Using reservoir computing for dynamics forecast of noise-perturbed FitzHugh-Nagumo system

Nikita Kulagin
Baltic Center for Neurotechnology and Artificial Intelligence Immanuel Kant Baltic Federal University
Kaliningrad, Russia
NDmKulagin@stud.kantiana.ru

Andrey Andreev
Baltic Center for Neurotechnology and Artificial Intelligence Immanuel Kant Baltic Federal University
Kaliningrad, Russia
andvreevandrei1993@gmail.com

Alexander Hramov
Baltic Center for Neurotechnology and Artificial Intelligence Immanuel Kant Baltic Federal University
Kaliningrad, Russia
hramovae@gmail.com

***Abstract*—We investigate the capability of reservoir computing to predict the dynamics of excitable FitzHugh-Nagumo model, exposed to Gaussian white noise, and reproduce the phenomenon of coherence resonance in the reservoir. We train the neural network on the dynamics of the system with three noise amplitudes and then test in on different noises. We show that the reservoir computing can exhibit coherence resonance under external stimulus.**

***Index Terms*—Reservoir computing, prediction, FitzHugh-Nagumo model, noise, coherence resonance**

I. Introduction

Nowadays, artificial neural networks are widely used to study mathematical models of biological neurons [1]–[3], know for their nonlinear nature. In particular, they were used to predict the occurrence of spikes in the Hodgkin-Huxley model [4], as well as spike features [5] and the dynamic behavior of the neuron model [6]. Also artificial neural networks were used to accurately predict parameters of FitzHugh-Nagumo model [7].

A convenient tool for learning nonlinear dynamical systems is a reservoir computer, a type of artificial recurrent neural network, that has been successfully used to predict chaotic time series [8], [9]. There was a number of works applying reservoir computers for studying mathematical neuron models, including chaotic and periodic time series prediction of Hindmarsh-Rose model [10]–[12] and prediction of extreme events in a system of coupled FitzHugh-Nagumo neurons [13], [14].

A phenomenon called "coherence resonance" is observed in excitable dynamical systems driven by external noise, corresponding to existence of noise intensity which causes highest degree of coherence of noise-induced oscillations [15]. Coherence resonance is observed in mathematical models of biological neurons, inlcuding FitzHugh-Nagumo [16], Hodgkin-Huxley [17] and Hindmarsh-Rose [18] models, and cortical networks [19]–[21].

In this study we investigate the possibility of exhibition of coherence resonance in a reservoir computer, as reservoir computer's hidden layer itself is an excitable system, by training a type of reservoir computer called "echo state network" on FitzHugh-Nagumo model time series with different levels of white noise intensity. We show that predictions of a manually optimized reservoir computer display varying degrees of coherence under varying noise amplitudes, sharing these features with the original FitzHugh-Nagumo time series.

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II. Methods

A. FitzHugh-Nagumo model

As a model neuron we use the FitzHugh-Nagumo model describes by the following system of differential equations:

$$\varepsilon\dot{x} = x - \frac{x^3}{3} - y, \quad \dot{y} = x + a + D\xi[t], \tag{1}$$

with $\varepsilon = 0.001$ and $a = 1.05$ are the constant parameters, $\xi[t]$ is a zero-mean Gaussian noise with standard deviation equal to 100, and D is a parameter that controls the intensity of the noise.

For solving the differential equations we use the Euler method with time step $\Delta t_0 = 0.0001$. The time step was further increased up to $\Delta t = 0.001$ by saving only every tenth data point, the other points were discarded.

For RC training we use the time series of duration $T = 240$. This time series was divided into three parts:

- $T_1 = 0$–80, with $D = 0.008$;
- $T_2 = 80$–160, with $D = 0.064$;
- $T_3 = 160$–240, with $D = 0.512$.

Training data also contained corresponding Gaussian noise $D\xi[t]$, however, the noise values weren't predicted by the RC. Thus, the reservoir computer was trained to distinguish signals with three different levels of noise amplitude.

All data for training and testing were normalized using z-score normalization.

B. Reservoir computing architecture

We use a type of reservoir computer known as an echo state network (ESN). It uses a hidden layer (called a reservoir) with sparse recurrent connections between neurons.

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The reservoir dynamics are described by the following formula:

$$h[t+1] = \tanh(W_h \times h[t] + W_{in} \times u[t]) \tag{2}$$

, where $h[t]$ is the reservoir state vector containing all activations of reservoir neurons at time t, W_h is an adjacency matrix containing connection weights values between neurons of the hidden layer, W_{in} is a matrix carrying input neurons' connection weights, $u[t]$ is a three-dimensional input signal.

Output data at a given time step is evaluated as follows:

$$y[t+1] = W_{out}\tilde{h}[t] \tag{3}$$

, where W_{out} is an output weights matrix, $\tilde{h}[t]$ is the reservoir state vector with half of its values squared.

W_{in} matrix values are randomly chosen from the interval $[-1, 1]$ with uniform distribution. W_{in} is formed in such a way that each input dimension is connected to $\frac{N}{3}$ reservoir nodes, where N is the total number of individual neurons in reservoir fixed to a constant value of 1002.

During W_h initialization, its values are chosen from the uniformly distributed interval $[0, 1]$. The sparsity of W_h is calculated as $\frac{d}{N}$, where d is a hyperparameter. Spectral radius ρ_0 of W_h is then rescaled to a set value of ρ.

Connections between reservoir neurons and output neurons, stored in W_{out}, are the only trainable connections of the RC. Tikhonov regularization is used to find the values of W_{out}.

III. Results

We manually explored the hyperparameters' space by trial and error, changing values of ρ, d and regularization parameter α. The RC's prediction performance was determined by its dynamics similarity with the original three testing signals. Those signals included FHN time series with $T = 200$ and $D = 0.008, 0.064, 0.512$ respectively. The prediction results are shown on Fig. 1, Fig. 2 and Fig. 3.

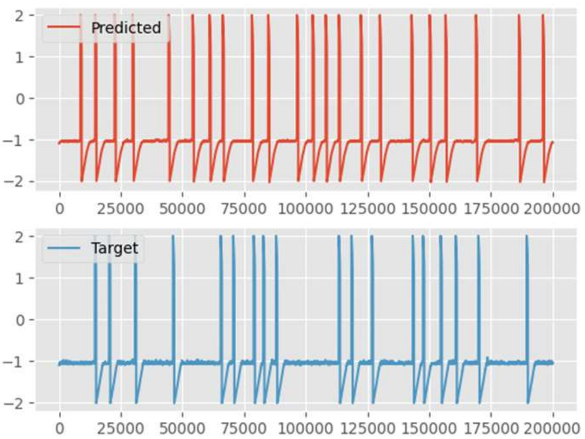

Fig. 1. Results of FitzHugh-Nagumo time series prediction with $D = 0.008$.

The results of prediction of the signal with $D = 0.008$ are shown on Figure 1. The RC has partially succeeded in reproducing the original signal's dynamics. The predicted

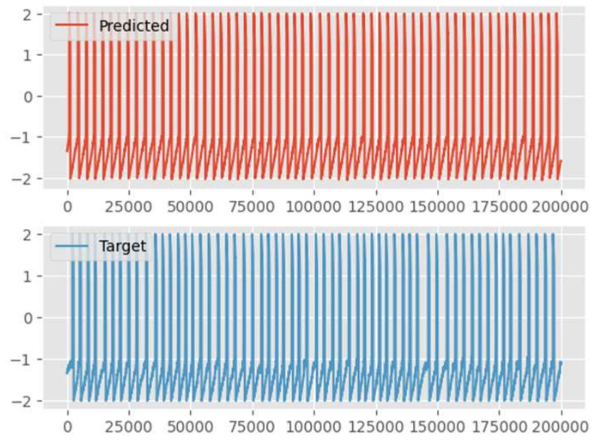

Fig. 2. Results of FitzHugh-Nagumo time series prediction with $D = 0.064$.

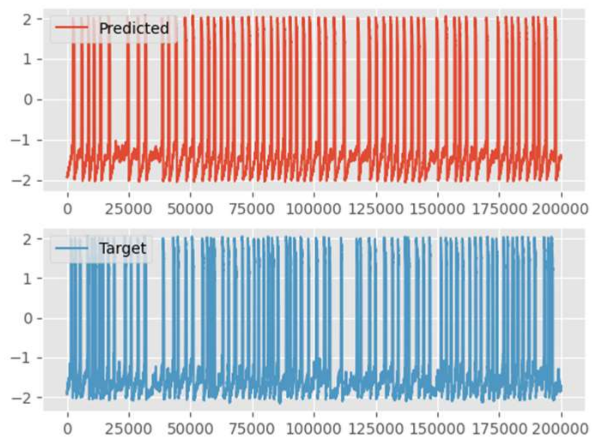

Fig. 3. Results of FitzHugh-Nagumo time series prediction with $D = 0.512$.

signal is more coherent, but the rare spike generation and variable interspike intervals, both features of the original signal, are still present.

Figure 2 shows the prediction results of the signal with $D = 0.064$. The reservoir computer's output signal dynamics are similar to the original model. However, the predicted oscillations are almost periodic, they are more coherent compared to the oscillations of the original signal.

The prediction results of the signal with high noise amplitude ($D = 0.512$) are presented on Figure 3. The predicted signal's spikes are frequent and intervals between the spikes are variable, these features are found in the original signal. However, the original signal is more noisy and its interspike intervals are even more variable.

IV. Conclusions

We trained a reservoir computer to replicate the dynamics of excitable FitzHugh-Nagumo model's signals with different levels of noise amplitude. Spike frequencies and interspike interval variations of RC's predicted signals are similar to corresponding original model signals, though the predicted signals are not exact replications of the original ones. Varying spike coherence of these predictions shows that reservoir computer exhibits the phenomenon of coherence resonance. Further reserch is needed in order to improve RC's performance in forecasting noisy FitzHugh-Nagumo model's signals.

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