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A Discrete Time Model System with "Intermittent" Intermittency

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Abstract—A model system with discrete time is developed in which there are simultaneously two different types of intermittency: intermittency of type 1 and eyelet intermittency. Statistical characteristics for this type of behavior have been found. The calculation data are compared to the derived theoretical regularities. A good agreement between analytical and numerical results is obtained.

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Study of intermittent behavior has always been an intriguing challenge for researchers. Primarily, this is due to the fact that this phenomenon is typical for a wide class of systems, having a fundamental nature [1]. However, all works known to date (see, for example [2, 4]) have aimed at considering cases in which the system with fixed control parameters has two types of alternating behavior, i.e., in which one particular type of intermittency is realized. However, recently it was found that a situation may occur in a nonlinear system where, for fixed values of the control parameters, there are simultaneously two types of intermittency, i.e., where the so-called "intermittent" intermittency mode is implemented [5]. This situation is observed, in particular, in dynamic systems near the boundaries of emergence of synchronous modes on boundary time scales of observation (see, for instance, [5, 6]). However, in systems with discrete time, this phenomenon has yet to be found. This work aims at finding a discrete-time system that can simultaneously demonstrate two alternating types of behavior for fixed values of parameters. This problem is of great interest from the fundamental point of view, since these studies will make it possible to better understand and comprehend the mechanisms and the nature of the phenomena of intermittency and chaotic synchronization.

Searching for a model system with discrete time exhibiting the phenomenon of intermittent intermittency will be performed by constructing this system from the reference models of discrete mapping. For this purpose, we consider two mappings of the circle related to one another,

$$\begin{aligned} x_{n+1} &= x_n + 2\Omega(1-\cos x_n) - \varepsilon, \quad \mathrm{mod}\,2\pi, \\ y_{n+1} &= y_n + 2\Omega(1-\cos y_n) - \gamma + \kappa\cos(\alpha/x_n^3), \quad \mathrm{mod}\,2\pi, \end{aligned} \tag{1}$$

where ε, γ, Ω, α, κ are the control parameters of the system. It must be noted that the connection between the mappings of the circle is realized by means of addend $\kappa\cos(\alpha/x_n^3)$ being added in the second equation of system (1). Thus, a unidirectional bond is realized in system (1). In order to observe simultaneously two alternating types of behavior in this system, it is necessary to introduce new variable z_n,

$$z_n = \sqrt{x_n^2 + y_n^2}. \tag{2}$$

Variable z_n allows one to consider system (1) as a two-dimensional dynamic system with discrete time exhibiting simultaneously two types of alternating behavior. This is due to the fact that, if variable x_n is considered separately, it may exhibit one known type of intermittency (type 1), depending on the value of ε. When considering variable y_n separately and changing parameter γ, we can find that this variable can exhibit another type of intermittency, since it undergoes the action of additional signal $\kappa\cos(\alpha/x_n^3)$. Therefore, variable z_n, being determined by (2), depends on variables x_n and y_n and exhibits simultaneously two types of alternating behavior. From the above, it can be argued that system (1) with discrete time is capable to demonstrate the existence of two types of intermittent behavior.

We set the values of control parameters $\Omega = 0.1$, $\alpha = 0.1$, $\kappa = 0.025$, $\varepsilon = -0.0001$, and $\gamma = 0.0005$ in studying the dynamics of system (1). Figures 1a and 1b show temporal realizations of x_n and y_n, respectively. One can see that, for both variables, one can allocate two characteristic types of behavior: type 1, when the variable is close to zero, and type 2, when the variable rises sharply. Therefore, one can say that variables x_n and y_n demonstrate complex alternating behavior, where laminar regions correspond to nearly zero values of variables, while turbulent areas correspond to a sharp increase of these variables. From qualitative comparison of Figs. 1a and 1b, one can see that the types of intermittent behavior that exhibit variables x_n

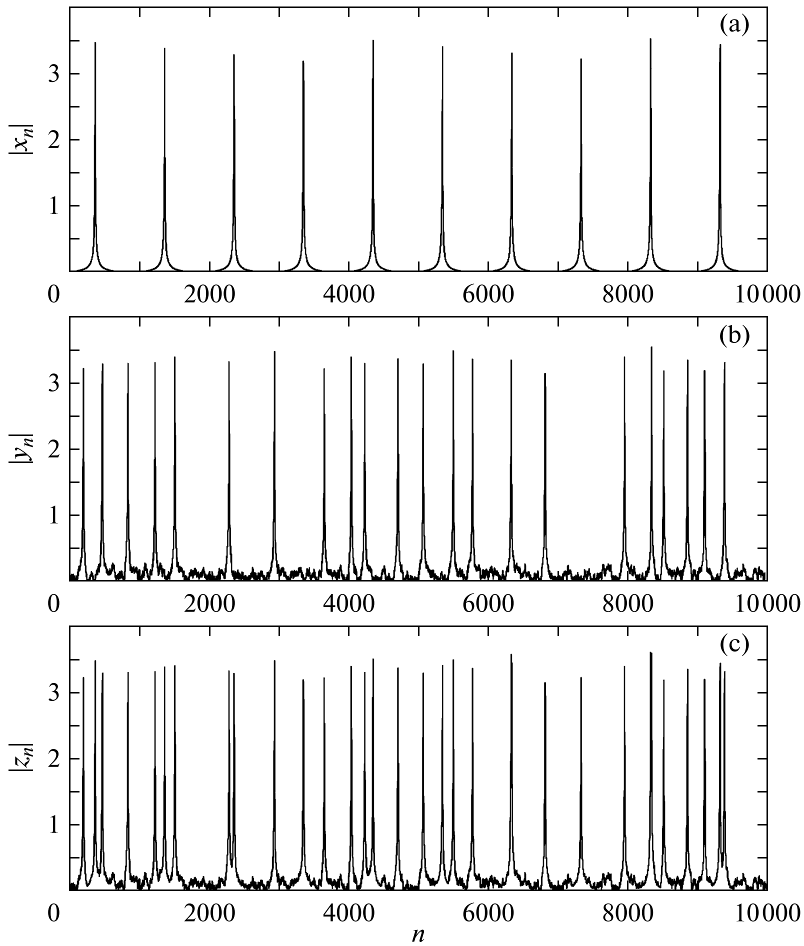

Fig. 1. Temporal dependences of (a) x_n, (b) y_n, and (c) z_n of system (1)–(2) in the (a, b) mode of intermittency and (c) coexistence mode of two different types of intermittency.

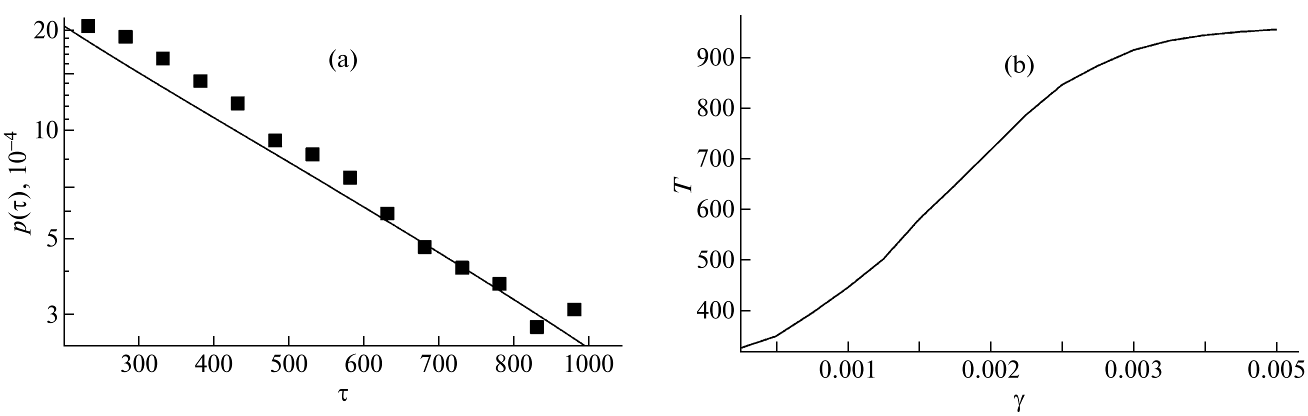

Fig. 2. Duration distribution of laminar phases at fixed values of the controlling parameters (a) and dependence of the average duration of laminar phase on parameter γ (b) for system (1)–(2). The numerical simulation data are shown by points; theoretical approximations corresponding to Eqs. (5) and (6) are shown by solid lines. The fitting parameters in Fig. 2 are $T_1 = 534$, $T_2 = 965$, and $x = 65$.

and y_n are different. Statistical analysis of the characteristics of these types of intermittent behavior shows that variables x_n and y_n exhibit type 1 of intermittency [2] and eyelet intermittency, respectively [4, 7] (the latter can be considered as type 1 noised intermittency [7]).

Using variable z_n determined by (2), let us consider the whole dynamics of system (1) with the aforementioned two types of intermittency (intermittency of type 1 and eyelet intermittency). Figure 1c shows the behavior of variable z_n with the same values of control parameters. It is evident that, as for variables x_n and y_n, a laminar phase is observed at close to zero values of variable z_n, while the moment of its sharp increase corresponds to the onset of the turbulent dynamic phase.

Based on the general theory of coexistence of two various types of intermittency [5], we obtain the theoretical dependence of the duration distribution of laminar phases for type 1 and eyelet intermittency. We take into account the fact that, in the mode of eyelet intermittency, the distribution of laminar phase duration is described by the exponential law [7]

$$p_1(\tau) = \frac{1}{KT_1}\exp\left(-\frac{\tau}{T_1}\right), \tag{3}$$

while, in the case of type 1 intermittency, it can approximately be described by the δ function [2]

$$p_2(\tau) = \delta(\tau - T_2), \tag{4}$$

where $K = e^{-x/T_1}$ is the normalization coefficient, x is the minimum value of laminar phase duration described by (3), and $T_{1,2}$ denotes the average durations of laminar phases for the two types of intermittency. The distributions of laminar phase duration when both type 1 and eyelet intermittency exist simultaneously then take the form

$$p(\tau) = \frac{\delta(\tau - T_2)\left(\exp\left[\frac{x-\tau}{T_1}\right]T_1 - \tau\exp\left[\frac{x}{T_1}\right]\Gamma\left[0, \frac{\tau}{T_1}\right]\right) + \exp\left[\frac{x-\tau}{T_1}\right]\left(\frac{T_1 + T_2 - \tau}{T_1} + \exp\left[\frac{\tau}{T_1}\right]\Gamma\left[0, \frac{\tau}{T_1}\right]\right)}{T_1 + T_2 + x\left(\exp\left[\frac{x}{T_1}\right]\mathrm{Ei}\left[-\frac{x}{T_1}\right] - 1\right)}, \tag{5}$$

where $\mathrm{Ei}(z) = -\int_{-z}^{\infty}(e^{-t}/t)dt$ is the exponential integral function and $\Gamma(a, z)$ is the incomplete γ function.

Substituting (5) into the expression for the average duration of the laminar phase $T = \int_x^{\infty}\tau\, p(\tau)d\tau$, we obtain the following expression for the average laminar phase duration in alternating cases of type 1 and eyelet intermittency:

$$T = \frac{\exp\left[\frac{x-T_2}{T_1}\right]\left(T_1\left(T_1\left(1 - \exp\left[\frac{T_2}{T_1}\right]\right) + T_2\left(1 + 2\exp\left[\frac{T_2}{T_1}\right]\right)\right) - T_2^2\exp\left[\frac{T_2}{T_1}\right]\left(\mathrm{Ei}\left[-\frac{T_2}{T_1}\right] - 2\Gamma\left[0, \frac{T_2}{T_1}\right]\right)\right)}{2\left(T_1 + T_2 + x\left(\exp\left[\frac{x}{T_1}\right]\mathrm{Ei}\left[-\frac{x}{T_1}\right] - 1\right)\right)}. \tag{6}$$

Let us analyze numerically the obtained statistical characteristics of variable z_n—namely, the duration distribution of laminar phases at fixed values of the control parameters and the average dependence of laminar phase duration on the supercriticality parameter. Figure 2a shows the duration distribution of laminar phases at the aforementioned values of the control parameters, while Fig. 2b shows the dependence of the average laminar phase duration on parameter γ. One can see that theoretical dependences agree well with the results of numerical modeling. This signifies the correctness of the proposed theory [5] in describing coexistence of alternating type 1 and eyelet intermittency in systems with discrete time. We developed a theory for these types of intermittent intermittency and obtained the distribution law of laminar phase duration and dependence of the average durations of laminar phases on the supercriticality parameter. The theoretical regularities have been compared to the results of numerical simulations, and good agreement between them is demonstrated.

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