Pattern formation in adaptive multiplex network in application to analysis of the complex structure of neuronal network of the brain

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ABSTRACT

In this paper we investigate the impact of competition between layers of adaptive multiplex network on pattern formation in the system under study and discuss the possibility of the further application of the obtained results for the analysis of the neural network of brain. To describe the dynamics of interacting nodes we use the Kuramoto model of coupled phase oscillators. To understand the macroscopic processes that take place in this system we calculate and compare the values of layer and global order parameter, which describe the degree of coherence between the nodes in each layer and over whole network, respectively. We find that in such adaptive network the low values of order inside layers corresponding to the formation of similar topologies among them. Nevertheless, the cluster synchronization results in divergence of layer structures from each other.

Keywords: Complex network, multilayer network, parallel computing, neural network of brain

1. INTRODUCTION

Study of the feedback between the dynamics of the individual elements, arranged in the network, and the evolution of the network topology represent itself as the key to the understanding of many processes, taken place in the real systems, referred to the networks of cities and populations,[1] electronic[2] and social systems,[3] biological networks,[4] and neural networks of brain.[5] In this context, along with the analysis of interaction between elements within the single network, study of the interaction between the adaptive networks, which can be arranged according to co-operative and competitive mechanisms, is an important task, which is associated with a more holistic view on the processes occurring in real systems including neuronal network of the brain.

The dynamics of the complex dynamical system, which consist of the number of the element, arranged in the set of interaction subnetworks, can be described in the framework of the multilayered network model. In the simplest case the different layers contain an identical set of the nodes, but have different topologies of the links between them. The more complicated models deal with the nonidentical nodes, which properties, as well, as the topology, can vary both within the single layer and within the different layers according to the specific features of the task. In the analysis of the neural network of the brain the multilayered models can be used for the study of the local synchronization between the neurons, located in the different parts of the brain, and the global synchronization, associated with the interaction between the remote regions of the brain. Moreover, the multilayered models can be effectively used for the analysis of the networks, where the elements interacts with each other via the several links simultaneously. In this case, the each layer can be associated with the interaction between the nodes through the each type of the links.

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In this paper we consider the fundamental multiplex model of adaptive network of phase oscillators, where the topology of the links between nodes is controlled by the feedback with the dynamic of the elements in the accordance with the additivity principles[3] and homeostasis.[6] The interaction between the layers of the considered network model is based on the principles of competition, leading to the dynamical redistribution of the link strength within each layer.

2. MATHEMATICAL MODEL UNDER STUDY

The model under study is a multilayer adaptive network of Kuramoto oscillators, which is a recognized tool used for studies of various forms of collective dynamics.[7–9] In our study, we created a model based on the adaptive model proposed by S. Boccaletti with colleagues[10] This model reflects the two key of features of natural networks, namely, scale-free distribution of the weight of bonds and formation of mesoscale structures. Such phenomena are caused by the above mentioned mechanisms: the homophily associated with the strengthening of ties between synchronized nodes, and the homeostasis — the mechanism of competition, by which increasing a connection from one network element is balanced by the weakening of other bonds of the same node in the network, implemented by holding the condition:

$$\sum_{j \neq i}^{N} \omega_{ij}^{l} = 1 \quad (1)$$

which means that the sum of all the weights within the node holds at all times; i.e., the sum of the weights of all incoming connections at each node is conserved. ω_{ij}^{l}–coefficient that determines the strength of the link connecting the nodes j and i on the layer l of the network.

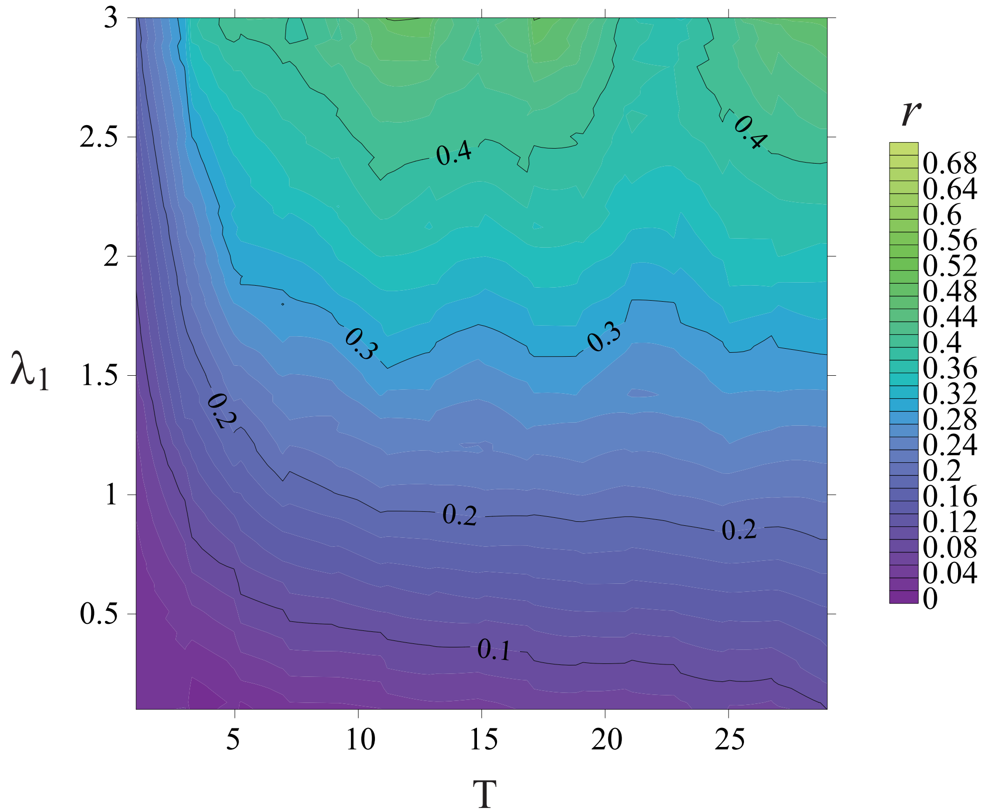

Figure 1. The 2D parameter plot of global order parameter r_{global} in dependence of the coupling strength λ and the characteristic adaptation time T

The model consists of M layers, each one containing N oscillators. In each layer $l = 1, \ldots, M$ each node $i = 1, \ldots, N$ interacts with all other nodes, that form the set:

$$\dot{\varphi_i^l} = \omega_i + \lambda \sum_{j=1}^{N} \omega_{ij}^l \sin(\varphi_i^l - \varphi_j^l), \tag{2}$$

where φ_i^l is the phase of i–th Kuramoto oscillator on the layer l, ω_i is the randomly selected frequency circular frequency phase Kuramoto oscillators, w_{ij}^l is the weight of the connection between nodes j and i within layer l as stated above and λ is a coupling strength.

The value w_{ij}^l changes over time according to the law:

$$\dot{w}_{ij}^l(t) = p_{ij}^l(t) - \left(\sum_{k \in N^l} p_{ik}^l(t) \right) w_{ij}^l(t) - \left(\sum_{k \neq l} p_{ij}^k(t) \right) w_{ij}^l(t), \tag{3}$$

where

$$p_{ij}^l(t) = \left| \frac{1}{T} \int_{t-T}^{t} e^{\sqrt{-1}[\varphi_j(\tau) - \varphi_i(\tau)]} d\tau \right|. \tag{4}$$

is the degree of coherence between the local oscillator i and j are averaged over time interval $[t - T, t]$. Thus, larger values p_{ij}^l increase the weight of the connection between the corresponding nodes. Together with the value (4) the second term of equation (3) describes the adaptive interaction between elements within the layers, while the third is a competition of layers for the optimal topology.

3. NUMERICAL RESULTS

The following are the results of a numerical calculation of the competition process in researched model (1). In our study we investigate the network, which contains two layers ($M = 2$), ($N = 300$) oscillators in each layer, the value of angular frequencies of the Kuramoto oscillators are assigned randomly in the range $[-\pi, \pi]$. Initially, the phases are also selected randomly in the interval $[-\pi; \pi]$, all weights are set to $\frac{1}{N}$. The coupling parameter between the oscillators is being changed between $0 \leq \lambda \leq 3$ with the constant step $\Delta\lambda = 0.05$, and the characteristic time T – in the interval $0 \leq T \leq 100$ with a constant step $\Delta T = 5$

First, the phases of elements were calculated at a certain period of time, greater than the characteristic time interval T. Then, the adaptive links were taken into consideration and the equation, described the change in the weights of connections began to be integrated together with the equations of the Kuramoto model. For the selection of the stationary structures we have calculated parameter, which represents the total change of the matrix relations related to the one step of integration:

$$\gamma = \frac{1}{M} \sum_{l=1}^{M} \sqrt{\sum_{i,j}^{N} \left[\omega_{ij}^l(t) - \omega_{ij}^l(t-1)\right]^2} \tag{5}$$

Using the developed model were number of calculated dynamics of the competition process in a multilayer network of oscillators with changing control parameters of the adaptation time, T, and the coupling strength, λ.

To obtain a complete picture of the evolution of the network topology, we determined the order parameter, which actually measures the level of global synchronization in the whole multilayer network:

$$r_{global}(t) = \frac{1}{MN} \left| \sum_{l=1}^{M} \sum_{i=1}^{N} e^{\sqrt{-1}\varphi_i^l} \right| \tag{6}$$

and the second order parameter, which characterizes the degree of synchronization of nodes within the single level, averaged over all levels of the network:

$$r_{layer}(t) = \frac{1}{MN} \sum_{l=1}^{M} \left| \sum_{i=1}^{N} e^{\sqrt{-1}\varphi_i^l} \right| \tag{7}$$

We have calculated the two-parameter dependencies of global and intra-layer order parameters, that are shown in Fig. 1 and Fig. 2 respectively. One can see, that while the interaction between nodes inside the layer is weak, both parameters demonstrate the quite similar values (low T and λ). The increasing of T and λ leads to growth of order in the system. But, at moderate values of controlling parameters intra-layer order parameter stars to exceed the global order significantly. Generally, the global order demonstrates more irregular behaviour than order inside layers. The dynamics described above can indicate that strengthening of intra-layer node interactions results in formation of different inter-layer topologies.

To illustrate it we calculated the following parameter:

$$w_{ij}^d = \sum_{i=1}^{N} \sum_{j=1}^{N} |w_{ij}^1 - w_{ij}^2| \quad (8)$$

which is the total difference between the weights of the connections between two nodes in the last moment of time (Fig. 3). Presented dependence shows that the small values of the intra-layer coupling result in formation of non-homogenous but identical structures in both layers. At the same time, the multiplex structure exists then the values of system parameters became moderate. This illustration has a irregularity character, because the state of the system is highly dependent on the initial conditions.

We also plot the graphical visualization of the stationary structure (5) of the studied network using open source software Gephi, that is represented in Fig. 3. One can see, that layers exhibit quite different topologies, at the same time, containing various strustural features as cluster formation and non-homogenous distribution of links. This is consistent with two-parameter dependencies above, described above.

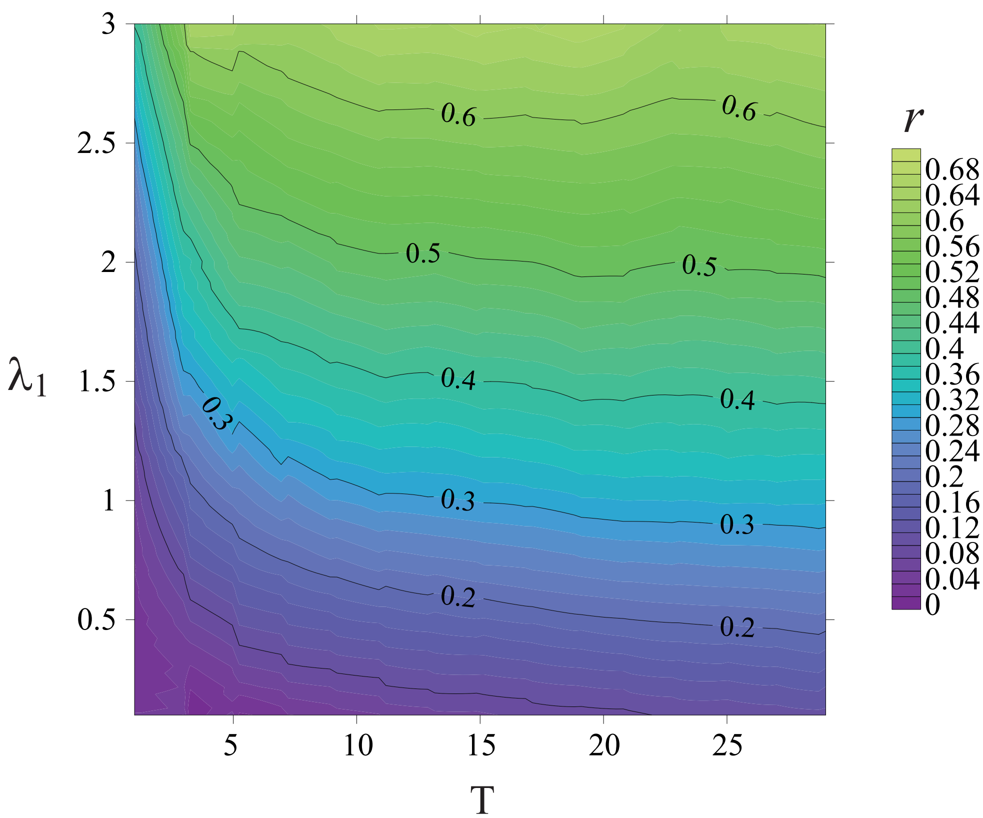

Figure 2. Two-parametric dependence on the coupling strength λ of the characteristic adaptation time T: global order parameter $r_{global}(a)$ and the layer order parameter $r_{local}(b)$

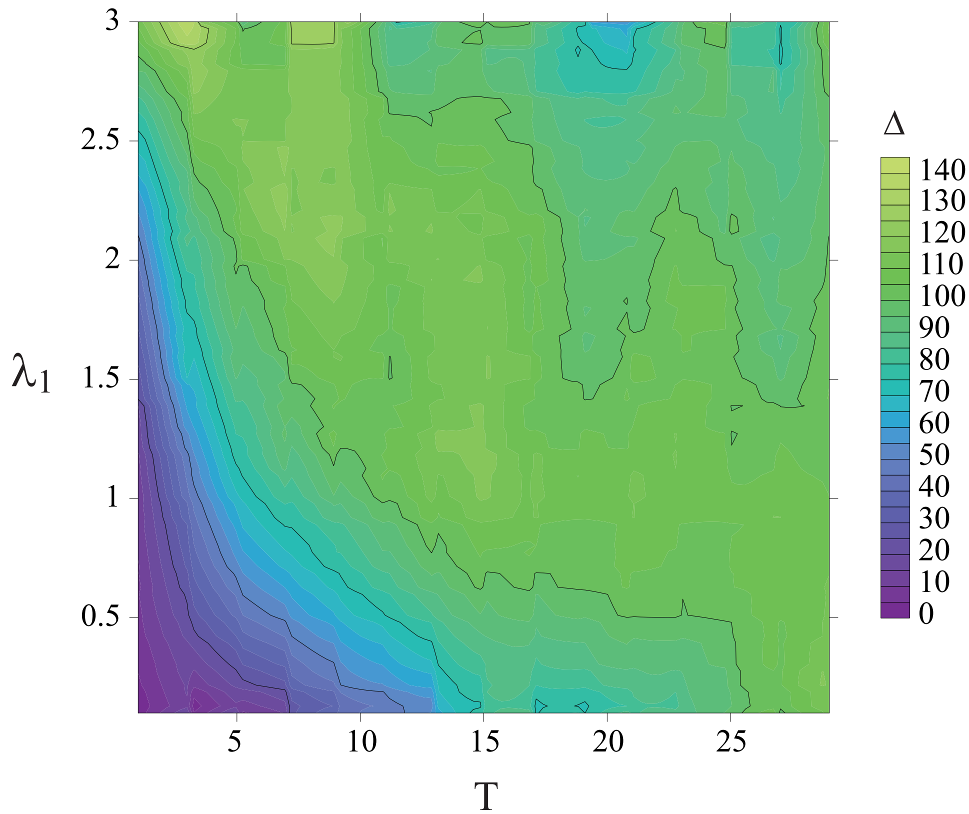

Figure 3. The total difference between the weights of the connections between two points in time.

4. CONCLUSION

We considered the fundamental multi-layer model of the adaptive network of phase oscillators, where the topology of the links between elements is controlled by the feedback with the dynamic of the elements in the accordance with the additivity principles[3] and homeostasis.[6] Using the developed model we numerically studied the processes of competition and synchronization between the coupled nodes, belonging to the different layers depending on the value of control parameters, represented themselves as the adaptation time, T, and the intra-layer coupling strength, λ.

We found that weak interaction between nodes inside layers is accompanied by the formation of identical topologies of the interacting layers and a inhomogeneous distribution of the link strength within each layer. We also shown that the cluster synchronization occurred for the moderate values of the coupling strength, led to the emergence of the natural multiplex structure within the system under study.

The obtained results can be considered as the universal phenomena which can take place in the wide range of the real system, including the neural networks of the brain.[11] In particular, the observed dynamical state, which is characterized by the global synchronization between the layers can by associated with the hyper-synchronous dynamics of the neural network,[12–16] which take place during the onset of the epileptic seizure due to the high degree of the interaction between the cortex and thalamus.[17–19] The regime of the cluster synchronization can be vise-versa associated with the background brain activity, where the different areas of the brain interact weakly, while take part in the different form of the cognitive activity and precursor activity of epileptic seizure.[20, 21]

It should be noted that despite the simplicity of the the considered network model, it demonstrates the effects, which reflects the key properties of the real networks, in which the nature of the nodes and links is much more complicated. We believe that the proposed model of adaptive network can be used as the base model for the development of the more specific and realistic structures, according to the key features of the concrete objects,

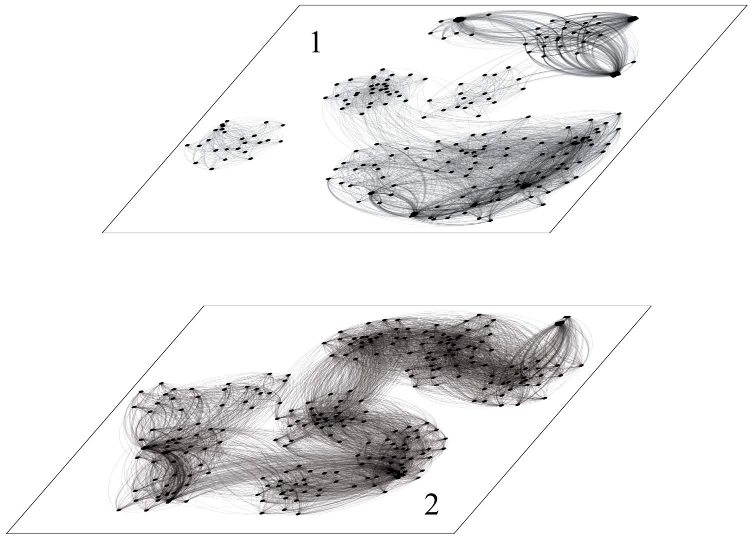

Figure 4. The structure of the network layers visualized via open source software Gephi. The coupling strength $\lambda = 0.5$, characteristic adaptation time $T = 15$

which provide the possibility to study and reveal new phenomena in the real-world networks including brain neuronal network.

5. ACKNOWLEDGMENTS

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