circles represent the results of calculations using the model equations, while triangles show the values calculated using the point process.

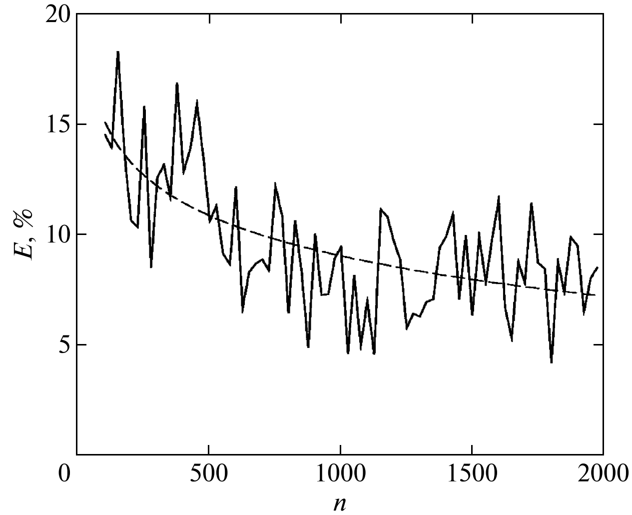

Fig. 2. Plot of the relative error of calculation of the maximum Lyapunov exponent vs. number of readings of the point process under consideration.

pling intensity parameter. The two systems are nonidentical, which is determined by the choice of oscillation frequencies $\omega_1 = \omega_0 + \Delta$ and $\omega_2 = \omega_0 - \Delta$ (differing by 2Δ). The present investigation was performed for the following values of control parameters: $a = 0.15$, $b = 0.2$, $\gamma = 0.02$, $\omega_0 = 1.0$, and $\Delta = 0.0092$, while μ was varied within [6.8, 7.2].

First, the two higher Lyapunov exponents $\lambda_{1,2}$ characterizing the dynamics of system (1) were calculated using the standard algorithm [10]. According to this, $\lambda_{1,2}$ values are determined by analysis of the temporal evolution of the axes of an infinitesimal sphere of initial conditions, the choice of a partial solution of model system (1) that is associated with the center of this sphere, and the Gram–Schmidt orthonormalization procedure. At this stage, no limitations were imposed on the computation time and the calculations were terminated upon attaining a preset accuracy of $\lambda_{1,2}$ estimations (when $\lambda_{1,2}$ variations changed by no more than 10^{-6} with increasing computational time). The corresponding values were treated as "exact" (Fig. 1, circles) and compared to the determinations of Lyapunov exponents obtained using I_i sequences.

For the diagnostics of a hyperchaotic dynamics of system (1) by the point process, it is necessary to select an appropriate input signal the crossing by which of a given threshold level determines T_i time moments. This signal must reflect the joint dynamics of both subsystems of model (1), since otherwise a correct diagnostics of the oscillatory regime based on a single sequence of interspike intervals I_i cannot be performed. Indeed, it has been shown [8] that, by setting the input signal as $x_1(t)$ or $y_1(t)$, it is only possible to calculate the maximum Lyapunov exponent so that the hyperchaotic dynamics of system (1) is wrongly classified as chaotic.

Let us consider an input signal in the form of a sum of the dynamic variables of two subsystems, e.g., $u(t) = x_2(t) + y_1(t)$, and record the moments of time when signal $u(t)$ crosses the level of $\Theta = 1$. The results of calculation of the two higher Lyapunov exponents by method [6] applied to the obtained point process in the form of sequences containing 1000 I_i intervals are presented in Fig. 1 by triangles. Note the good agreement of the obtained results with theoretically predicted values, which is indicative of the efficiency of the proposed approach. For the comparison, it can be recalled that alternative methods of hyperchaos diagnostics [12, 13] employed sample sets containing on the order of 5000 points and only diagnosed the transition from chaos to hyperchaos without quantitative description of the dynamics corresponding to the hyperchaotic regime. The approach used in the present work provides the obtaining of more informative characteristics of complex dynamics accompanying the appearance of hyperchaotic oscillations.

Moreover, the method of simultaneous determination of the two higher Lyapunov exponents allows us to use a reduced amount of data. Figure 2 shows the dependence of the relative error of calculation of the maximum Lyapunov exponent λ_1 on the number of data points used in the calculations, which is approximated by a logarithmic curve. According to these data, an acceptable accuracy of calculations (on the order of 12%) can be achieved for a sample set of about 500 I_i intervals. If a preset accuracy of estimation of the Lyapunov exponents is not required and it is only necessary to identify the transition from chaotic to hyperchaotic oscillations or vice versa, the sample set volume can be reduced to about 300 I_i intervals.

Thus, it has been demonstrated that the diagnostics of hyperchaotic dynamics of an oscillatory system can be based on a single sequence of time intervals between the moments of time at which the response signal crosses a threshold level. The diagnostics employs a relative short sequence that allows, e.g., separating the regions of chaotic and hyperchaotic dynamics for systems with characteristics varying in time.

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