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# Cooperation of deterministic and stochastic mechanisms resulting in the intermittent behavior

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ABSTRACT

Intermittent behavior near the boundary of chaotic phase synchronization in the presence of noise (when deterministic and stochastic mechanisms resulting in intermittency take place simultaneously) is studied. The noise of small intensity is shown to do not affect on the characteristics of intermittency whereas the noise of large amplitude induces new effects near the boundary of the synchronous regime. In the first case the eyelet intermittency takes place near the boundary of the synchronous regime, in the second one the ring intermittency or coexistence of both types of intermittency is realized. Main results are illustrated using the example of two unidirectionally coupled Rössler systems. Similar effects are shown to be observed in coupled spatially distributed Pierce beam–plasma systems.



## 1. Introduction

Intermittency is an ubiquitous phenomenon in nonlinear sciences. It is observed, in particular, at transition from periodic oscillations to the chaotic ones as well as at the boundaries of the synchronous regime onset [1]. Intermittent behavior precedes almost all known types of chaotic synchronization, with the mechanisms of their arising and characteristics of intermittency being different for various synchronization types. In particular, near the boundaries of lag [4], generalized [5] and noise-induced synchronization [6] on–off intermittency takes place. These types of intermittent synchronous behavior are also called as intermittent lag synchronization, intermittent generalized synchronization and intermittent noise-induced synchronization, respectively. In turn, near the boundary of phase synchronization eyelet [2] or ring [3] intermittencies (called also as intermittent phase synchronization) are realized. Eyelet intermittency corresponds to the small values of the control parameter detuning of interacting systems whereas the ring intermittency is observed for the relatively large values of the control parameter mismatch. These types of intermittency are characterized by different mechanisms of the arising and different statistical characteristics of the laminar phase lengths (see Section 3 for details). In particular, in the ring intermittency regime attractor of one of interacting systems is phase-incoherent whereas for the eyelet intermittency both attractors remain phase-coherent.

All mechanisms resulting in the intermittent behavior may be divided, generally, into two large classes, the deterministic and stochastic ones. The majority of the intermittent manifestations is caused by the deterministic mechanisms, e.g., all intermittencies observed in the vicinity of the chaotic synchronization boundaries refer to the deterministic class. One of the interesting types of deterministic intermittent synchronous behavior is the intermittent time scale synchronization [7] which is related closely to the phase synchronization regime [8]. The type of intermittency realized in this case depends both on

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the value of the control parameter mistuning and time scale of observation. If interacting systems are detuned slightly from each other, the eyelet intermittency takes place [2] near the onset of the synchronous regime. For the large values of parameter mismatch the ring intermittency [3] is realized. The same type of intermittency is observed for the time scale synchronization regime at the boundary time scales of observation independently on the value of the control parameter mismatch [7], whereas on the boundary of such regime the eyelet intermittency, the ring intermittency or the coexistence of both types of intermittency, i.e. the so-called intermittency of intermittencies, takes place [9].

External noise is known to influence sufficiently on the nonlinear regimes (see, e.g., [10–15]) and characteristics of intermittency [16]. In particular, the effect of noise on the non-autonomous periodic system being in the synchronous regime results in the appearance of intermittency (the type I intermittency in the presence of noise) [17,18] which characteristics are closely connected with the eyelet ones [19].

The present paper aims to consider the intermittency phenomenon when both mechanisms (the deterministic and stochastic ones) resulting in the intermittent behavior take place simultaneously. We analyze the influence of noise on the characteristics of intermittent phase synchronization in the case of a relatively small values of the control parameter mismatch between both interacting systems with a small number of degrees of freedom (coupled chaotic Rössler oscillators [20]) and spatially extended chaotic media (Pierce beam–plasma diodes [21,22]). In these cases two mechanisms resulting in the intermittent behavior are presented. One of them is connected with the deterministic dynamics of the system, whereas the second one is the manifestation of the stochastic behavior. As would be shown below the noise of small intensity does not practically affect on the characteristics of intermittency whereas the noise of large amplitude is able to induce new features in the characteristics of intermittency.

## 2. System under study

Let us analyze the characteristics of intermittent phase synchronization in the presence of noise using the example of two unidirectionally coupled chaotic Rössler oscillators. The system under study is given by

$$\begin{aligned}
\dot{x}_1 &= -\omega_1 y_1 - z_1,\\
\dot{y}_1 &= \omega_1 x_1 + a y_1,\\
\dot{z}_1 &= p + z_1(x_1 - c),\\
\dot{x}_2 &= -\omega_2 y_2 - z_2 + \varepsilon(x_1 - x_2),\\
\dot{y}_2 &= \omega_2 x_2 + a y_2 + D\xi,\\
\dot{z}_2 &= p + z_2(x_2 - c),
\end{aligned} \tag{1}$$

where $\mathbf{x}_{1,2}(t) = (x_{1,2}, y_{1,2}, z_{1,2})^T$ are state vectors of the drive and response systems, respectively, $a = 0.15, p = 0.2, c = 10, \omega_1 = 0.93, \omega_2 = 0.95$ are the control parameter values, $\xi$ is a random Gaussian process with zero mean and unit variance, $D$ is a noise intensity.

To integrate the stochastic differential equations (1) we have used the four order Runge–Kutta method adapted for the stochastic differential equations [23] with time step $\Delta t = 0.001$. Detection of the phase synchronization regime has been performed by the analysis of the time dependencies of the phase differences of interacting systems and testifying the phase locking condition

$$|\Delta\phi| = |\phi_1(t) - \phi_2(t)| < \text{const}. \tag{2}$$

The phases $\phi_{1,2}(t)$ of chaotic signals have been introduced into consideration in traditional way as rotation angles on $(x_{1,2}, y_{1,2})$-planes [8].

## 3. Intermittency near the boundary of the phase synchronization regime in coupled Rössler oscillators

First of all we analyze the influence of the noise intensity on the boundary value of the phase synchronization regime onset in system (1). The results of our calculations show that if the noise intensity exceeds the certain critical value the synchronous regime starts destructing due to the loss of the phase coherence of the response system attractor. It is clear that in the fields where the boundary of the synchronous regime is not changed dramatically ($D \leqslant 9$) the noise will not affect sufficiently both on the boundary of the synchronous regime onset and characteristics of intermittency taking place near that boundary. At the same time, in the field of the loss of the phase-coherence of the response system attractor ($D \geqslant 9$) the noise is able to bring new features in the characteristics of intermittency.

To define the type of intermittency realized in the system under study we use the rotating plane approach initially proposed in [3] and used successfully in recent papers [24,7,9]. We analyze the behavior of the response system (1) on the plane

$$\begin{aligned}
x' &= x_2 \cos\phi_1 + y_2 \sin\phi_1,\\
y' &= -x_2 \sin\phi_1 + y_2 \cos\phi_1,
\end{aligned} \tag{3}$$

rotating with the frequency of the drive system. Here $\phi_1 = \phi_1(t)$ is the phase of the drive system. In Fig. 1 the phase differences $\Delta\phi(t)$ (*a*,*b*), rotating planes (*c*,*e*,*g*,*i*) and phase portraits (*d*,*f*,*h*,*j*) of the response system (1) for different values of the coupling parameter $\varepsilon$ and noise intensity $D$ are shown. Fig. 1*a*, *c*, *d*, curve 1 corresponds to the case when in two unidirectionally coupled Rössler systems (1) the synchronous dynamics is observed. In this case the phase difference is locked, the response system attractor is phase-coherent and phase trajectory of the response oscillator looks like a smeared fixed point which does not envelop the origin on the rotating plane. Below the boundary of the synchronous regime in the case when the noise intensity is small enough as well as in the case of the absence of noise the eyelet intermittency takes place (see Fig. 1*a*, *e*, *f*, curve 2). The phase difference in such case contains time intervals of synchronized motion persistently and intermittently interrupted by sudden phase slips during which the value of $|\Delta\phi|$ jumps up $2\pi$. The response system attractor in this regime is also phase-coherent but the trajectory on the rotating plane is represented by a smeared limit cycle [3]. The growth of the noise intensity

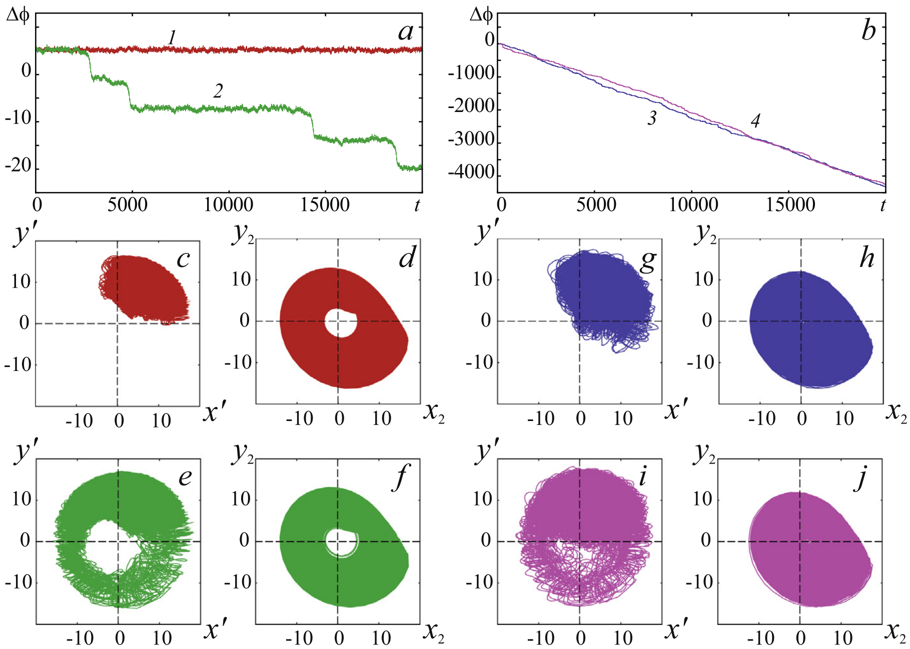


**Fig. 1.** Phase differences $\Delta\phi(t)$ (*a,b*) and phase trajectories of the response Rössler system on the rotating plane $(x', y')$ (*c,e,g,i*) and phase portraits of the same response system on $(x_2, y_2)$-plane (*d,f,h,j*): (*a,c,d*), curve 1—the synchronous regime ($\varepsilon = 0.045, D = 1.5$), (*a,e,f*), curve 2—the eyelet intermittency ($\varepsilon = 0.038, D = 1.5$), (*b,g,h*), curve 3—the ring intermittency ($\varepsilon = 0.045, D = 10$), (*b,i,j*), curve 4—the intermittency of eyelet and ring intermittencies ($\varepsilon = 0.038, D = 10$).

changes the properties of the response system attractor remarkably, i.e. it becomes principally phase-incoherent (see Fig. 1*h*, *j*) and the phase trajectory on the rotating plane starts enveloping origin (Fig. 1*g*, *i*). The phase slips in such regimes become more frequent (see Fig. 1*b*) that is connected with a strong noise influence. At the same time, the envelop of the origin by the phase trajectory can be performed in two different ways. If the coupling parameter exceeds the boundary value of the phase synchronization regime onset in the absence of noise but for the selected value of the noise intensity the phase synchronization does not exist the phase trajectory is represented by a smeared fixed point enveloping origin (Fig. 1*g*). In that case in the system under study the ring intermittency is realized [3]. If the coupling parameter is less than the threshold value of the phase synchronization in the absence of noise the phase trajectory on the rotating plane looks like a smeared limit cycle enveloping origin (Fig. 1*i*). In this case the intermittency of eyelet and ring intermittencies takes place simultaneously [9]. It should be noted that in both considered cases the dynamics of the phase difference is qualitatively identical to each other (compare Fig. 1*b* curves 3 and 4). This may be connected with the fact that the most part of phase slips in the regime of intermittency of intermittencies is associated with the ring intermittency.

To confirm the presence of the types of intermittency mentioned above near the boundary of the phase synchronization in the presence of noise we analyze the statistical characteristics of intermittency such as the distributions of the laminar phase lengths for the fixed values of the control parameters and dependence of the mean length of the laminar phases on the coupling parameter. Both eyelet and ring intermittencies are known to be characterized by the exponential distributions of the laminar phase lengths

$$p(\tau) = \frac{1}{T_{1,2}} \exp\left(-\frac{\tau}{T_{1,2}}\right) \tag{4}$$

where $T_{1,2}$ are mean lengths of the laminar phases for eyelet $T_1$ and ring $T_2$ intermittencies [19,3], whereas the dependence of the mean length of the laminar phases for eyelet intermittency obeys the law

$$T_1 = K \exp \kappa(\varepsilon_c - \varepsilon)^{-1/2}, \tag{5}$$

or, in the other form,

$$\ln 1/T_1 = C - \kappa(\varepsilon_c - \varepsilon)^{-1/2}, \tag{6}$$

where $\varepsilon_c$ is a critical value of the coupling parameter corresponding to the onset of the phase synchronization, $K$, $C = \ln 1/K$ and $\kappa$ are the parameters of approximation [25,26]. In Ref. [9] we have shown that in the regime of intermittency of eyelet and ring intermittencies the laminar phase length distribution should be written as

$$\begin{aligned} p(\tau) = {} & \frac{\exp(-\tau/T_1)}{(T_1+T_2)}\left(1-\frac{\tau}{T_1}\right)\Gamma\left(0,\frac{\tau}{T_2}\right) \\ & + \frac{T_1^2+T_2^2}{T_1T_2(T_1+T_2)}\exp\left(-\frac{\tau}{T_1}-\frac{\tau}{T_2}\right) \\ & + \frac{\exp(-\tau/T_2)}{(T_1+T_2)}\left(1-\frac{\tau}{T_2}\right)\Gamma\left(0,\frac{\tau}{T_1}\right). \end{aligned} \tag{7}$$

Here $\Gamma(a,z)$ is incomplete $\Gamma$-function, at that the mean length of the laminar phases for this type of intermittent behavior obeys relation

$$T = -\frac{T_1^2\log\left(\frac{T_1+T_2}{T_1}\right) - 2T_1T_2 + T_2^2\log\left(\frac{T_1+T_2}{T_2}\right)}{T_1+T_2}, \qquad (8)$$

where $T_{1,2}$ can be obtained numerically for the regimes when the only one type of intermittent behavior should exist [9].

In Fig. 2 the distributions of the laminar phase lengths for all types of intermittency mentioned above and their theoretical approximations by relations (4) and (7) are shown. It is clearly seen that the numerically obtained data are in a good agreement with the results of theoretical predictions that confirms the possibility of realization of different types of intermittency near the boundary of the phase synchronization in the presence of noise. The additional proof of the presence of eyelet intermittency, ring intermittency or intermittency of intermittency (coexistence of eyelet and ring intermittencies) near the synchronization boundary is the dependence of the mean length of the laminar phases on the coupling parameter. Due to the fact that the appearance of turbulent phases associated with the ring intermittency does not practically depend on the coupling parameter we analyze the statistics of the mean lengths of the laminar phases for only two types of intermittency, namely, the eyelet intermittency and intermittency of eyelet and ring intermittencies. In Fig. 3 the dependencies of the mean lengths of the laminar phases on the coupling parameter obtained numerically for both types of intermittency mentioned above as well as their theoretical approximations (5) and (8) are shown. It is clearly seen that the data obtained numerically are in excellent agreement with the theoretical relations. The coefficients for the regularity given by (5) have been obtained by means of application of the least square method to the dependence of $\ln 1/T$ on $(\varepsilon_c-\varepsilon)^{-1/2}$ shown in the frame. One can see that such dependence obeys relation (6) in full agreement with the theory of eyelet intermittency.

## 4. Intermittency of intermittencies in coupled spatially distributed beam–plasma systems

So, in two unidirectionally coupled Rössler systems near the boundary of the phase synchronization in the presence of noise depending on the value of the coupling parameter and noise intensity the eyelet intermittency, the ring intermittency or coexistence of eyelet and ring intermittencies can exist. To extend obtained results on other systems capable to demonstrate the phase synchronization regime in the presence of noise let us analyze the intermittent behavior near the boundary of the synchronous regime in two unidirectionally coupled spatially distributed beam–plasma systems (Pierce diodes) which dynamics in the fluid electronic approximation is described by the self-congruent system of dimensionless Poisson, continuity and motion equations

$$\begin{aligned}
&\frac{\partial^2\varphi_{1,2}}{\partial x^2} = -(\alpha_{1,2})^2(\rho_{1,2}-1),\\
&\frac{\partial\rho_{1,2}}{\partial t} = -\frac{\partial(\rho_{1,2}v_{1,2})}{\partial x},\\
&\frac{\partial v_{1,2}}{\partial t} = -v_{1,2}\frac{\partial v_{1,2}}{\partial x} + \frac{\partial\varphi_{1,2}}{\partial x},
\end{aligned} \qquad (9)$$

with the boundary conditions

$$v_{1,2}(0,t)=1, \quad \rho_{1,2}(0,t)=1, \quad \varphi_{1,2}(0,t)=0, \qquad (10)$$

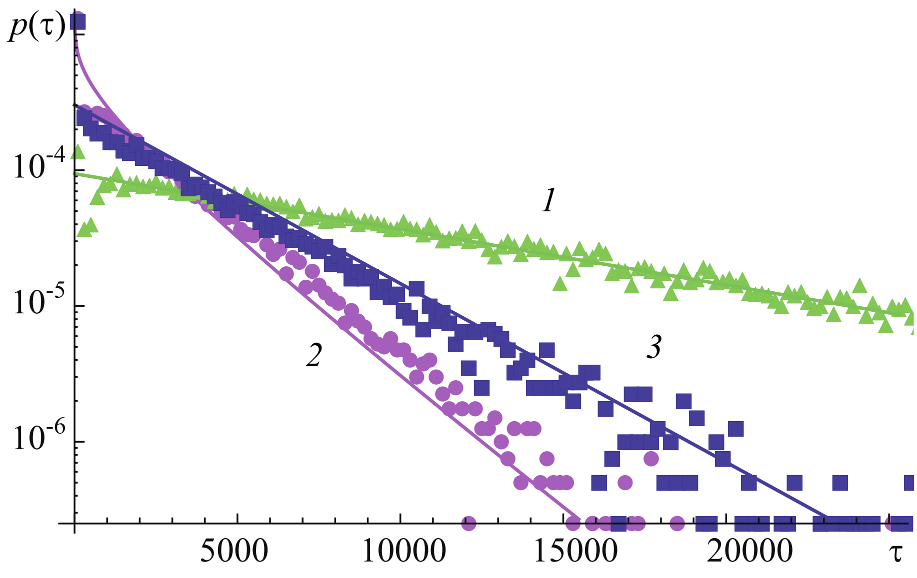


**Fig. 2.** Distributions of the laminar phase length in the regimes of eyelet intermittency ($D = 1.5, \varepsilon = 0.038$, curve 1), intermittency of eyelet and ring intermittencies ($D = 10, \varepsilon = 0.038$, curve 2) and ring intermittency ($D = 10, \varepsilon = 0.045$, curve 3) and their theoretical approximations. The numerical data are marked by points, their theoretical approximations are shown by solid lines. The parameters of approximations are the following: $1 - T_1 = 10593$, $2 - T_1 = 12533, T_2 = 3300$, $3 - T_2 = 3300$.

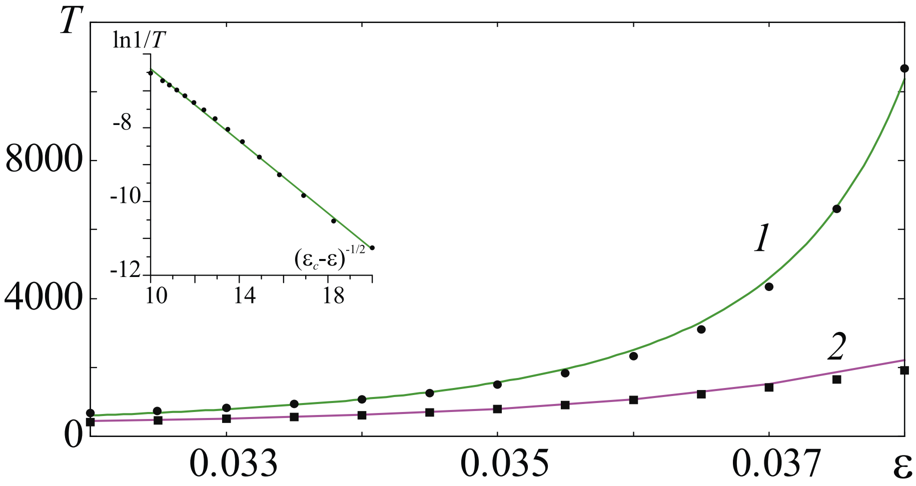


**Fig. 3.** Dependence of the mean length of the laminar phases on the coupling parameter in the regimes of eyelet intermittency ($D = 1.5$, curve 1) and intermittency of eyelet and ring intermittencies ($D = 10$, curve 2) and their theoretical approximations. The numerical data are marked by points, their theoretical approximations are shown by solid lines. In the frame the dependence 1 in the form (6), i.e. the dependence of $\ln(1/T)$ on $(\varepsilon_c - \varepsilon)^{-1/2}$ is shown. The parameters of approximations are the following: $K = 4.545, \kappa = 0.489, \varepsilon_c = 0.042, C = -1.514$.

where $\varphi_{1,2}(x,t)$ is the dimensionless potential of the electric field, $\rho_{1,2}(x,t)$ and $v_{1,2}(x,t)$ are the dimensionless density and velocity of the electron beam ($0 \leqslant x \leq 1$), the indices "1" and "2" correspond to the drive and response coupled beam–plasma systems, respectively [21,27]. The unidirectional coupling between such systems is realized by the modification of the boundary conditions on the right boundary of the systems, in the same way as it has been done in [28]

$$\begin{cases} \varphi_1(1,t) = 0, \\ \varphi_2(1,t) = \varepsilon(\rho_2(x=1,t) - \rho_1(x=1,t)) + D\xi(t). \end{cases} \tag{11}$$

The term $D\xi(t)$ corresponds to the noise influence on the system, where $\xi(t)$ is stochastic Gaussian process with zero mean and unit variance, $D$ is the noise intensity. Continuity and motion equations of (9) have been integrated numerically with the help of the one-step explicit two-level scheme with upstream differences and the Poisson equation has been solved by the method of the error vector propagation [29]. The time and space integration steps have been taken as $\Delta t = 0.003$ and $\Delta x = 0.005$, respectively. The control parameters of Pierce diodes have been chosen as $\alpha_1 = 2.858\pi$ and $\alpha_2 = 2.860\pi$. As in the case of Rössler systems described above the phase synchronization has been detected by the verification of phase locking condition (2). The phases of the drive and response Pierce diodes have been introduced into consideration as rotation angles on $(\rho_{1,2}(x=0.2,t), \rho_{1,2}(x=0.6,t))$-plane as well as it has been done in [30].

The numerical simulation of system (9) with the boundary conditions 10,11 shows that as in the case of system (1) the synchronous regime in two unidirectionally coupled Pierce diodes starts destructing quickly with the growth of the noise intensity. At the same time, due to the specificity of the system itself, the Pierce diodes are more sensible to the influence of noise in comparison with the Rössler systems. Therefore, the new effects for such spatially extended media are revealed for a relatively small values of the noise intensity.

Fig. 4 illustrates the dynamics of the phase differences $\Delta\phi(t)$ ($a$,$b$) and behavior of the response Pierce diode both on the plane $(x', y')$ rotating with the frequency of the drive Pierce diode defined by (3) with $x_2 = \rho_2(x=0.2,t)$, $y_2 = \rho_2(x=0.6,t)$ ($c,e,g,i$) and on $(\rho_2(x=0.2,t), \rho_2(x=0.6,t))$-plane ($d,f,h,j$) in different regimes. It is clearly seen that Fig. 4 is qualitatively identical to Fig. 1. In particular, if the noise intensity is small enough in the synchronous regime the phase difference is bounded (see Fig. 4*a*, curve 1) and the response system attractor on the rotating plane looks like a smeared fixed point which does not envelop the origin (see Fig. 4*c* and compare it with the Fig. 1*c*). The reconstructed attractor in this case is phase-coherent (Fig. 4*d*). Near the boundary of the phase synchronization regime in the case of the small values of the noise intensity the eyelet intermittency takes place. In this case the phase difference contains sudden rare enough phase slips (Fig. 4*a*, curve 2), the response system attractor is also phase-coherent (Fig. 4*f*), and the phase trajectory on the rotating plane represents the smeared limit cycle which does not touch the origin (Fig. 4*e*).

The behavior of the Pierce diodes is changed dramatically if the noise intensity becomes a big enough (Fig. 4*b*, *g*–*j*). Both for the coupling parameter values corresponding to the synchronous regime in the absence of noise ($\varepsilon = 0.058$) as well as for the asynchronous one ($\varepsilon = 0.006$) the dynamics of the phase difference in the presence of noise is characterized by sharp decrease of its value (Fig. 4*b*) and the response system attractor is phase-incoherent (Fig. 4*h*, *j*). At that, if in the absence of noise in the system under study the synchronous regime is realized in the same system in the presence of noise of a relatively large amplitude the ring intermittency takes place. The response system attractor on the rotating plane looks like a smeared fixed point enveloping origin in this case (Fig. 4g). For the coupling parameter value

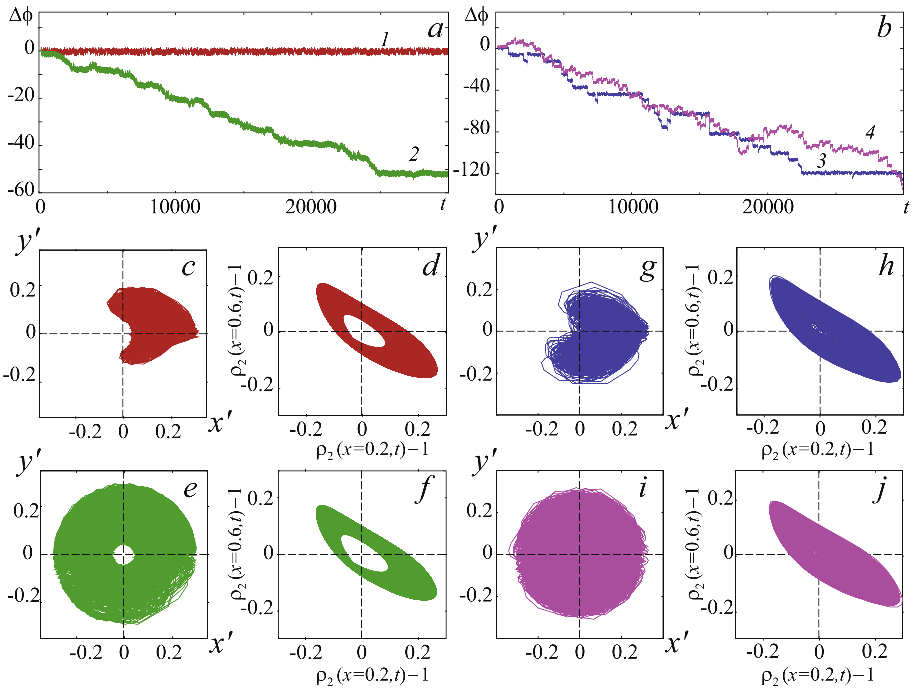


**Fig. 4.** Phase differences $\Delta\phi(t)$ (*a*,*b*) and phase trajectories of the response Pierce diode on the rotating plane $(x', y')$ (*c*,*e*,*g*,*i*) and phase portraits of the same response system on $(\rho_{1,2}(x = 0.2, t), \rho_{1,2}(x = 0.6, t))$-plane (*d*,*f*,*h*,*j*): (*a*,*c*,*d*), curve 1—the synchronous regime ($\varepsilon = 0.058, D = 10^{-5}$), (*a*,*e*,*f*), curve 2—the eyelet intermittency ($\varepsilon = 0.006, D = 10^{-5}$), (*b*,*g*,*h*), curve 3—the ring intermittency ($\varepsilon = 0.058$, $D = 0.03$), (*b*,*i*,*j*), curve 4—the intermittency of eyelet and ring intermittencies ($\varepsilon = 0.006, D = 0.03$).

corresponding to the eyelet intermittency in the absence of noise in the Pierce diodes subjected to the strong noise influence the coexistence of eyelet and ring intermittencies takes place. The response system attractor in such regime is represented by a smeared limit cycle enveloping origin (Fig. 4*i*).

## 5. Conclusion

In conclusion, both in systems with a small number of degrees of freedom (two unidirectionally coupled Rössler oscillators) and spatially extended beam–plasma media (two Pierce diodes in the case of unidirectional coupling) the intermittent behavior near the boundary of the phase synchronization in the presence of noise is observed. At that, in all considered cases the noise of small intensity does not almost influence on characteristics of intermittency, i.e. in the case of the small values of parameter mismatch near the boundary of phase synchronization the eyelet intermittency takes place. The increase of the noise intensity results in the growth of the threshold value of the synchronous regime onset stipulated by the loss of the phase coherence of the response system attractor. Consequently, on the boundary of the phase synchronization in the supercritical region of the control parameters the ring intermittency comes into being whereas in the subcritical one the coexistence of ring and eyelet intermittencies takes place. The found behavior is assumed to possess a high level of generality. One can expect that similar regularities would be observed for the wide class of real systems.

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## References

[1] Berge P, Pomeau Y, Vidal C. Order within chaos. New York: John Wiley and Sons; 1984.
[2] Pikovsky AS, Osipov GV, Rosenblum MG, Zaks M, Kurths J. Attractor–repeller collision and eyelet intermittency at the transition to phase synchronization. Phys Rev Lett 1997;79(1):47–50.
[3] Hramov AE, Koronovskii AA, Kurovskaya MK, Boccaletti S. Ring intermittency in coupled chaotic oscillators at the boundary of phase synchronization. Phys Rev Lett 2006;97:114101.
[4] Boccaletti S, Valladares DL. Characterization of intermittent lag synchronization. Phys Rev E 2000;62(5):7497–500.
[5] Hramov AE, Koronovskii AA. Intermittent generalized synchronization in unidirectionally coupled chaotic oscillators. Europhysics Lett 2005;70(2):169–75.
[6] Moskalenko OI, Koronovskii AA, Shurygina SA. Intermittent behavior on the boundary of the noise-induced synchronization. Tech Phys 2011;56(9):1369–72.
[7] Zhuravlev MO, Koronovskii AA, Moskalenko OI, Ovchinnikov AA, Hramov AE. Ring intermittency near the boundary of the synchronous time scales of chaotic oscillators. Phys Rev E 2011;83:027201.
[8] Rosenblum MG, Pikovsky AS, Kurths J. Phase synchronization of chaotic oscillators. Phys Rev Lett 1996;76(11):1804–7.
[9] Hramov AE, Koronovskii AA, Moskalenko OI, Zhuravlev MO, Ponomarenko VI, Prokhorov MD. Intermittency of intermittencies. CHAOS 2013;23(3):033129.

[10] Pikovsky AS, Kurths J. Coherence resonance in a noise-driven excitable system. Phys Rev Lett 1997;78(5):775–8.
[11] Neiman A, Silchenko A, Anishchenko VS, Schimansky-Geier L. Stochastic resonance: noise-enhanced phase coherence. Phys Rev E 1998;58:7118–25.
[12] Zhou CS, Kurths J. Noise-induced phase synchronization and synchronization transitions in chaotic oscillators. Phys Rev Lett 2002;88:230602.
[13] Zhou CS, Kurths J, Allaria E, Boccaletti S, Meucci R, Arecchi FT. Noise-enhanced synchronization of homoclinic chaos in a $CO_2$ laser. Phys Rev E 2003;67:015205(R).
[14] Hramov AE, Koronovskii AA, Moskalenko OI. Are generalized synchronization and noise-induced synchronization identical types of synchronous behavior of chaotic oscillators? Phys Lett A 2006;354(5–6):423–7.
[15] Goldobin DS, Pikovsky AS. Synchronization and desynchronization of self-sustained oscillators by common noise. Phys Rev E 2005;71(4):045201(R).
[16] Eckmann JP, Thomas L, Wittwer P. Intermittency in the presence of noise. J Phys A: Math Gen 1981;14:3153–68.
[17] Kye W-H, Kim C-M. Characteristic relations of type-I intermittency in the presence of noise. Phys Rev E 2000;62(5):6304–7.
[18] Hramov AE, Koronovskii AA, Kurovskaya MK, Ovchinnikov AA, Boccaletti S. Length distribution of laminar phases for type-I intermittency in the presence of noise. Phys Rev E 2007;76(2):026206.
[19] Hramov AE, Koronovskii AA, Kurovskaya MK, Moskalenko OI. Type-i intermittency with noise versus eyelet intermittency. Phys Lett A 2011;375:1646–52.
[20] Rössler OE. An equation for continuous chaos. Phys Lett A 1976;57(5):397–8.
[21] Godfrey BB. Oscillatory nonlinear electron flow in Pierce diode. Phys Fluids 1987;30:1553.
[22] Hramov AE, Rempen IS. Investigation of the complex dynamics and regime control in Pierce diode with the delay feedback. Int J Electron 2004;91(1):1–12.
[23] Wilkie J. Numerical methods for stochastic differential equations. Phys Rev E (Stat Nonlinear Soft Matter Phys) 2004;70(1):017701.
[24] Hramov AE, Koronovskii AA, Kurovskaya MK. Two types of phase synchronization destruction. Phys Rev E 2007;75(3):036205.
[25] Grebogi C, Ott E, Yorke JA. Fractal basin boundaries, long lived chaotic transients, and unstable–unstable pair bifurcation. Phys Rev Lett 1983;50(13):935–8.
[26] Boccaletti S, Allaria E, Meucci R, Arecchi FT. Experimental characterization of the transition to phase synchronization of chaotic $CO_2$ laser systems. Phys Rev Lett 2002;89(19):194101.
[27] Matsumoto H, Yokoyama H, Summers D. Computer simulations of the chaotic dynamics of the Pierce beam–plasma system. Phys Plasmas 1996;3(1):177.
[28] Filatov RA, Hramov AE, Koronovskii AA. Chaotic synchronization in coupled spatially extended beam–plasma systems. Phys Lett A 2006;358:301–8.
[29] Rouch PJ. Computational fluid dynamics. Albuquerque: Hermosa Publishers; 1976.
[30] Moskalenko OI, Koronovskii AA, Hramov AE, Boccaletti S. Generalized synchronization in mutually coupled oscillators and complex networks. Phys Rev E 2012;86:036216.