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Reservoir computing shows partial statistical dynamics prediction of two coupled stochastic FitzHugh-Nagumo neurons

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***Abstract*—We train a reservoir computing-based echo state network to predict the stochastic dynamics of two coupled FitzHugh-Nagumo neurons excited by external Gaussian noise. We describe the design principles of the echo state network and the coupled oscillator network. Results show that prediction quality of the dynamics of each neuron varies despite their high coupling strength.**

***Index Terms*—Reservoir computing, prediction, FitzHugh-Nagumo model, noise, coherence resonance, coupled oscillators**

I. Introduction

Study of coupled oscillator networks is challenging due to inherent complexity of their structure and dynamics. Physical systems with such behavior are omnipresent: from electronic circuits to fish schooling and human brain neural oscillations [1], [2]. Different aspects of coupled oscillator networks are being explored, such as exhibition of self-organized bistability in a globally coupled Kuramoto network [3], synchronization of FitzHugh-Nagumo (FHN) [4], Rulkov and Hodgkin-Huxley (HH) networks [5]. Systems of coupled HH neurons with different topologies also demonstrated transition to a chimera state [6], [7].

Under external noise driving a single FHN neuron exhibits coherence resonance, a state of maximal coherence of induced oscillations for intermediate noise amplitude [8]. This phenomenon was studied in populations of FHN neurons with different topologies [9], [10] and multiplex FHN networks [11], [12]. Coherence resonance was also found in Rulkov network [13] and even in experiments with visual perception [14].

Due to complexity of coupled oscillator networks, prediction of their dynamics is a particularly difficult task. Reservoir computing (RC) network [15] is a recurrent neural network type that was used to forecast a macroscopic signal of adaptive network of Kuramoto phase oscillators [16] and to reconstruct the dynamics of a coupled Lorenz network [17]. RC was also successfully trained to predict dynamics of a single FHN neuron with various noise intensities while being trained on noise level that causes coherence resonance [18]. However, the usage of RC for prediction of dynamics of a coupled FHN network, driven by external noise, was not yet explored.

Here, we train a RC network on signals from two coupled FHN neurons to predict dynamics of both neurons and compare the correspondence of statistical characteristics, namely interspike interval (ISI) probability density, of predicted signals with target ones.

II. Methods

A. System of coupled FitzHugh-Nagumo neurons

We use a coupled pair of FHN neurons as a model system. Each of two neurons is in excitable steady state, driven by external Gaussian noise. The coupled system is defined as follows [19]:

$$\begin{aligned} \frac{dx_i}{dt} &= x_i - \frac{x_i^3}{3} - y_i + I + \frac{\sigma}{N-1}\sum_{j=1}^{N}(x_j - x_i), \\ \frac{dy_i}{dt} &= \frac{x_i + a - by_i}{\tau} + D\xi[t], \end{aligned} \tag{1}$$

where x_i and y_i represent a membrane potential variable and a recovery variable, respectively. $I = 0.3$ denotes injected electric current, $\sigma = 0.4$ is the system's coupling strength between $N = 2$ neurons. $\tau = 12.5$ serves to separate the time scales of fast variable x_i and slow variable y_i. White Gaussian noise with zero-mean and unit variance $\xi_i[t]$ drives two excitable coupled neurons, resulting in spike generation, parameter D sets amplitude of $\xi_i[t]$. $a = 0.7$, $b = 0.8$ are system parameters ensuring excitability of the coupled FHN system.

We employed the forward Euler method with time step $\Delta t = 0.1$ to numerically solve the system.

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B. Echo state network architecture

We constructed a type of RC network called an echo state network (ESN). It consists of an input layer, recurrent neurons layer (the reservoir), and an output layer.

Connection strengths between the input layer and the reservoir are stored in W_{in} adjacency matrix with size $[n, m]$, where $n = 500$ is the number of recurrent neurons in the reservoir and $m = 6$ is the number of dimensions of input time series data including x_1, x_2, y_1, y_2, $D\xi_1$ and $D\xi_2$.

$[n, n]$ sparse adjacency matrix W_r contains connection strengths between recurrent neurons of the reservoir. Tunable hyperparameter ρ sets the spectral radius of this matrix, with higher ρ values corresponding to stronger connections between neurons. Expected reservoir neuron degree is set through another hyperparameter d.

Output weights matrix W_{out} defines connections between recurrent neurons of the reservoir and the output layer. Connections in W_{out} are the only trainable parameters in our RC network, ridge regression is used to determine the connection strengths while preventing overfitting.

The RC network is trained to predicted the values of x_1, x_2, y_1 and y_2. Therefore, during prediction stage, the RC network uses known testing $D\xi_1$ and $D\xi_2$ values as input. x_1, x_2, y_1 and y_2, on the contrary, are predicted by the RC network, and these predicted values are used for further prediction.

III. Results

The RC network was trained on FHN network time series with noise amplitude $D = 0.2$. We have optimized the network to minimize the root mean squared error (RMSE) during prediction of signals with three different D values: $D = 0.2$, which was used for RC network training, and signals with noise amplitudes $D = 0.05$ and $D = 1.0$. This approach was used to enable the reservoir computer network to model the FHN network dynamics at different noise levels. Hyperparameters ρ and d were tuned during the optimization process.

After the optimization process, the trained model was used to predict the FHN system dynamics at different noise levels in range $D \in [0.05, 1.0]$ with step $\Delta D = 0.05$. Probability density functions of target and predicted signal ISIs were evaluated. The results for $D = 0.2$, $D = 0.8$ and $D = 1.0$ are shown on Figures 1, 2 and 3, respectively.

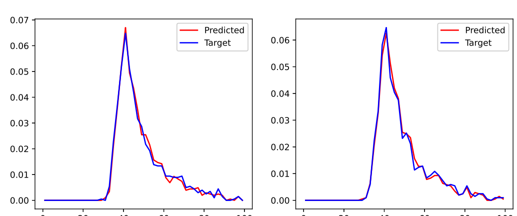

Fig. 1. Probability density functions of interspike intervals of first (left) and second (right) model FitzHugh-Nagumo neuron signal with $D = 0.2$ and echo state network predicted signal.

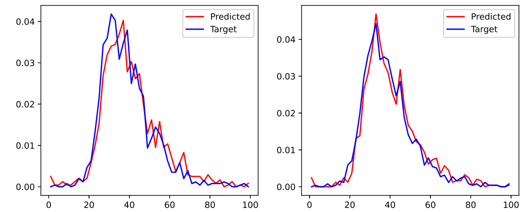

Fig. 2. Probability density functions of interspike intervals of first (left) and second (right) model FitzHugh-Nagumo neuron signal with $D = 0.8$ and echo state network predicted signal.

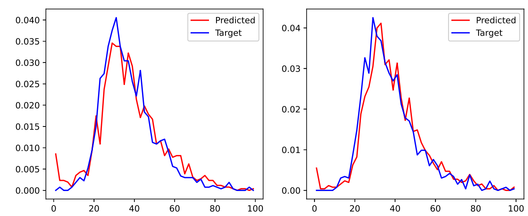

Fig. 3. Probability density functions of interspike intervals of first (left) and second (right) model FitzHugh-Nagumo neuron signal with $D = 1.0$ and echo state network predicted signal.

In case $D = 0.2$, the ISI statistical characteristics of both coupled FHN neurons dynamics predictions match corresponding characteristics of the original system. Such high prediction accuracy is expected since the RC network was trained on a signal with the same noise amplitude.

However, probability density of ISIs of prediction time series with $D = 0.8$ is deviating from the the target curve. Moreover, the prediction accuracy of the first FHN neuron is significantly less accurate than prediction accuracy of the second one. The PDF curve of the second prediction, unlike the first one, is notably closer to the target PDF curve.

Dynamics prediction of the FHN network with $D = 1.0$ shows significant deviation from the target probability density function curve for both neurons, which can be explained by a high noise level, making precise modeling of the FHN network dynamics a challenging task.

IV. Conclusions

We have trained the RC network to model the behavior of two coupled FHN neurons, driven by external Gaussian noise, and evaluated RC network's prediction performance based on predicted time series ISI distribution. RC network has successfully modeled the FHN system's behavior at $D = 0.2$, noise level that was used for training the RC network, for both neurons, but in case of $D = 0.8$ the statistical dynamics prediction quality was different for two coupled neurons: while the second neuron's dynamics prediction was rather successful, although not as precise as at $D = 0.2$, the ISI probability density function has significantly deviated from the target curve, showing partial statistical dynamics prediction of the

FHN system as the whole. But with further increase of D up to 1.0, statistical prediction of both neurons was not precise.

These results show of existence of three different prediction modes of systems of two coupled excitable oscillators: successful prediction of dynamics of both neurons, successful prediction of only one of neural dynamics, and absence of successful prediction of both dynamics. These prediction modes can depend on the amplitude of the driving noise. Further research is necessary to determine the dependence of these prediction modes on noise level, coupling strength and other system parameters.

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