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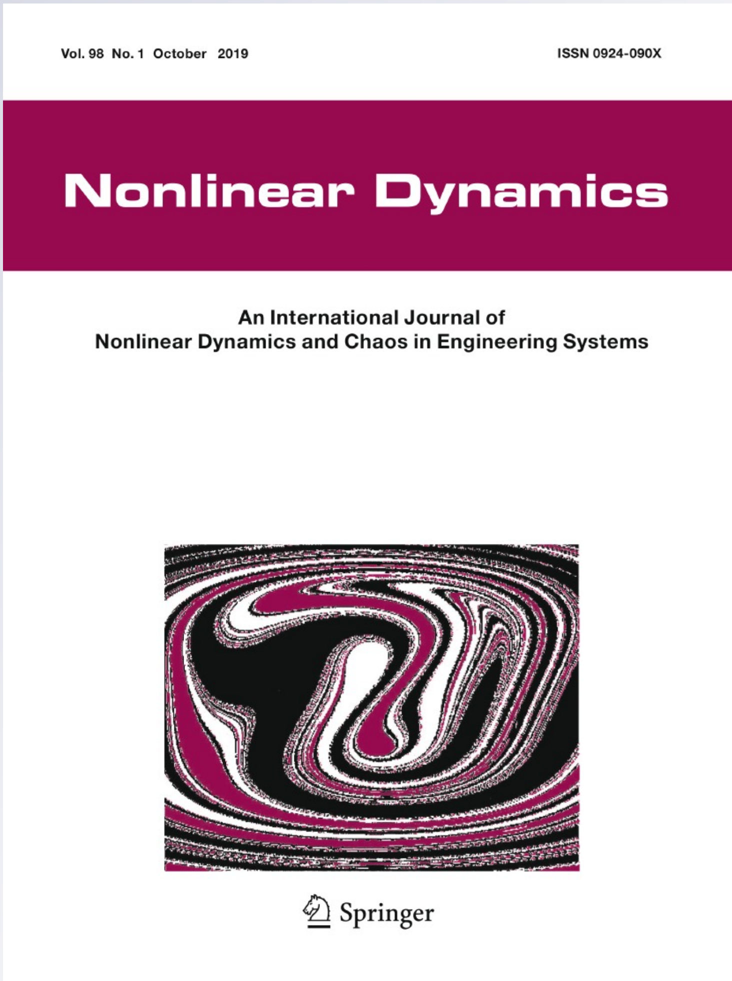
Nonlinear Dynamics
An International Journal of Nonlinear Dynamics and Chaos in Engineering Systems

ISSN 0924-090X
Volume 98
Number 1

Nonlinear Dyn (2019) 98:735-748
DOI 10.1007/s11071-019-05224-x



Springer

Nonlinear Dyn (2019) 98:735–748
https://doi.org/10.1007/s11071-019-05224-x

ORIGINAL PAPER

Synchronization in ensembles of delay-coupled nonidentical neuronlike oscillators

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Received: 14 May 2019 / Accepted: 28 August 2019 / Published online: 31 August 2019

Abstract We study both numerically and experimentally the synchronization in an ensemble of nonidentical neuronlike oscillators described by the FitzHugh–Nagumo equations. The cases of constant values of time-delayed couplings between the oscillators and adaptively controlled values of time-delayed couplings are considered. For the experimental study of the ensemble of neuronlike oscillators, we construct a radio engineering setup, in which the ability to specify both constant values and adaptively tuned values of couplings between the oscillators is implemented. Moreover, it is possible to specify an arbitrary architecture and type of dynamical couplings between oscillators in the setup. By the example of a system of two bidirectionally coupled nonidentical oscillators and a ring consisting of ten unidirectionally coupled nonidentical FitzHugh–Nagumo systems, it is shown that the using of an adaptively controlled time-delayed coupling allows one to achieve the in-phase synchronization of all oscillators in the ensemble even in the case of a large parameter mismatch. The results obtained in the physical experiment are in good agreement with the results of the numerical simulation.

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Keywords Ensembles of neuronlike oscillators · Time-delayed coupling · Synchronization · Adaptive control

1 Introduction

The study of synchronization in ensembles of coupled oscillators attracts the attention of many researchers in various scientific disciplines [1–3]. The adjustment of rhythms of self-sustained oscillators due to their interaction is typical in many natural and human-made nonlinear oscillators. In particular, many biological and physiological systems exhibit the ability to synchronize. In some cases, synchronization plays a positive role; for example, it is necessary for the realization of motor activity [4,5] and functioning of the internal organs of the body [6–8], while in other cases synchronization has a negative effect. For example, abnormal synchronization of brain neurons can lead to epilepsy [9–11], schizophrenia [12], and Parkinson's disease [13].

The study of synchronization is of special interest in neural networks, which are used to model the processes of interaction of brain neurons [14–16]. To simulate the activity of individual neurons, models in the form of nonlinear dynamical systems are widely used. The most famous among them are the Hodgkin–Huxley model [17], FitzHugh–Nagumo model [18,19], Morris–Lecar model [20], and Hindmarsh–Rose model [21]. Since the nerve impulses between brain neurons propagate at a finite speed, and the time of information exchange between neurons is comparable to the characteristic period of their oscillations, it is necessary to take into account the delay in couplings between the neural network elements [22–24]. Along with the topology and intensity of couplings between the network oscillators, the presence of delay in couplings has a great influence on the occurrence of synchronization in the network [25–27].

The phenomenon of synchronization was studied in most detail for the case of identical oscillators, the coupling between which is constant. However, in real networks, oscillators usually differ in parameters and, therefore, have different frequencies and amplitudes of oscillations. The parameter mismatch impedes the synchronization of oscillators [28,29]. Moreover, a more realistic situation is when couplings between oscillators in an ensemble do not remain constant, but change over time [30–32]. Therefore, when studying the synchronization in neural networks, it is important to simultaneously take into account the nonidentity of oscillators, the presence of delay in couplings between them, and the change of couplings in time.

Control of synchronization in networks of coupled oscillators is an important task for many applications [33]. Various methods have been proposed for its solution, including the methods of adaptive control of synchronization, which have been greatly developed in recent years [34–38]. Such methods allow changing the control parameters depending on the state of the oscillator or external disturbances acting on it. The methods of adaptive control can not only adjust the strength of couplings between the network elements, but also change the topology and direction of couplings [39]. Some methods of adaptive synchronization use pinning control, in which the control action is applied not to all network oscillators, but only to a small fraction of all nodes [40–42].

Most of the methods for adaptive control of synchronization have been applied to model networks of coupled identical oscillators. Only a small number of papers are devoted to adaptive control of synchronization in model ensembles of heterogeneous oscillators [43–45]. At the same time, the need for adaptive control of collective dynamics of coupled nonidentical neurooscillators also arises in real networks. For example, when developing central pattern generators in robotics, it is important to ensure the synchronization of ensemble elements in a wide range of control parameters [46]. In this paper, for the first time, we study experimentally the adaptively controlled synchronization in a large ensemble of nonidentical oscillators.

To control synchronization in a physical experiment, we build an original radio engineering setup consisting of nonidentical neuronlike oscillators described by the delay-coupled FitzHugh–Nagumo equations. The type of dynamical couplings necessary for controlling the synchronization of oscillators is specified in the experimental setup using the LabView programming language, which makes it possible to easily and quickly vary the couplings in real time. The setup implements a simple linear coupling between electronic oscillators, simulating an electrical synaptic coupling between neurons. In analog modeling, this coupling corresponds to the coupling of two electronic oscillators via a resistor [47,48].

Note that, in addition to resistors, elements such as a capacitor, an inductor, or a memristor can be used for coupling of electronic oscillators. As was recently shown using the circuit simulator Multisim, Chua systems [49] can be synchronized using electric field coupling via a capacitor [50] or magnetic field coupling via an induction coil [51]. It was also shown numerically that FitzHugh–Nagumo neuroscillators can be synchronized if they are coupled using memristors [52,53]. The neuroscillators described by the Hindmarsh–Rose equations are also synchronized via a memristor-based coupling [54–58].

Our original approach to the experimental study of large ensembles of coupled oscillators allows one to specify almost any type of couplings between the oscillators. Since the signals responsible for the coupling of electronic oscillators are generated programmatically in the experimental setup, it is possible to couple the network oscillators via a resistor, capacitor, inductor, or memristor, making appropriate changes to the program. As the base oscillator of the ensemble, one can use various oscillators with an arbitrary architecture of couplings between them. Thus, the proposed approach is promising for the experimental study of synchronization in various networks of coupled oscillators.

The paper is organized as follows: Sect. 2 describes the model system and its experimental implementation with electronic oscillators. In Sect. 3, we investigate both numerically and experimentally the synchronization of two bidirectionally delay-coupled nonidentical FitzHugh–Nagumo systems for the cases of constant coupling and adaptively controlled coupling and construct a partition of the parameter plane of the coupled system into the regions of typical oscillation regimes. In Sect. 4, the possibility of adaptive control of the in-phase oscillation regimes in a ring of ten unidirectionally coupled nonidentical FitzHugh–Nagumo systems is demonstrated in both numerical and physical experiments. Section 5 contains a discussion of the obtained results and a conclusion.

2 Model and experimental systems

As the base element of the ensemble, we consider a neuronlike oscillator described by the simplified FitzHugh–Nagumo differential equations [22,45]:

$$\begin{aligned}\varepsilon\dot{u}(t) &= u(t) - \frac{u^3(t)}{3} - v(t),\\ \dot{v}(t) &= u(t) + a,\end{aligned} \tag{1}$$

where $u(t)$ and $v(t)$ denote the activator and inhibitor variables, respectively, ε is the time-scale parameter, which is usually a small value, and a is the threshold parameter. The FitzHugh–Nagumo equations are the standard model for excitable dynamics of neurons. For $a > 1$, the oscillator (1) is in a locally stable equilibrium point and is excitable, while for $a < 1$ it exhibits self-sustained periodic firing beyond the Hopf bifurcation at $a = 1$. Oscillations of the variable $u(t)$ qualitatively reproduce the spikes generated by real neurons.

Due to its simplicity, on the one hand, and an adequate reflection of the basic properties of a neuron, on the other hand, the FitzHugh–Nagumo model is very popular among researchers. As a model of excitable dynamics, the FitzHugh–Nagumo model is also used to describe the dynamics of a number of other systems, such as a tunnel diode [59] and cardiac tissue [60]. Furthermore, it can be quite simply implemented in a radio physical experiment [19,61–63]. Using the ideology of analog modeling, we proposed a radio engineering circuit for the experimental implementation of the FitzHugh–Nagumo system (1), which differs from the experimental circuits [19,61–63] constructed for another form of FitzHugh–Nagumo equations. A schematic diagram of the constructed FitzHugh–Nagumo electronic oscillator is shown in Fig. 1.

The circuit contains two operational amplifiers U4B and U3A that play the role of integrators. Their output voltages are denoted as U and V, respectively. The cubic transformation is performed by multipliers U1 and U2. A repeater is implemented using the operational amplifier U3B, and an amplifier with a gain of -1 is implemented using the operational amplifier U4A, so that we have $-U$ at its output. The output signal U is fed to the output Out1, and the output signal V is fed to the output Out2. The potentiometer $R10$ voltage is equal to A. The time-scale parameter takes the value $\varepsilon_{\rm ex} = R7C2$. The coupling of the considered base oscillator with other oscillators in the ensemble is realized by applying a coupling signal to the input In of the operational amplifier U4B.

The dynamics of the considered electronic oscillator is described by the dimensionless equations (1), in which $u = \frac{U}{u'}$, $v = \frac{V}{v'}$, $a = \frac{A}{a'}$, $t = \frac{T}{t'}$, and $\varepsilon = \frac{\varepsilon_{\rm ex}}{t'}$, where $u' = 1$ V, $v' = 1$ V, $a' = 1$ V, $t' = R13C1 = 1$ ms, the experimental voltages U, V, and A are measured in volts, and the time T in the experiment is measured in ms.

An experimental study of a network consisting of real radio engineering oscillators is a more difficult task than modeling analog circuits using circuit simulators [50,51,54]. However, such a study allows one to check the robustness of the results in a real radio physical experiment, in which noises are inevitably present, not all of which can be adequately taken into account in the simulation. Moreover, the proposed circuit contains analog multipliers and operational amplifiers, for the description of which approximate mathematical models are used in circuit simulators. Even simple circuit elements, such as resistors and capacitors, have parameters whose values may differ from the nominal ones and may vary during the real experiment. These factors increase the significance of the experiment carried out using a real radio engineering setup.

Let us consider an ensemble consisting of a ring of unidirectionally coupled neuronlike oscillators (1), with each oscillator described by the following equations:

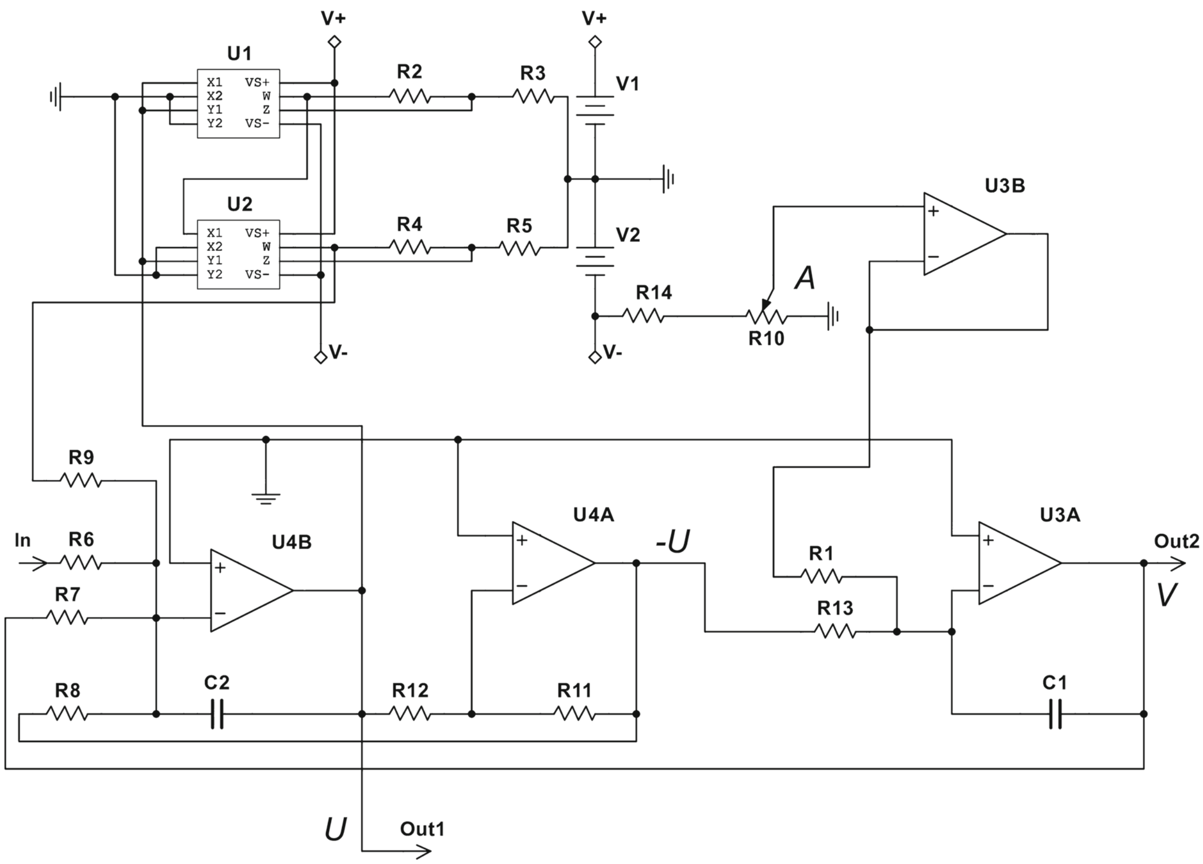

Fig. 1 Schematic diagram of the FitzHugh–Nagumo electronic oscillator. U1 and U2 are analog multipliers, U3A, U3B, U4A, and U4B are operational amplifiers, and V1 and V2 are DC voltage sources. Other elements are capacitors $C1 = 10$ nF and $C2 = 1$ nF, resistors $R1 = R6 = R7 = R8 = R9 = R11 = R12 = R13 = 100$ kΩ, $R2 = R4 = 1$ kΩ, $R3 = 9$ kΩ, $R5 = 2$ kΩ, and $R14 = 5$ kΩ, and potentiometer $R10 = 1$ kΩ

$$
\begin{aligned}
\varepsilon \dot{u}_i(t) &= u_i(t) - \frac{u_i^3(t)}{3} - v_i(t) \\
&\quad + C_i(t)\left(u_{(i+1) \bmod N}(t-\tau) - u_i(t)\right), \\
\dot{v}_i(t) &= u_i(t) + a_i,
\end{aligned} \tag{2}
$$

where $i = 1, \ldots, N$, with N being the number of oscillators, τ is the delay, which characterizes the time the signal needs to propagate between the two neighbor oscillators in the ring, and $C_i(t)$ describes the strength of coupling. In general case, all oscillators in the ensemble are nonidentical. Note that the time τ in the dimensionless equations (2) and the delay time τ_{ex} in coupling of the experimental electronic oscillators are related by the following equation: $\tau = \frac{\tau_{\text{ex}}}{t'}$. Neurooscillators (2) are coupled by a linear coupling, which models the electrical synaptic coupling between neurons. In analog modeling, this coupling corresponds to the coupling of electronic oscillators via a resistor [47,48].

In the presence of coupling between the FitzHugh–Nagumo systems, they can exhibit oscillations even in the excitable state with $a_i > 1$. If oscillators (2) are nonidentical, their complete synchronization in the form $(u_1, v_1) = \cdots = (u_N, v_N)$ is unattainable, but a situation is possible in which the oscillations of all elements in the ensemble are close to each other. This situation corresponds to the in-phase synchronization of oscillators, in which the phase shift between their oscillations is close to 0, but the amplitudes of oscillations are different.

For the experimental study of the system (2), we constructed an original radio engineering setup, the block diagram of which is depicted in Fig. 2.

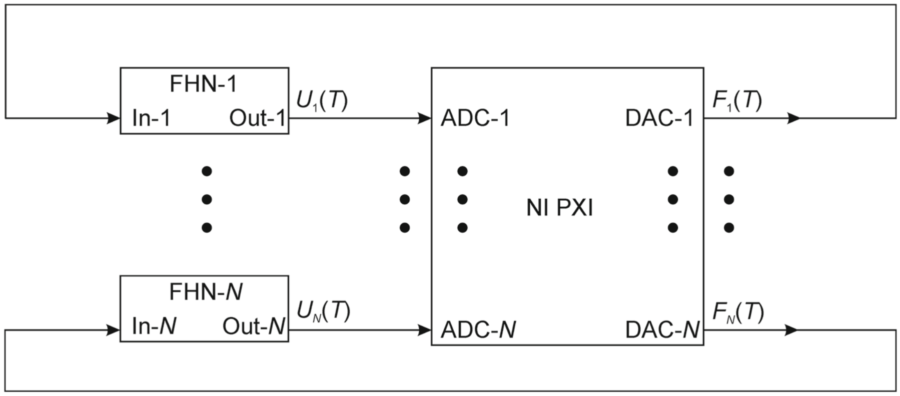

Fig. 2 Block diagram of the experimental setup. The first (FHN-1) and Nth (FHN-N) FitzHugh–Nagumo electronic oscillators are shown. Out-1 and Out-N are the oscillator output signals corresponding to the variables $U_1(T)$ and $U_N(T)$, respectively. In-1 and In-N are the oscillator inputs to which the signals $F_1(T)$ and $F_N(T)$, respectively, responsible for the coupling are fed. NI PXI is the National Instruments PXI system. Its analog inputs are denoted as ADC-1 and ADC-N, and the analog outputs are denoted as DAC-1 and DAC-N

For coupling between the FitzHugh–Nagumo electronic oscillators, the schematic diagram of which is shown in Fig. 1, a National Instruments PXI multichannel input-output system was used, which includes a chassis, a controller, an analog input block, and an analog output block. The signals $U_i(t)$ from the output of each FHN-i oscillator are fed to the analog inputs of the multichannel analog-to-digital converter (ADC) and are digitized at a frequency $f = 100$ kHz for further processing. Then, nonlinear conversion of the signals $U_i(t)$ is carried out using the LabView programming language and the signals $F_i(T) = C_i(T)\left(U_{(i+1) \bmod N}(T - \tau_{ex}) - U_i(T)\right)$ are generated that are responsible for the coupling of oscillators. From the outputs of the multichannel digital-to-analog converter (DAC), these signals $F_i(T)$ are fed to the inputs of oscillators.

The constructed experimental setup allows us to specify an arbitrary architecture of couplings between the oscillators and ensure the adaptive adjustment of coupling signals $F_i(T)$ during the experiment. Since the signals responsible for the coupling of oscillators are generated programmatically in the setup, it is possible to implement almost any type of couplings between the oscillators, including the coupling via a resistor, capacitor, inductor, or memristor. The proposed approach allows one to study experimentally the large ensembles of coupled oscillators, including the ensembles with a complex architecture of couplings. Other oscillators can also be used as the base oscillator of the ensemble.

3 Synchronization of two bidirectionally coupled FitzHugh–Nagumo systems

First, we consider the simplest case of two delay-coupled FitzHugh–Nagumo systems, corresponding to the case $N = 2$ in Eqs. (2). Figure 3 shows the partition of the parameter plane (A_1, A_2) of the experimental system (2) into the regions of typical oscillation regimes at $\varepsilon_{ex} = 0.1$ ms, $\tau_{ex} = 1.5$ ms, and constant coupling coefficients $C_1(T) = C_2(T) = 0.3$. The experimentally determined boundaries of the transitions between the regimes are marked by dots through which solid lines are drawn for clarity.

In the region denoted by LL, the first and second oscillators exhibit qualitatively similar oscillations with close amplitudes. With the chosen parameter values, the oscillators perform anti-phase oscillations in the region LL. In Fig. 3, the region of this anti-phase synchronization is denoted by LL_{AP}. Figure 3 also shows the typical time series of $U_1(T)$, $U_2(T)$, $V_1(T)$, and $V_2(T)$ in the areas marked by boxes in the parameter plane (A_1, A_2). The time series of $U_1(T)$ and $U_2(T)$ in the region LL_{AP} are shifted in phase by approximately π, as well as the time series of $V_1(T)$ and $V_2(T)$.

When $A_1 \neq A_2$, the amplitudes of $U_1(T)$ and $U_2(T)$ in the region LL_{AP} are different as well as the ampli-

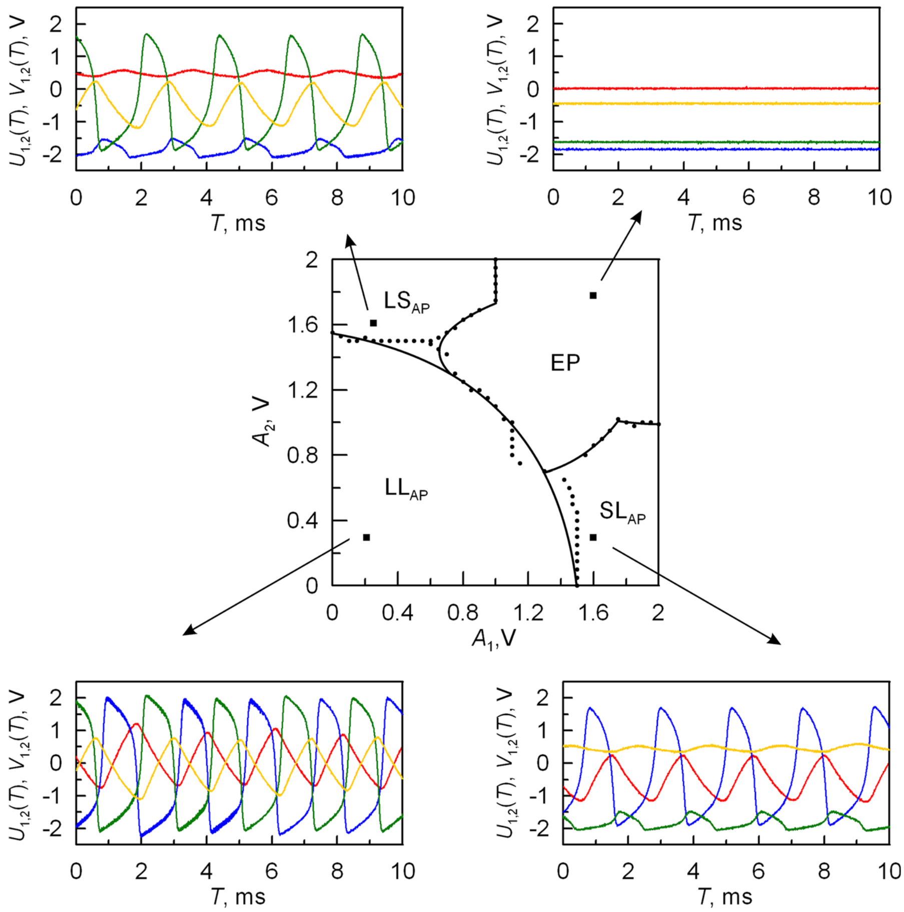

Fig. 3 Regions of typical oscillation regimes in the plane (A_1, A_2) and time series of $U_1(T)$ (green), $U_2(T)$ (blue), $V_1(T)$ (yellow), and $V_2(T)$ (red) in the system of two delay-coupled FitzHugh–Nagumo electronic oscillators for the case of constant strength of coupling. (Color figure online)

tudes of $V_1(T)$ and $V_2(T)$. As the mismatch of A_1 and A_2 increases, the difference in the amplitudes of oscillations of the first and second oscillators becomes more pronounced. Let $A_1 = A_2 = A$, where $A < 1$. We will increase A_2 under the constant $A_1 = A$. In this case, the amplitudes of $U_2(T)$ and $V_2(T)$ smoothly decrease. Finally, for a certain critical value of A_2, an abrupt decrease in the amplitude of oscillations of the second oscillator takes places.

We introduce the parameter $r = (U_{1\max}(T) - U_{1\min}(T))/(U_{2\max}(T) - U_{2\min}(T))$, where $U_{i\max}(T)$ and $U_{i\min}(T)$ are the maximum and minimum values of $U_i(T)$, respectively, and denote by LS the region in the plane (A_1, A_2) in which $r > 3$. At all points of the region LS, the first oscillator is in the oscillatory regime and has large amplitude, while the second oscillator is in the excitable regime and has small amplitude of oscillations. In the region LS in Fig. 3, the oscillators perform non-phase oscillations that are similar to anti-phase ones. This region of anti-phase synchronization is denoted by LS$_{AP}$ in Fig. 3.

Similarly, if we fix the value of $A_2 = A$, where $A < 1$, and will increase A_1, then at a certain critical value of A_1, a sharp decrease in the amplitude of oscillations of the first oscillator takes place. We denote by SL the region in the parameter plane (A_1, A_2) in which $r < 1/3$. At all points of the region SL, the first oscillator is in the excitable regime and has small amplitude of oscillations, while the second oscillator is in the oscillatory regime and has large amplitude. As seen from the time series, the oscillators in this region exhibit non-phase oscillations that are similar to anti-phase ones. In Fig. 3, this region of anti-phase synchronization is marked as SL$_{AP}$. For small constant values of coupling coefficients C_1 and C_2, the location of the regions of non-phase regimes in the parameter plane (A_1, A_2) remains almost unchanged in a wide range of τ_{ex} values.

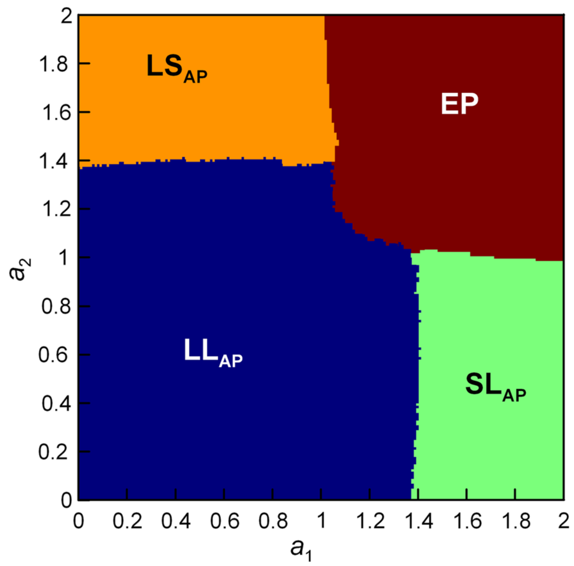

Fig. 4 Regions of typical oscillation regimes in the plane (a_1, a_2) in the model system of two delay-coupled FitzHugh–Nagumo systems for the case of constant strength of coupling. (Color figure online)

In the region denoted by EP, both oscillators are in the stable equilibrium point. As seen in Fig. 3, the values of the variables in this region remain constant in time. Note that in [64] we studied the oscillation regimes in the experimental system of two delay-coupled FitzHugh–Nagumo electronic oscillators with higher values of coupling coefficients, at which there was a bistability in the region LL in the parameter plane (A_1, A_2). In this region of bistability, the anti-phase oscillations $\mathrm{LL_{AP}}$ coexisted with the in-phase oscillations $\mathrm{LL_{IP}}$.

Figure 4 shows the partition of the parameter plane (a_1, a_2) of the model system of two delay-coupled FitzHugh–Nagumo systems (2) into the regions of characteristic regimes at $\varepsilon = 0.1$, $\tau = 1.5$, and constant coupling coefficients $C_1(t) = C_2(t) = 0.3$. The model parameter values are chosen the same as in the considered above case of the experimental system. In the regions $\mathrm{LL_{AP}}$, $\mathrm{LS_{AP}}$, $\mathrm{SL_{AP}}$, and EP in Fig. 4, the oscillation regimes are similar to the regimes observed in the corresponding regions depicted in Fig. 3. For clarity, the regions of different regimes are shown in Fig. 4 by different colors. The experimental parameter plane (A_1, A_2) and the model parameter plane (a_1, a_2) presented in Figs. 3 and 4, respectively, have a qualitatively similar structure.

In a number of problems, for example, when developing central pattern generators in robotics [46], it is important to ensure the in-phase synchronization of ensemble elements in a wide range of control parameters. To achieve this goal, we will vary the coupling strength $C_i(t)$ in the system (2) according to the adaptive law proposed in [45] based on the speed gradient method [65]. The idea of the speed gradient method is that for a nonlinear dynamical system $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{g}, t)$, where $\mathbf{g}$ are the control variables, a control goal $Q(\mathbf{x}, t) \leq \Delta$ for $t \geq t^*$ is defined, where $Q(\mathbf{x}, t)$ is a smooth scalar goal function and Δ is the desired level of precision. Then, the function $\dot{Q} = \omega(\mathbf{x}, \mathbf{g}, t)$ is calculated that is the speed at which $Q(\mathbf{x}, t)$ is changing along the trajectories of the dynamical system:

$$\omega(\mathbf{x}, \mathbf{g}, t) = \frac{\partial Q(\mathbf{x}, t)}{\partial t} + [\nabla_{\mathbf{x}} Q(\mathbf{x}, t)]^{\mathrm{T}} \mathbf{F}(\mathbf{x}, \mathbf{g}, t). \tag{3}$$

The gradient of function (3) with respect to the variables $\mathbf{g}$ is evaluated as

$$\nabla_{\mathbf{g}}\omega(\mathbf{x}, \mathbf{g}, t) = \nabla_{\mathbf{g}} [\nabla_{\mathbf{x}} Q(\mathbf{x}, t)]^{\mathrm{T}} \mathbf{F}(\mathbf{x}, \mathbf{g}, t). \tag{4}$$

As a result, the control function takes the following form: $\mathbf{g}(t) = \mathbf{g}^0 - \psi(\mathbf{x}, \mathbf{g}, t)$, where $\psi(\mathbf{x}, \mathbf{g}, t) = \gamma \nabla_{\mathbf{g}}\omega(\mathbf{x}, \mathbf{g}, t)$, γ characterizes the strength of adaptive coupling, and $\mathbf{g}^0 = \text{const}$ is an initial control value [45,65]. Several analytic conditions exist, guaranteeing that the control goal $Q(\mathbf{x}, t)$ can be achieved in the system. The main condition is the existence of a constant value of the parameter $\mathbf{g}^*$, ensuring attainability of the goal in the system $\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{F}(\mathbf{x}, \mathbf{g}^*, t)$ [45].

The term $-\nabla_{\mathbf{g}}\omega(\mathbf{x}, \mathbf{g}, t)$ points to the direction in which the value of $\dot{Q}$ decreases with the highest speed. If one changes $\mathbf{g}$(t) in this direction, the value of $\dot{Q}$ will decrease and finally become negative. When $\dot{Q} < 0$, the goal function $Q(\mathbf{x}, t)$ will decrease, tending to zero.

As it is mentioned in Sect. 2, if oscillators (2) are nonidentical, their complete synchronization in the form $(u_1, v_1) = \cdots = (u_N, v_N)$ is unattainable. However, it is possible to ensure the in-phase synchronization of oscillators, in which the phase shift between their oscillations is close to 0, but the amplitudes of oscillations are different. To achieve the in-phase synchronization of two bidirectionally coupled nonidentical FitzHugh–Nagumo oscillators (2), the following goal function is introduced:

$$Q(\mathbf{x}, t) = \frac{1}{2}(u_1(t) - u_2(t) + a_1 - a_2)^2. \tag{5}$$

The choice (5) ensures that the coupled system follows trajectories for which, for $t \geq t^*$, the following conditions are satisfied:

$$u_1(t) - u_2(t) \approx -a_1 + a_2,$$
$$v_1(t) - v_2(t) \approx c, \tag{6}$$

where c is a constant. Thus, the goal function (5) yields synchronization with a shift in the values of the inhibitor and activator variables of the two oscillators.

Using the speed gradient method with $\mathbf{g} = (C_1, C_2)$, the following adaptive law for changing the coupling coefficients was obtained in [45]:

$$C_{1,2}(t) = C^0_{1,2} + \frac{\gamma}{\varepsilon}\big(u_{1,2}(t) - u_{2,1}(t) + a_{1,2} - a_{2,1}\big)\big(u_{1,2}(t) - u_{2,1}(t-\tau)\big), \tag{7}$$

where $C_1(t)$ and $C_2(t)$ describe the adaptive coupling for the first and second oscillators, respectively, $C^0_{1,2}$ are the initial values of coupling coefficients, and γ characterizes the strength of adaptive coupling. At $\gamma = 0$, we have a constant strength of coupling. The possibility of achieving the in-phase synchronization of two nonidentical FitzHugh–Nagumo oscillators has been shown numerically in [45] using the adaptive tuning of the coupling strength (7). We have shown that the approach proposed in [45] can be successfully applied to achieve the in-phase synchronization of nonidentical FitzHugh–Nagumo electronic oscillators in a radio physical experiment.

Figure 5 shows the partition of the parameter plane (A_1, A_2) of the experimental system (2) into the regions of typical oscillation regimes at $\varepsilon_{\text{ex}} = 0.1$ ms, $\tau_{\text{ex}} = 1.5$ ms, and the coupling strength varying according to (7) with $C^0_1 = C^0_2 = 0$ and $\gamma = 0.02$. From the comparison of Figs. 3 and 5, it is seen that the use of adaptive coupling has led to the disappearance of the anti-phase oscillation regimes LL_{AP}, LS_{AP}, and SL_{AP}. Instead of them, the in-phase oscillation regimes LL_{IP}, LS_{IP}, and SL_{IP} appeared in approximately the same regions of the parameter plane (A_1, A_2). The time series of these in-phase oscillation regimes are also shown in Fig. 5. The phase shift between $U_1(T)$ and $U_2(T)$ is close to 0, but the amplitudes of $U_1(T)$ and $U_2(T)$ are different. The difference $U_1(T) - U_2(T)$ is approximately equal to $A_2 - A_1$. Thus, the use of adaptively controlled time-delayed coupling allows us to solve the problem of achieving the in-phase synchronization of nonidentical oscillators. Note that for small γ values ($\gamma < 0.01$), the in-phase synchronization of oscillators can be achieved not for every choice of initial conditions.

Let us consider the change in the dynamics of the experimental system (2) at the transition from the constant time-delayed coupling to the adaptively controlled time-delayed coupling. The experimental time series of $U_1(T)$, $U_2(T)$, $V_1(T)$, and $V_2(T)$ are presented in Fig. 6 for $A_1 = 0.5$ V and $A_2 = 0.7$ V. At $T < 3$ ms, the oscillators are coupled by the delayed coupling, the value of which remains constant, $C_1(T) = C_2(T) = 0.3$. The oscillators exhibit anti-phase oscillations at $T < 3$ ms. At $T = 3$ ms, the coupling is switched from the constant value to the adaptive coupling (7) with the parameters $C^0_1 = C^0_2 = 0$ and $\gamma = 0.02$. After a short transient process, the duration of which is about 1–2 characteristic periods of oscillations, the in-phase regime takes place in the coupled system.

Figure 7 shows the partition of the parameter plane (a_1, a_2) of the model system (2) into the regions of typical oscillation regimes at $\varepsilon = 0.1$, $\tau = 1.5$, and the coupling strength varying according to the adaptive law (7) with $C^0_1 = C^0_2 = 0$ and $\gamma = 0.02$. The experimental and model parameter planes in Figs. 5 and 7, respectively, have a qualitatively similar structure. Note that in this paper, for the first time, we have constructed the partition of the parameter plane (a_1, a_2) of the model system of two delay-coupled FitzHugh–Nagumo oscillators into the regions of characteristic regimes. In [45], the adaptive control of in-phase synchronization of two nonidentical FitzHugh–Nagumo oscillators has been shown numerically only for some fixed values of the parameters a_1 and a_2.

4 Synchronization of FitzHugh–Nagumo systems coupled in a ring

Let us consider a ring ensemble consisting of a large number of unidirectionally coupled nonidentical FitzHugh–Nagumo systems (2). In the case of a constant strength of coupling, non-phase oscillation regimes prevail in the ring. Figure 8a, b shows the time series of $u_i(t)$ and $v_i(t)$, respectively, for all oscillators of the model system consisting of ten nonidentical oscillators (2) with the constant time-delayed coupling $C_1(t) = \cdots = C_N(t) = 0.3$ and the parameters $\varepsilon = 0.1$ and $\tau = 1.5$. The model parameters a_i are specified as follows: $a_i = 0.1 + 0.05i$. As seen

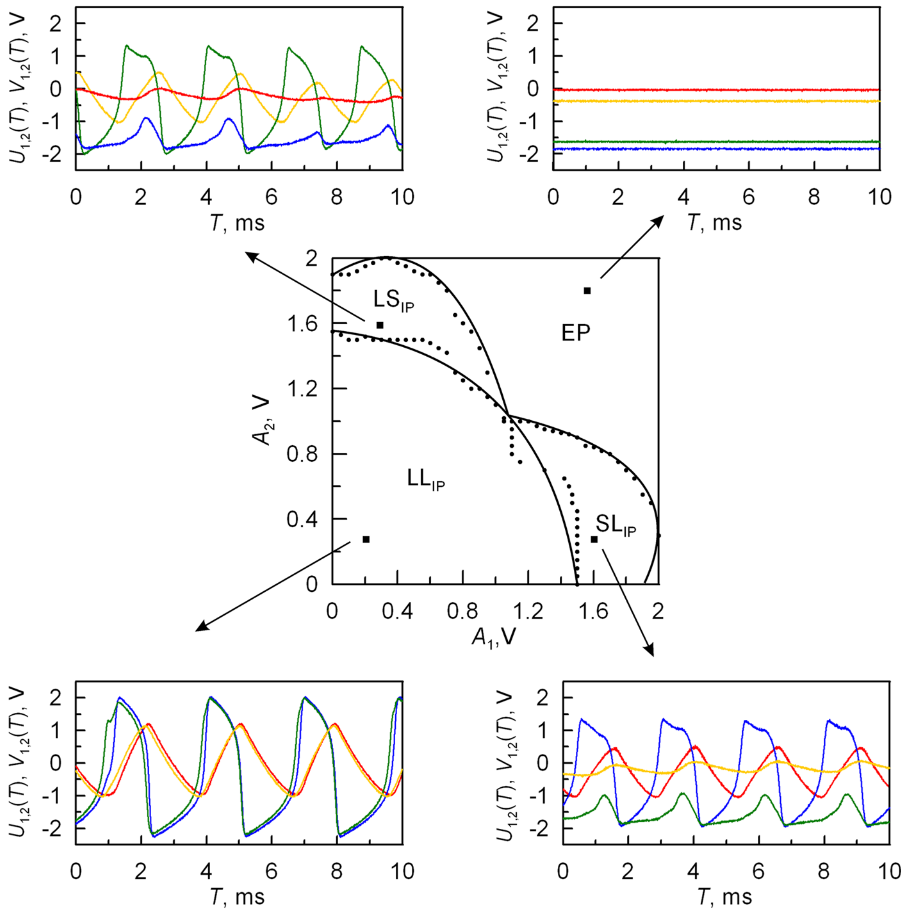

Fig. 5 Regions of typical oscillation regimes in the plane (A_1, A_2) and time series of $U_1(T)$ (green), $U_2(T)$ (blue), $V_1(T)$ (yellow), and $V_2(T)$ (red) in the system of two delay-coupled FitzHugh–Nagumo electronic oscillators for the case of adaptively controlled coupling. (Color figure online)

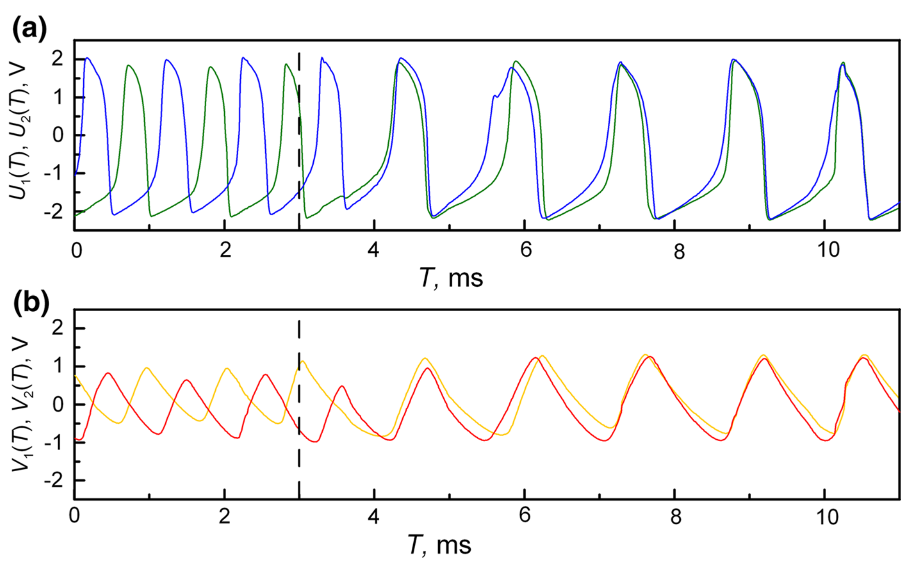

Fig. 6 Experimental time series of $U_1(T)$ (green) and $U_2(T)$ (blue) in (**a**) and $V_1(T)$ (yellow) and $V_2(T)$ (red) in (**b**) with the constant time-delayed coupling at $T < 3$ ms and adaptively controlled time-delayed coupling at $T \geq 3$ ms. The moment of switching of coupling from the constant value to the adaptive coupling is shown by vertical dash lines. (Color figure online)

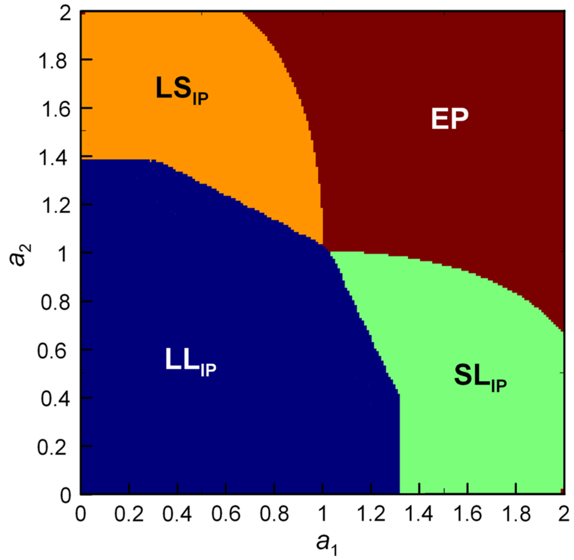

Fig. 7 Regions of typical oscillation regimes in the plane (a_1, a_2) in the model system of two delay-coupled FitzHugh–Nagumo systems for the case of adaptively controlled coupling. (Color figure online)

in Fig. 8a, b, the oscillations of both $u_i(t)$ and $v_i(t)$ have close amplitudes, but they are not synchronized in phase. The in-phase synchronization of all oscillators in the ring cannot be achieved even with large constant values of $C_i(t)$.

To achieve the in-phase synchronization of nonidentical oscillators (2) coupled in a ring, the following goal function was introduced in [45]:

$$Q(\mathbf{x}, t) = \frac{1}{2}\sum_{i=1}^{N}\Big(u_i(t) - u_{(i+1) \bmod N}(t) + a_i - a_{(i+1) \bmod N}\Big)^2. \tag{8}$$

Then, using the speed gradient method with $\mathbf{g} = (C_1, \ldots, C_N)$, the following adaptive law for changing the coupling coefficients $C_i(t)$ was obtained in [45]:

$$C_i(t) = C_i^0 + \frac{\gamma}{\varepsilon}\big(u_i(t) - u_{(i+1) \bmod N}(t-\tau)\big) \times \big(2u_i(t) - u_{(i-1) \bmod N}(t) - u_{(i+1) \bmod N}(t) + 2a_i - a_{(i-1) \bmod N}(t) - a_{(i+1) \bmod N}(t)\big). \tag{9}$$

The coupling strength of each oscillator in the ensemble is controlled separately. The control algorithm adjusts only the coupling strength between the oscillators and does not change the parameters of the local dynamics of oscillators, which makes it easy to implement this algorithm in our experimental setup.

Figure 8c, d shows the time series of $u_i(t)$ and $v_i(t)$, respectively, for all oscillators of the model ring system for the case of adaptively controlled time-delayed coupling (9) with $C_1^0 = \cdots = C_N^0 = 0$ and $\gamma = 0.02$. It is clearly seen that all oscillators exhibit in-phase oscillations. However, the amplitudes of oscillations are slightly different. The difference in the amplitudes of $u_i(t)$ depends on the values of a_i. Qualitatively similar results were obtained in [45].

For the first time, we studied experimentally the synchronization in the ring of ten unidirectionally coupled nonidentical FitzHugh–Nagumo electronic oscillators for the cases of constant time-delayed coupling and adaptively controlled time-delayed coupling. We choose $A_i = (0.1 + 0.05i)$ V, $\varepsilon_{ex} = 0.1$ ms, and $\tau_{ex} = 1.5$ ms. Figure 9 illustrates the change in the experimental time series of $U_i(T)$ at the transition from the constant coupling to the adaptive coupling. At $T < 3.55$ ms, the oscillators are coupled by the delayed coupling, the value of which remains constant, $C_1(T) = \cdots = C_N(T) = 0.3$. In this case, the oscillators exhibit non-phase oscillations. At $T = 3.55$ ms, the coupling is switched from the constant value to the adaptive coupling (9) with the parameters $C_1^0 = \cdots = C_N^0 = 0$ and $\gamma = 0.02$. After a transient process, the duration of which is about six characteristic periods of oscillations, the in-phase regime takes place in the ring.

In Fig. 10, the space–time plot of the ring of ten delay-coupled FitzHugh–Nagumo electronic oscillators is presented for the case depicted in Fig. 9. The oscillators are denoted by the numbers from 1 to 10. The oscillators exhibit non-phase oscillations at $T < 3.55$ ms. After the switching of the time-delayed coupling from the constant value to the adaptive coupling at $T = 3.55$ ms, a transient process is observed in the ensemble. After the completion of the transient process, all oscillators in the ring show in-phase oscillations. Thus, the adaptive control ensures the in-phase synchronization of nonidentical oscillators.

Control of synchronization of nonidentical oscillators was studied both theoretically and numerically in various model networks [66–70]. However, the problem of adaptive control of synchronization in networks of nonidentical oscillators is still poorly studied. To solve this problem, a method based on the Lyapunov stability theory was proposed in [43]. In [44], the method of adaptive intermittent pinning control was

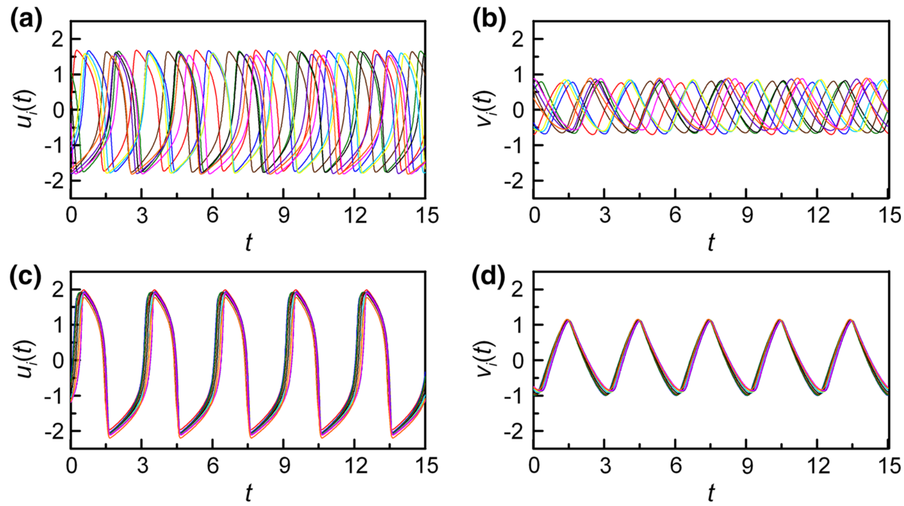

Fig. 8 Time series of $u_i(t)$ and $v_i(t)$ in the model ring system of ten nonidentical FitzHugh–Nagumo systems for the cases of constant time-delayed coupling (**a**) and (**b**) and adaptively controlled time-delayed coupling (**c**) and (**d**). (Color figure online)

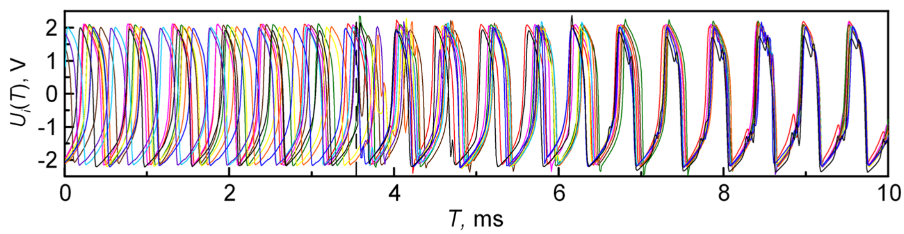

Fig. 9 Experimental time series of $U_i(T)$ in the ring of delay-coupled nonidentical FitzHugh–Nagumo electronic oscillators with the constant time-delayed coupling at $T < 3.55$ ms and adaptively controlled time-delayed coupling at $T \geq 3.55$ ms. The moment of switching of coupling from the constant value to the adaptive coupling is shown by vertical dash line. (Color figure online)

used for the synchronization of heterogeneous dynamical networks. The speed gradient method applied for the adaptive control of synchronization of nonidentical oscillators in [45] and in this paper is quite promising. It has been successfully used to synchronize delay-coupled identical Stuart–Landau oscillators [35] and delay-coupled identical Rössler systems [71] and can be developed for the case of various nonidentical oscillators.

5 Conclusion

We have investigated both experimentally and numerically the synchronization in the ensembles of delay-coupled nonidentical neuronlike oscillators described by the FitzHugh–Nagumo equations. We have considered the case of constant coupling of oscillators, as well as the case more typical for neural networks, in which the couplings between oscillators do not remain constant, but change over time.

For the experimental study of the ensemble of FitzHugh–Nagumo systems, we have constructed the original radio engineering setup, which allows us to specify any architecture of couplings between the oscillators and implement an arbitrary type of adaptive couplings. The type of dynamical couplings required for controlling the synchronization of oscillators is specified programmatically in the experimental setup, and the adaptive adjustment of coupling strength is implemented in real time. Using the constructed setup, we have studied the location of the regions of typical oscillation regimes in the parameter plane of two bidirectionally coupled nonidentical oscillators. For the first time, we have studied experimentally the synchronization in the ring of ten unidirectionally coupled nonidentical FitzHugh–Nagumo electronic oscillators. The results obtained in the physical experiment are in good

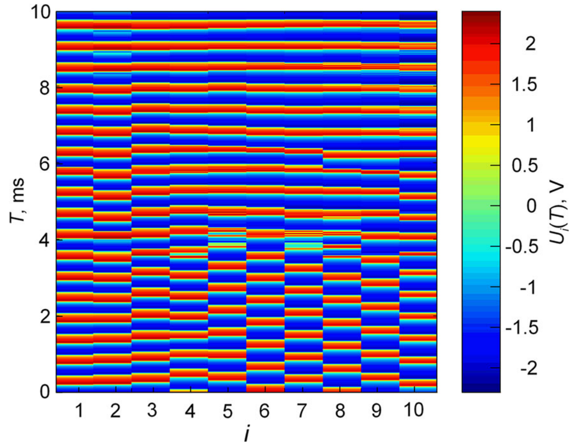

Fig. 10 Space–time plot of the ring of ten delay-coupled non-identical FitzHugh–Nagumo electronic oscillators. At $T = 3.55$ ms, the coupling is switched from the constant value to the adaptive coupling. (Color figure online)

agreement with the results of the numerical simulation. The proposed approach allows one to study experimentally the large ensembles with different base oscillators, different topologies of couplings, and different types of couplings between oscillators.

We have considered in detail the case of adaptively controlled coupling leading to the in-phase synchronization of all oscillators in the ensemble. It has been shown that the using of the adaptively controlled time-delayed coupling allows one to pass from non-phase oscillations of nonidentical oscillators to their in-phase synchronization. Adaptively controlled time-delayed coupling allows one to achieve the in-phase synchronization of all nonidentical oscillators in the ensemble even in the case of a large parameter mismatch and in the case when the constant coupling cannot ensure the in-phase synchronization even at large values of coupling coefficients. Thus, the adaptive coupling of oscillators can be promising in robotics when developing central pattern generators responsible for the control of locomotion.

To implement various robot gaits and switching between them, it is necessary to ensure different types of synchronization in the ensemble, differing by the value of phase lag between the oscillations of the ensemble elements. Therefore, in addition to the control of the in-phase synchronization of oscillators considered in this paper, it is important to learn how to control other types of synchronization. In further research, we plan to realize the possibility of generating the couplings between the ensemble oscillators not programmatically using the LabView language, but at the hardware level using FPGA. This will increase the quickness of the control system and make it more compact that is important when developing a robot locomotion control system.

Acknowledgements This study was funded by the Russian Science Foundation (Grant No. 17-72-30003).

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Pikovsky, A., Rosenblum, M., Kurths, J.: Synchronization: A Universal Concept in Nonlinear Sciences. Cambridge University Press, Cambridge (2001)
2. Glass, L.: Synchronization and rhythmic processes in physiology. Nature **410**, 277–284 (2001)
3. Osipov, G.V., Kurths, J., Zhou, C.: Synchronization in Oscillatory Networks. Springer, Berlin (2007)
4. Goulding, M.: Circuits controlling vertebrate locomotion: moving in a new direction. Nat. Rev. Neurosci. **10**, 507–518 (2009)
5. Maksimenko, V.A., Pavlov, A., Runnova, A.E., Nedaivozov, V., Grubov, V., Koronovskii, A., Pchelintseva, S.V., Pitsik, E., Pisarchik, A.N., Hramov, A.E.: Nonlinear analysis of brain activity, associated with motor action and motor imaginary in untrained subjects. Nonlinear Dyn. **91**, 2803–2817 (2018)
6. Prokhorov, M.D., Ponomarenko, V.I., Gridnev, V.I., Bodrov, M.B., Bespyatov, A.B.: Synchronization between main rhythmic processes in the human cardiovascular system. Phys. Rev. E **68**, 041913 (2003)
7. Maksimenko, V.A., Lüttjohann, A., Makarov, V.V., Goremyko, M.V., Koronovskii, A.A., Nedaivozov, V., Runnova, A.E., van Luijtelaar, G., Hramov, A.E., Boccaletti, S.: Macroscopic and microscopic spectral properties of brain networks during local and global synchronization. Phys. Rev. E **96**, 012316 (2017)
8. Karavaev, A.S., Kiselev, A.R., Runnova, A.E., Zhuravlev, M.O., Borovkova, E.I., Prokhorov, M.D., Ponomarenko, V.I., Pchelintseva, S.V., Efremova, TYu., Koronovskii, A.A., Hramov, A.E.: Synchronization of infra-slow oscillations of brain potentials with respiration. Chaos **28**, 081102 (2018)
9. Blume, W.T., Pillay, N.: Electroencephalographic and clinical correlates of secondary bilateral synchrony. Epilepsia **26**, 636–641 (1985)
10. van Luijtelaar, G., Lüttjohann, A., Makarov, V.V., Maksimenko, V.A., Koronovskii, A.A., Hramov, A.E.: Methods of automated absence seizure detection, interference by stimulation, and possibilities for prediction in genetic absence models. J. Neurosci. Methods **260**, 144–158 (2016)
11. Maksimenko, V.A., van Heukelum, S., Makarov, V.V., Kelderhuis, J., Lüttjohann, A., Koronovskii, A.A., Hramov,

A.E., van Luijtelaar, G.: Absence seizure control by a brain computer interface. Sci. Rep. **7**, 2487 (2017)
12. Uhlhaas, P., Pipa, G., Lima, B., Melloni, L., Neuenschwander, S., Nikolic, D., Singer, W.: Neural synchrony in cortical networks: History, concept and current status. Front. Integr. Neurosci. **3**, 17 (2009)
13. Levy, R., Hutchison, W.D., Lozano, A.M., Dostrovsky, J.O.: High-frequency synchronization of neuronal activity in the subthalamic nucleus of parkinsonian patients with limb tremor. J. Neurosci. **20**, 7766–7775 (2000)
14. Golomb, D., Rinzel, J.: Clustering in globally coupled inhibitory neurons. Physica D **72**, 259–282 (1994)
15. Abarbanel, H.D., Rabinovich, M.I., Selverston, A., Bazhenov, M.V., Huerta, R., Sushchik, M.M., Rubchinskii, L.L.: Synchronisation in neural networks. Phys. Usp. **39**, 337–362 (1996)
16. Sun, H., Cao, H.: Complete synchronization of coupled Rulkov neuron networks. Nonlinear Dyn. **84**, 2423–2434 (2016)
17. Hodgkin, A.L., Huxley, A.F.: A quantitative description of membrane current and its application to conduction and excitation in nerve. J. Physiol. **117**, 500–544 (1952)
18. FitzHugh, R.: Impulses and physiological states in theoretical models of nerve membrane. Biophys. J. **1**, 445–466 (1961)
19. Nagumo, J., Arimoto, S., Yoshizawa, S.: An active pulse transmission line simulating nerve axon. Proc. IRE **50**, 2061–2070 (1962)
20. Morris, C., Lecar, H.: Voltage oscillations in the barnacle giant muscle fiber. Biophys. J. **35**, 193–213 (1981)
21. Hindmarsh, J.L., Rose, R.M.: A model of neuronal bursting using three coupled first order differential equations. Proc. R. Soc. Lond. B. Biol. Sci. **221**, 87–102 (1984)
22. Dahlem, M.A., Hiller, G., Panchuk, A., Schöll, E.: Dynamics of delay-coupled excitable neural systems. Int. J. Bifurc. Chaos **19**, 745–753 (2009)
23. Lehnert, J., Dahms, T., Hövel, P., Schöll, E.: Loss of synchronization in complex neuronal networks with delay. Europhys. Lett. **96**, 60013 (2011)
24. Mao, X., Wang, Z.: Stability, bifurcation, and synchronization of delay-coupled ring neural networks. Nonlinear Dyn. **84**, 1063–1078 (2016)
25. Flunkert, V., Yanchuk, S., Dahms, T., Schöll, E.: Synchronizing distant nodes: a universal classification of networks. Phys. Rev. Lett. **105**, 254101 (2010)
26. Klinshov, V.V., Nekorkin, V.I.: Synchronization of delay-coupled oscillator networks. Phys. Usp. **56**, 1217–1229 (2013)
27. Otto, A., Radons, G., Bachrathy, D., Orosz, G.: Synchronization in networks with heterogeneous coupling delays. Phys. Rev. E **97**, 012311 (2018)
28. Sun, J., Bollt, E.M., Nishikawa, T.: Master stability functions for coupled nearly identical dynamical systems. Europhys. Lett. **85**, 60011 (2009)
29. Sysoev, I.V., Ponomarenko, V.I., Prokhorov, M.D.: Reconstruction of ensembles of nonlinear neurooscillators with sigmoid coupling function. Nonlinear Dyn. **95**, 2103–2116 (2019)
30. Makarov, V.V., Koronovskii, A.A., Maksimenko, V.A., Hramov, A.E., Moskalenko, O.I., Buldu, J.M., Boccaletti, S.: Emergence of a multilayer structure in adaptive networks of phase oscillators. Chaos Solitons Fractals **84**, 23–30 (2016)
31. Maslennikov, O.V., Nekorkin, V.I.: Adaptive dynamical networks. Phys. Usp. **60**, 694–704 (2017)
32. Pitsik, E., Makarov, V., Kirsanov, D., Frolov, N., Goremyko, M., Li, X., Wang, Z., Hramov, A., Boccaletti, S.: Inter-layer competition in adaptive multiplex network. New J. Phys. **20**, 075004 (2018)
33. Sun, J.Q., Ding, G.: Advances in Analysis and Control of Time-Delayed Dynamical Systems. World Scientific, Singapore (2013)
34. Lu, X., Qin, B.: Adaptive cluster synchronization in complex dynamical networks. Phys. Lett. A **373**, 3650–3658 (2009)
35. Selivanov, A.A., Lehnert, J., Dahms, T., Hövel, P., Fradkov, A.L., Schöll, E.: Adaptive synchronization in delay-coupled networks of Stuart–Landau oscillators. Phys. Rev. E **85**, 016201 (2012)
36. Yuan, W.-J., Zhou, J.-F., Sendiña-Nadal, I., Boccaletti, S., Wang, Z.: Adaptive control of dynamical synchronization on evolving networks with noise disturbances. Phys. Rev. E **97**, 022211 (2018)
37. Novičenko, V., Ratas, I.: In-phase synchronization in complex oscillator networks by adaptive delayed feedback control. Phys. Rev. E **98**, 042302 (2018)
38. Fan, D., Zhang, L., Wang, Q.: Transition dynamics and adaptive synchronization of time-delay interconnected corticothalamic systems via nonlinear control. Nonlinear Dyn. **94**, 2807–2825 (2018)
39. Lehnert, J., Hövel, P., Selivanov, A.A., Fradkov, A.L., Schöll, E.: Controlling cluster synchronization by adapting the topology. Phys. Rev. E **90**, 042914 (2014)
40. Wang, X., She, K., Zhong, S., Cheng, J.: Synchronization of complex networks with non-delayed and delayed couplings via adaptive feedback and impulsive pinning control. Nonlinear Dyn. **86**, 165–176 (2016)
41. Shi, L., Zhu, H., Zhong, S., Shi, K., Cheng, J.: Cluster synchronization of linearly coupled complex networks via linear and adaptive feedback pinning controls. Nonlinear Dyn. **88**, 859–870 (2017)
42. Zhou, P., Cai, S.: Pinning synchronization of complex directed dynamical networks under decentralized adaptive strategy for aperiodically intermittent control. Nonlinear Dyn. **90**, 287–299 (2017)
43. Guo, X., Li, J.: Stochastic adaptive synchronization for time-varying complex delayed dynamical networks with heterogeneous nodes. Appl. Math. Comput. **222**, 381–390 (2013)
44. Cai, S., Jia, Q., Liu, Z.: Cluster synchronization for directed heterogeneous dynamical networks via decentralized adaptive intermittent pinning control. Nonlinear Dyn. **82**, 689–702 (2015)
45. Plotnikov, S.A., Lehnert, J., Fradkov, A.L., Schöll, E.: Adaptive control of synchronization in delay-coupled heterogeneous networks of FitzHugh–Nagumo nodes. Int. J. Bifurc. Chaos **26**, 1650058 (2016)
46. Lodi, M., Shilnikov, A., Storace, M.: Design of synthetic central pattern generators producing desired quadruped gaits. IEEE Trans. Circuits Syst. I Regul. Pap. **65**, 1028–1039 (2018)
47. Shchapin, D.S.: Dynamics of two neuronlike elements with inhibitory feedback. J. Commun. Technol. Electron. **54**, 175–184 (2009)

48. Sysoev, I.V., Prokhorov, M.D., Ponomarenko, V.I., Bezruchko, B.P.: Reconstruction of ensembles of coupled time-delay systems from time series. Phys. Rev. E **89**, 062911 (2014)
49. Chua, L., Komuro, M., Matsumoto, T.: The double scroll family. IEEE Trans. Circuts Syst. **33**, 1072–1118 (1986)
50. Liu, Z., Ma, J., Zhang, G., Zhang, Y.: Synchronization control between two Chua's circuits via capacitive coupling. Appl. Math. Comput. **360**, 94–106 (2019)
51. Yao, Z., Ma, J., Yao, Y., Wang, C.: Synchronization realization between two nonlinear circuits via an induction coil coupling. Nonlinear Dyn. **96**, 205–217 (2019)
52. Xu, Y., Jia, Y., Ma, J., Alsaedi, A., Ahmad, B.: Synchronization between neurons coupled by memristor. Chaos Solitons Fractals **104**, 435–442 (2017)
53. Zhang, J., Liao, X.: Effects of initial conditions on the synchronization of the coupled memristor neural circuits. Nonlinear Dyn. **95**, 1269–1282 (2019)
54. Ren, G., Xu, Y., Wang, C.: Synchronization behavior of coupled neuron circuits composed of memristors. Nonlinear Dyn. **88**, 893–901 (2017)
55. Guo, S., Xu, Y., Wang, C., Jin, W., Hobiny, A., Ma, J.: Collective response, synapse coupling and field coupling in neuronal network. Chaos Solitons Fractals **105**, 120–127 (2017)
56. Xu, Y., Jia, Y., Ma, J., Hayat, T., Alsaedi, A.: Collective responses in electrical activities of neurons under field coupling. Sci. Rep. **8**, 1349 (2018)
57. Lv, M., Ma, J., Yao, Y.G., Alzahrani, F.: Synchronization and wave propagation in neuronal network under field coupling. Sci. China Technol. Sci. **62**, 448–457 (2019)
58. Usha, K., Subha, P.A.: Collective dynamics and energy aspects of star-coupled Hindmarsh–Rose neuron model with electrical, chemical and field couplings. Nonlinear Dyn. **96**, 2115–2124 (2019)
59. Heinrich, M., Dahms, T., Flunkert, V., Teitsworth, S.W., Schöll, E.: Symmetry breaking transitions in networks of nonlinear circuit elements. New J. Phys. **12**, 113030 (2010)
60. Keener, J.P., Sneyd, J.: Mathematical Physiology. Springer, New York (1998)
61. Luchinsky, D.G., McClintock, P.V.E., Dykman, M.I.: Analogue studies of nonlinear systems. Rep. Prog. Phys. **61**, 889–997 (1998)
62. Binczak, S., Kazantsev, V.B., Nekorkin, V.I., Bilbault, J.M.: Experimental study of bifurcations in a modified FitzHugh–Nagumo cell. Electron. Lett. **39**, 961–962 (2003)
63. Nouri, M., Karimi, G.R., Ahmadi, A., Abbott, D.: Digital multiplierless implementation of the biological FitzHugh–Nagumo model. Neurocomputing **165**, 468–476 (2015)
64. Ponomarenko, V.I., Kul'minskii, D.D., Prokhorov, M.D.: An experimental study of synchronization of nonidentical neuronlike oscillators with an adaptive delayed coupling. Tech. Phys. Lett. **44**, 761–764 (2018)
65. Fradkov, A.L.: Cybernetical Physics: From Control of Chaos to Quantum Control. Springer, Heidelberg (2007)
66. Yang, Z.Q., Liu, Z.X., Chen, Z.Q., Yuan, Z.Z.: Controlled synchronization of complex network with different kinds of nodes. J. Control Theory Appl. **6**, 11–15 (2008)
67. Zhao, J., Hall, D.J., Liu, T.: Synchronization of dynamical networks with nonidentical nodes: criteria and control. IEEE Trans. Circuits Syst. **I**(58), 584–594 (2011)
68. Ricci, F., Tonelli, R., Huang, L., Lai, Y.-C.: Onset of chaotic phase synchronization in complex networks of coupled heterogeneous oscillators. Phys. Rev. E **86**, 027201 (2012)
69. Isidori, A., Marconi, L., Casadei, G.: Robust output synchronization of a network of heterogeneous nonlinear agents via nonlinear regulation theory. IEEE Trans. Autom. Contr. **59**, 2680–2691 (2014)
70. DeLellis, P., di Bernardo, M., Liuzza, D.: Convergence and synchronization in heterogeneous networks of smooth and piecewise smooth systems. Automatica **56**, 1–11 (2015)
71. Guzenko, P.Y., Lehnert, J., Schöll, E.: Application of adaptive methods to chaos control of networks of Rössler systems. Cybern. Phys. **2**, 15–24 (2013)