Macroscopic dynamics prediction by reservoir computing

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***Abstract*—Nowadays, the problem of forecasting complex signals is significant and has many applications in real life. One of such applications is the prediction of neurophysiological signals, like EEG. Such signals are macroscopic signals of a group of neurons, and the connections between them adapt in time. Here, we investigate the possibility of forecasting the dynamics of the modulated adaptive network, which topology changes in time, using Reservoir Computing (RC). We show that the dynamics of the signal is chaotic, and RC cannot predict it, but reconstruction of the phase space by adding the delays improves the quality of the signal's prediction.**

***Index Terms*—Reservoir computing, prediction, complex network, adaptation, Kuramoto phase oscillator**

I. Introduction

Nowadays, investigation of nonlinear dynamics of complex systems such as neural networks [1]–[3], brain [4]–[11], functional networks [12]–[14], electron flows [15]–[20], etc. Complex systems are characterized by multiple, interacting spatiotemporal scales that challenge classical numerical methods for prediction and control. In real life, we face the challenges of predicting the dynamics of different natures like weather, climate [21], [22], medicine [23], [24], neuroscience [25]–[27], ocean and turbulent flows [28], [29], traffic flows [30]–[33], etc. One of the interesting tasks here is forecasting neurophysiological signals.

Forecasting neurophysiological signals is an important task due to allows diagnosing and reacting in time to a negative phenomenon, for example, epilepsy seizures. Recurrent Neural Networks (RNNs) offer a potential method for addressing these challenges. The most promising type of RNN for solving this task is Reservoir Computing (RC). The neurophysiological signal is a macroscopic signal from adaptive neural network.

An important task is to predict the macroscopic dynamics of complex network systems often encountered in real life. The difficulty here lies in the fact that signals from many separate objects through a complex communication structure merge into a single signal from the entire system, which significantly reduces the dimension of the signal, making it impossible to restore the sources of such a signal.

So, we address the question of using Reservoir Computing to forecast the dynamics of the modulated adaptive network. We show, that the dynamics of the signal is chaotic, and RC cannot predict it, but reconstruction of the phase space by adding the delays significantly improves the prediction's quality.

II. Methods

As a model, we use a network of 100 Kuramoto phase oscillators with an adaptation of couplings. The model and the adaptation mechanism are described in [34]. Each oscillator is described by the following equation:

$$\dot{\phi}_i(t) = \omega_i + \sum_{j \neq i} w_{ij}(t) \sin(\phi_j - \phi_i), \qquad (1)$$

where $i = 1, ..., N$, ω_i is a natural frequency, w_{ij} is the connection weight between elements i and j and it is allowed to evolve in time according to the rule from [34].

This model describes the adaptive network of phase oscillators with the competition between homophily and homeostasis.

Initially, all the weights w_{ij} and phases ϕ_i are random. At time the coupling between the oscillators are changing along with the topology. We analyze the signal averaged over all $N = 100$ phase oscillators:

$$X_{\text{avr}}(t) = \frac{1}{N} \sum_{i=1}^{N} \sin[\phi_i(t)]. \qquad (2)$$

To solve the differential equations, we use the Runge Kutta 4th order method with time step $\Delta t = 0.1$ s for $T = 7000$ s.

The schematic representation of Recurrent neural network with Reservoir is shown in Fig. 1. The network has the input, hidden (reservoir) and output layers. Every reservoir node has

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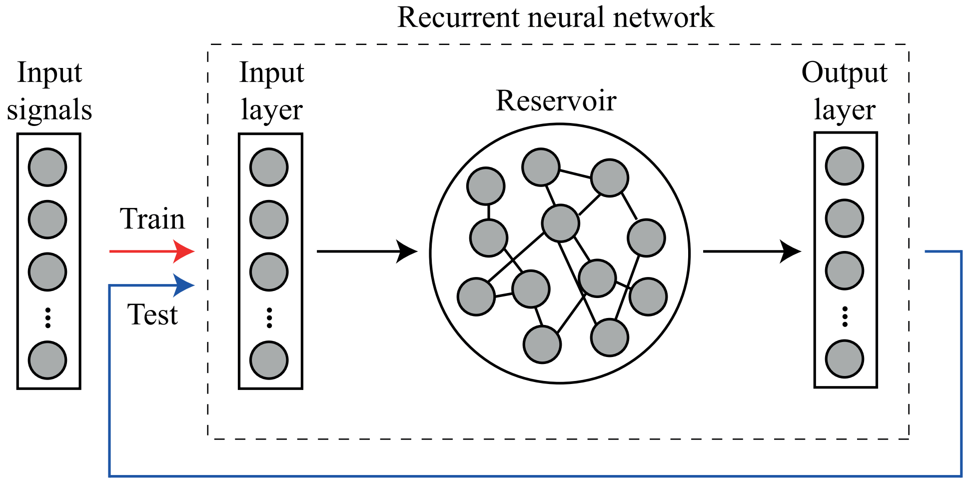

Fig. 1. The schematic representation of Recurrent neural network with Reservoir.

inputs drawn from other nodes in the reservoir or the input to the RC, and every input has an associated weight. Each reservoir node also has an output. The output of each reservoir node is fed into the output layer of the RC, which performs a linear operation of the node values to produce the output of the RC as a whole.

During training process we apply signal to the input of RC and receive output signal. The goal of RC is to approximate the desired outputs appropriate to the inputs. After training is complete, we start the testing process when RC try to predict the signal dynamics by itself. We apply only first point of the signal to reservoir's input, after that the output is fed into the input, and the reservoir system is run autonomously.

We use the reservoir with $D = 1000$ nodes and investigate RCs with a different number of input nodes. The number of output nodes in each case is the same as the number of input nodes.

III. Results

We analyze the capability of Reservoir computing to predict the dynamics of the macroscopic signal received from the network of 100 Kuramoto phase oscillators with adaptation.

Applying only the original macroscopic signal makes RC impossible predict it. To improve the prediction quality, we reconstruct the phase space of the investigated signal by adding the delayed signals to the original one. Such approach significantly improves the quality of the signal's prediction.

We investigate how a number of delayed signals influence the quality of prediction. We found that the maximal correlation between the original and the predicted signal is achieved for 2 delays, and further increase of the delays does not change it. The same effect was observed for the normalized root mean square error. Wherein the longest time during which correlation between the original and the predicted signal is more than 0.95 is achieved for 2 delays. This result correlates with the estimated embedding dimension of the original signal is 3.

We analyze the parameter's space $(D, R, \sigma_{\text{in}})$ of RC and found that increasing the number of the delays from 2 to 8 leads to decreases the number of optimal points in the parameter's space with high correlation. We suppose that it is connected to the original signal phase space dimension and the reservoir's capacity. Reconstruction of the phase space allows to predict the signal correctly but a further increase of the number of signals decreases the effectiveness of the reservoir's forecast, which manifests itself in reducing the number of optimal parameters.

IV. Conclusions

We analyzed the capability of Reservoir computing to predict the dynamics of the macroscopic signal. As a model, we used the network of Kuramoto phase oscillators with the adaptation of couplings. We have shown that the dynamics of the signal is chaotic, and RC cannot predict it. To improve the prediction quality, we reconstructed the phase space of the investigated signal by adding the delayed signals to the original one. Such approach improves the quality of the signal's prediction.

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