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Synchronization in four interacting networks of Hodgkin-Huxley neurons

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***Abstract*—We develop a mathematical model of a "network of networks" consisting of a small input network and four large subnetworks interacting with each other through inhibitory couplings. We show that synchronization indexes of subnetworks periodically change in time. Depending on the strength of the connection, the synchronization indexes of neurons in different subnetworks can change both in phase and in antiphase.**

***Index Terms*—Synchronization, Hodgkin-Huxley neuron, complex network**

I. Introduction

Nowadays, the use of methods of radiophysics, nonlinear dynamics and network analysis in problems of neurophysiology is an actual trend in modern science [1]–[3].

Various methods of network theory are used in the analysis of interactions between parts of the brain during cognitive activity, based both on the analysis of experimental data (for example, multichannel records of electrical [4]–[13], magnetic [14]–[17] and oxygenation [15], [18], [19] activity), and on numerical modeling of the interaction of individual neurons and their groups by constructing mathematical models of networks of nonlinear elements [20]–[23].

Currently, the application of the theory of complex networks to neuroscience is very promising for the analysis of the structural and functional connections of neurons in the brain [24]–[26]. Collective neural activity plays an important role in the functioning of the brain. According to studies of functional magnetic resonance imaging (fMRI), network activity of the whole brain is generated through the interaction of several functional subnets during a resting state or performing a task. Collective processes resulting from functional interactions between distant populations of cortical neurons support cognitive abilities when performing complex tasks. Modern understanding of neural communication emphasizes the vital role of phase coherence in functional interactions between distant neural ensembles.

In this work we develop a mathematical model of a "network of networks" consisting of a small input network and four large subnetworks interacting with each other through inhibitory couplings. We show that synchronization indexes of subnetworks periodically change in time. Depending on the strength of the connection, the synchronization indexes of neurons in different subnetworks can change both in phase and in antiphase.

II. Methods

We use the Hodgkin-Huxley (HH) model to describe the time evolution of the transmembrane potential of each neuron [27]. In this work, we consider the coupling via chemical synapses only.

Synchronization inside each network is quantified with the synchronization index defined as [29], [30]:

$$\Xi = \sqrt{\frac{1}{T-t_0}\int_{t_0}^{T}\eta(t)dt}, \tag{1}$$

where t_0 and T are durations of transients and total time series, and $\eta(t)$ is the standard deviation given as

$$\eta(t) = \frac{1}{N}\sum_{i=1}^{N}\left(x^{(i)}(t)\right)^2 - \left(\frac{1}{N}\sum_{i=1}^{N}x^{(i)}(t)\right)^2. \tag{2}$$

The lower the synchronization index Ξ, the better the synchronization, so that $\Xi = 0$ means complete synchronization.

The correlation between interacted N_1 and N_2 sub-networks can be found on the base of their synchronization indices Ξ_1 and Ξ_2. The Pearson's linear correlation coefficient is calculated as follows [31]

$$r = \frac{\int_{t_0}^{T}(\Xi_1(t)-\overline{\Xi}_1)(\Xi_2(t)-\overline{\Xi}_2)dt}{\sqrt{\int_{t_0}^{T}(\Xi_1(t)-\overline{\Xi}_1)^2(\Xi_2(t)-\overline{\Xi}_2)^2dt}}. \tag{3}$$

Here, $r = 1$ and $r = -1$ mean perfect positive and perfect negataive correlation, respectively.

III. Results

We develop the "network of networks". The external stimulus of constant current with amplitude A is applied to the input network of $N^{ex} = 5$ neurons. All of them are connected to each other with the coupling strength chosen randomly from the range [0,0.15]. This network is connected to the four other subnetworks of $N_1 = N_2 = N_3 = N_4 = 50$ neurons by one-directional excitatory couplings with coupling strength $g_c = 0.05$ and probability $p = 30\%$. The subnetworks

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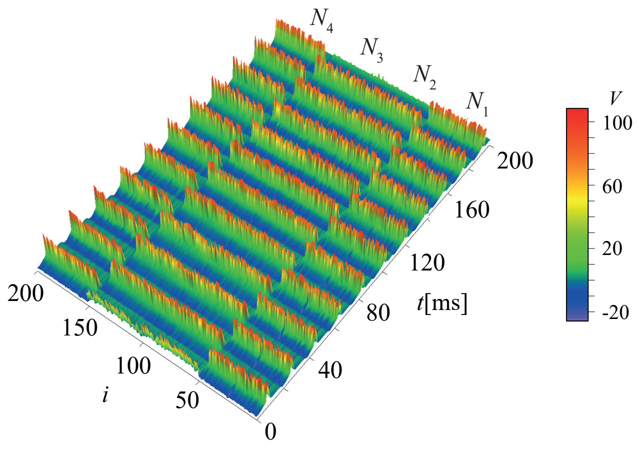

Fig. 1. Spatio-temporal diagram of the membrane potential V of neurons in the first N_1 ($i = 1, ..., 50$), the second N_2 ($i = 51, ..., 100$), the third N_3 ($i = 101, ..., 150$) and the fourth N_4 ($i = 151, ..., 200$) subnetwork.

are connected to each other by two-directional inhibitory couplings with coupling strength g_c^{ex} and probability $p = 30\%$. Inside them neurons are connected to each other according to "small-world" (SW) topology with coupling strength g_c^{in}.

We analyze the dynamics of the developed "network of networks". An example of a typical temporal implementation of the entire neural ensemble is shown in Fig. 1. Within each subnetwork, all neurons generate spikes at almost the same time due to the excitatory connection between them. Thus, the signals averaged over all neurons of each subnetwork will contain sequences of separate concentrated spikes.It can be seen that 4 subnets are divided into 2 clusters under the influence of the inhibitory connection between them: the first network is synchronized with the fourth, and the second with the third. Networks synchronized with each other generate spikes synchronously at the same time intervals, while the activity of networks from different clusters demonstrates antiphase dynamics. Thus, the two subnet groups are in constant out-of-phase with each other. Considering that these subnets are engaged in processing the signal received from a small input network, we can conclude that in order to achieve an optimal operation process, 2 groups of networks share the input information among themselves equally, in each group, the networks must be synchronous for efficient processing of the input signal.

It was found that the first subnet for the entire range of the considered values of the communication forces demonstrates almost zero correlation with all other subnets at low values of the inter-network communication strength, which decreases to $r = -0.2$ with an increase in inhibiting bonds.In this case, the other three subnets behave completely differently: with weak interconnection links, the correlation between their synchronization indices is close to 0, but with an increase in this connection, the correlation increases up to 1.0. It should also be noted that an increase in the strength of intra-network connections leads to an increase in the correlation between all networks. It has been shown that increasing couplings strength between neurons in a small input network affects synchronization in large networks. Given that the input network plays the role of processing low-level signals, communication between neurons in this network is necessary for efficient signal processing.

IV. Conclusions

A mathematical model of a "network of networks" was developed, consisting of a small input network and four large subnets. The external signal received by the input network was converted by it into a spike sequence, which was then transmitted to 4 subnets with a "small world" topology, interacting with each other through inhibitory communication, which interacted with each other to process the signal.

The dynamics of the developed network model was analyzed and it was shown that synchronization indices in subnets periodically fluctuate in time. They were found to exhibit either in-phase or anti-phase synchronization depending on the strength of the inhibitory coupling between the subnets. It is assumed that the mechanism underlying the antiphase dynamics is the redistribution of cognitive resources between neural ensembles in the brain.

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