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Noisy Signal Filtration Using Complex Wavelet Basis Sets

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Abstract—Methods of noisy signal filtration using a discrete wavelet transform (DWT) with real basis sets of the Daubechies family are compared to methods employing a double-density dual-tree complex wavelet transform (DDCWT) with excess (nonorthonormalized) basis sets. Recommendations concerning the choice of filter parameters for minimization of the error of noisy signal filtration are formulated.

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In recent years, methods of wavelet filtration have proved to be a reliable tool for digital processing of experimental data so as to rapidly clean noisy signals and images from additive noise and random fluctuations, including localized disturbances [1–5]. For this purpose, approaches based on orthonormalized wavelet basis sets such as functions of the Daubechies family, pyramid expansion algorithms, and subband coding [2] are used in various applications. These approaches have clear advantages, including high speed (providing online processing of audio and video signals) and signal expansion with a minimum number of coefficients, which provides more accurate representation and, which is especially important, correct reconstruction of signals after the filtration of noises [4]. However, progress in computing technologies led to a change in priorities. so that the quality of signal filtration (assessed by the root-mean-square (rms) error level [6, 7] or other criteria related to specific features of particular signals [8]) became important rather than the speed of data processing.

The aforementioned circumstances stimulated considerable interest in methods employing frames representing excess (nonorthonormalized) wavelet basis sets [2, 3]. These methods reduce distortions of the reconstructed informative signals in cases in which the filtration removes some essential coefficients of expansion or when the presence of significant background noise decreases the accuracy of signal presentation in the wavelet basis set [4]. Although various functions can be used as wavelet basis sets, complex wavelets are preferred such that eliminate main disadvantages of the standard method of filtration based on the discrete wavelet transform (DWT). These drawbacks include the absence of invariance with respect to shift of the basis set function, an oscillating character of expansion coefficients in the vicinity of singularities, and the appearance of artifacts in the signal reconstructed upon correction of the wavelet expansion coefficients [9]. It has been established [10, 11] that, for eliminating these disadvantages, it is effective to use complex functions with real and imaginary parts related through the Hilbert transform—that is, analytic or almost analytic wavelets. However, the choice of a "good" basis set does not guarantee that filtration based on this wavelet would ensure the reduction of error since the quality of signal cleaning from noise significantly depends on filter parameters such as the threshold-function setting and signal-to-noise ratio (SNR).

The present Letter puts an emphasis on tuning the parameters of wavelet filters employing complex basis sets and shows that their effective application requires proper control of the threshold level.

Traditionally, the wavelet coefficients are corrected by selecting one of two variants, "hard" and "soft," of threshold-function setting [6, 7]. In the former case of hard threshold setting, the coefficients not exceeding threshold C value are set zero, while in the case of soft

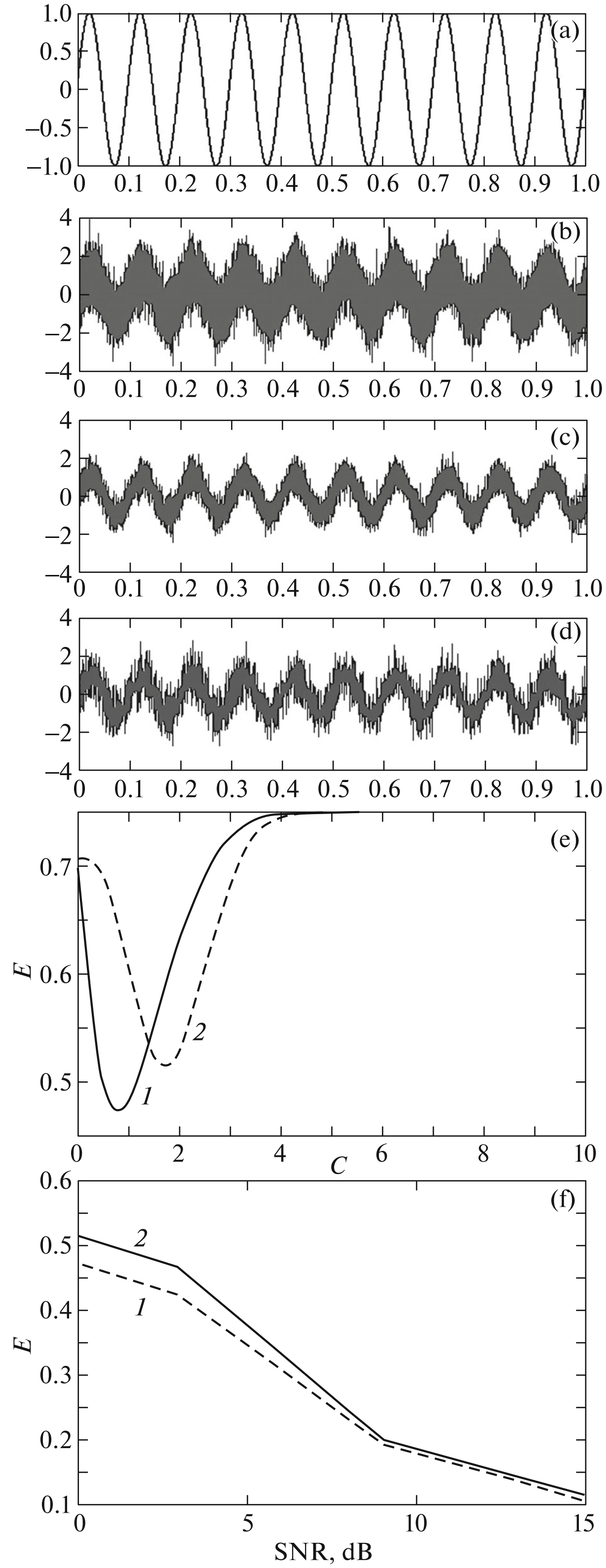

Fig. 1. Test-signal filtration based on a traditional DWT: (a) initial signal; (b) noisy signal (SNR = 0 dB); (c, d) signal upon filtration with soft and hard variants of threshold-function setting, respectively, for the D^{20} wavelet basis set of the Daubechies family); (e, f) rms error of filtration E vs. threshold level C and SNR, respectively, as calculated for (*1*) soft and (*2*) hard variants of threshold-function setting.

setting all coefficients are modified but to different degrees:

$$y(x) = \begin{cases} x - C, & x \geq C, \\ x + C, & x \leq -C, \\ 0, & |x| \leq C. \end{cases} \quad (1)$$

The latter variant of filtration allows us to avoid the $y(x)$ function discontinuities that lead to additional distortions of the reconstructed signal. According to conclusions that were drawn in previous investigations [7], the soft variant of threshold-function setting is preferred for the digital filtration of signals and images.

It is a more complicated task to select the optimum threshold level. Despite some recommendations being known, they have been mostly formulated for DWT-based filters, and their application to complex basis sets does not provide minimization of the filtration error. In order to illustrate this, let us consider the method of filtration based on the double-density dual-tree complex wavelet transform (DDCWT) [12, 13], which differs from the DWT in using two wavelet functions ψ_i with real and imaginary parts related through the Hilbert transform. As a result, "detailing" wavelet-expansion coefficients are completely retained upon scale change, while the "approximating" coefficients (in expansion over scaling functions φ) are two-fold rarefied. The scaling transformations are defined by the following relations:

$$\psi_{1,2}(t) = \sqrt{2}\sum_n h_{1,2}(n)\psi_{1.2}(2t - n), \quad h_2(n) = h_1(n-1),$$
$$\varphi(t) = \sqrt{2}\sum_n h_0(n)\varphi(2t - n), \quad (2)$$

where the filter coefficients are set according to tables calculated in [12].

For the comparative analysis of parameters ensuring the best quality of filtration, we have considered a test example of harmonic oscillations with additive white noise of large intensity (SNR = 0 dB). First, the model signal filtration was performed by a method based on the traditional DWT with Daubechies wavelets (Fig. 1). As can be seen, the quality of filtration using the soft variant of threshold-function setting is higher than that for the hard variant (cf. Figs. 1c, 1d). Calculations confirmed this conclusion both for the given example (Fig. 1e) and for other SNR values (Fig. 1f). These results are quite to be expected and agree with conclusions drawn in other investigations. However, one important circumstance to be noted is that the advantage of selecting threshold function (1) is only manifested for small C (fig. 1e). Once the threshold is chosen on a greater level, the situation dramatically changes and the hard variant of threshold-function setting will become preferred.

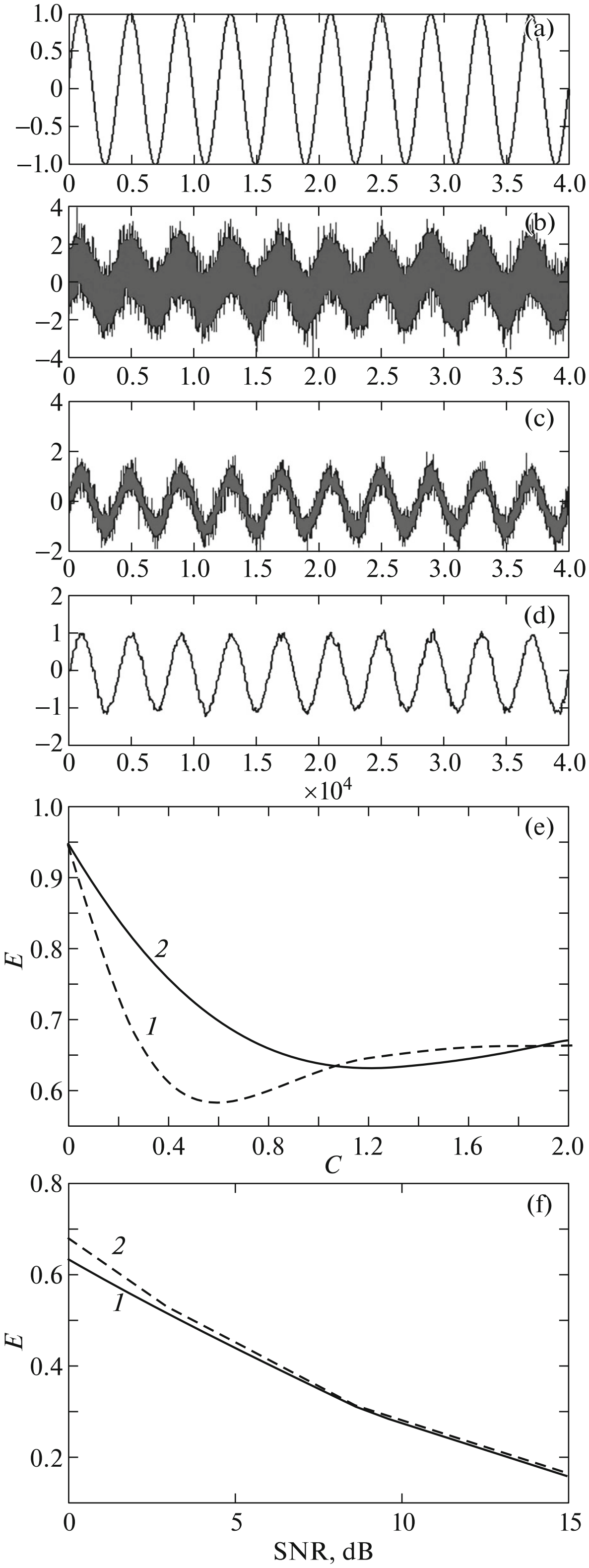

Fig. 2. Test-signal filtration based on a DDCWT: (a) initial signal; (b) noisy signal (SNR = 0 dB); (c, d) signal upon filtration using DWT (for the D^7 wavelet basis set of the Daubechies family and DDCWT, respectively; (e, f) rms error of filtration E vs. threshold level C and SNR, respectively, as calculated for (*1*) DDCWT and (*2*) DWT methods.

The calculation illustrated in Fig. 1 did not stipulate optimization of the choice of wavelet basis set in the Daubechies family. Now let us consider the next task and minimize the error of filtration by tuning the filter parameters (wavelet basis set and threshold level C). Figure 2 shows the obtained results, according to which a minimum rms error of 0.68 corresponds to the use of Daubechies wavelet set D^7 at a threshold level of $C = 1.22$. Note that, for the convenience of comparison, Fig. 2 (in contrast to Fig. 1) presents normalized values of the rms error of filtration.

Then, analogous calculations were performed for the same signal filtered by the DDCWT method. The quality of filtration can be visually assessed by comparing Figs. 2c and 2d, which shows the advantages of the algorithm employing the complex wavelet basis set. According to calculations performed for various threshold levels, the minimum rms error 0.63 was achieved for $C = 0.6$. Therefore, the DDCWT method not only provides for a decrease in the error of filtration, but also reduces the optimum threshold level approximately by half. Note that, if the threshold level were set according to the conventional recommendations developed for the DWT method (choice of a universal threshold, setting C using the SURE procedure, etc. [7]), the advantages of the DDCWT would not be provided and, moreover, the results could be even worse than those obtained by the standard DWT method. As can be seen from Fig. 2e, the DDCWT algorithm with a threshold optimum for the DWT ($C = 1.22$) leads to greater error of filtration. Note that the complex wavelet basis sets are especially effective at high noise levels, while the results of low noise filtration are comparable to those obtained by the standard DWT method (Fig. 2f).

Analogous comparison of the two filtration methods was also carried out for some other signals, in particular, for audio signals with additive noise. Despite individual specific features in the behavior of the error of filtration depending on the selected threshold level, the established qualitative correspondence was observed in all test examples. In particular, the general conclusions of the effectiveness of about twofold reduction in the optimum threshold level calculated for the DWT method with soft variant of threshold-function setting were confirmed. This decrease is among the important advantages of the DDCWT method, since it allows using a lower degree of correction of the most informative wavelet-expansion coefficients, thus reducing the risk of introducing accidental distortions at the stage of signal synthesis.

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