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SHORT COMMUNICATIONS

Application of Continuous Wavelet Transform to the Analysis of Structural Variations in Complex Networks

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Abstract—The dynamics of a network of phase oscillators is analyzed using the continuous wavelet transform. The adaptive network in which the synchronous dynamics leads to reinforcement of the links between interacting elements is considered as an example. It is shown that analysis of the network integral characteristic using the wavelet transform makes it possible to effectively detect changes in the network topology and clusterization processes.

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The investigation of the dynamics of objects consisting of a large number of interacting elements has recently attracted attention of the academic community. Such objects with a network structure can serve as models of the artificially obtained systems (networks of interacting radiophysical elements and artificial learning neural networks), as well as mathematical interpretations of actual biological [1], social [2], anthropogenic [3], and other objects.

The dynamics of actual networks is usually investigated by constructing mathematical models and their numerical analysis. However, such an approach can be employed if the laws describing the dynamics of individual elements of the network being considered are known, as well as the distribution and temporal evolution of the links between them. For example, mathematical models in which the dynamics of an individual node can be described by a harmonic oscillator with a given unique frequency and the links between the nodes are simulated according to the data of statistical analysis and sociological studies are widely used in analysis of social networks [4].

At the same time, it seems impossible to study a wide range of actual systems by constructing and analyzing mathematical models. In this case, the researchers face the lack of data on the network topology and dynamics of individual elements. First of all, analysis of synchronization and formation of structural patterns in the brain neural networks can serve as a specific example of such problems [5]. Information on the condition of the neural network and its evolution in time can be obtained from electroencephalograms (EEGs) and magnetoencephalograms (MEGs), which are the records of the total signals of electric activity, produced by large neuron ensembles.

The problems associated with analysis of integral characteristics of a neuron ensemble and interpretation of temporal variations of macroscopic characteristics of the network with evolution of its topology are now of great importance for fundamental research in the field of nonlinear dynamics, as well as applied aspects connected with the study of both normal and pathological brain activity.

In this work, we consider the possibility of studying structural changes in complex networks on the basis of analysis of their integral characteristics using the continuous wavelet transform.

As an object for investigation, we use a network of the Kuramoto phase oscillators proposed in 1975 [6] as a mathematical interpretation of the collective dynamics of chemical and biological oscillators [7]. Various modifications of this model of a phase oscillator network are now often used in analyzing clusterization and synchronization processes, including those in social systems [4].

The dynamic state of the ith node of the given network can be determined by the relation

$$\dot{\varphi}_i = \omega_i + \lambda\sum_{j=1}^{N} w_{ij}(t)\sin(\varphi_j - \varphi_i), \qquad (1)$$

where ω_i are the natural frequencies preset at random in the range [2π Hz, 20π Hz], $w_{ij}(t)$ is the weight of connection between nodes j and i, and λ is the connection strength. The initial phases of interacting ele-

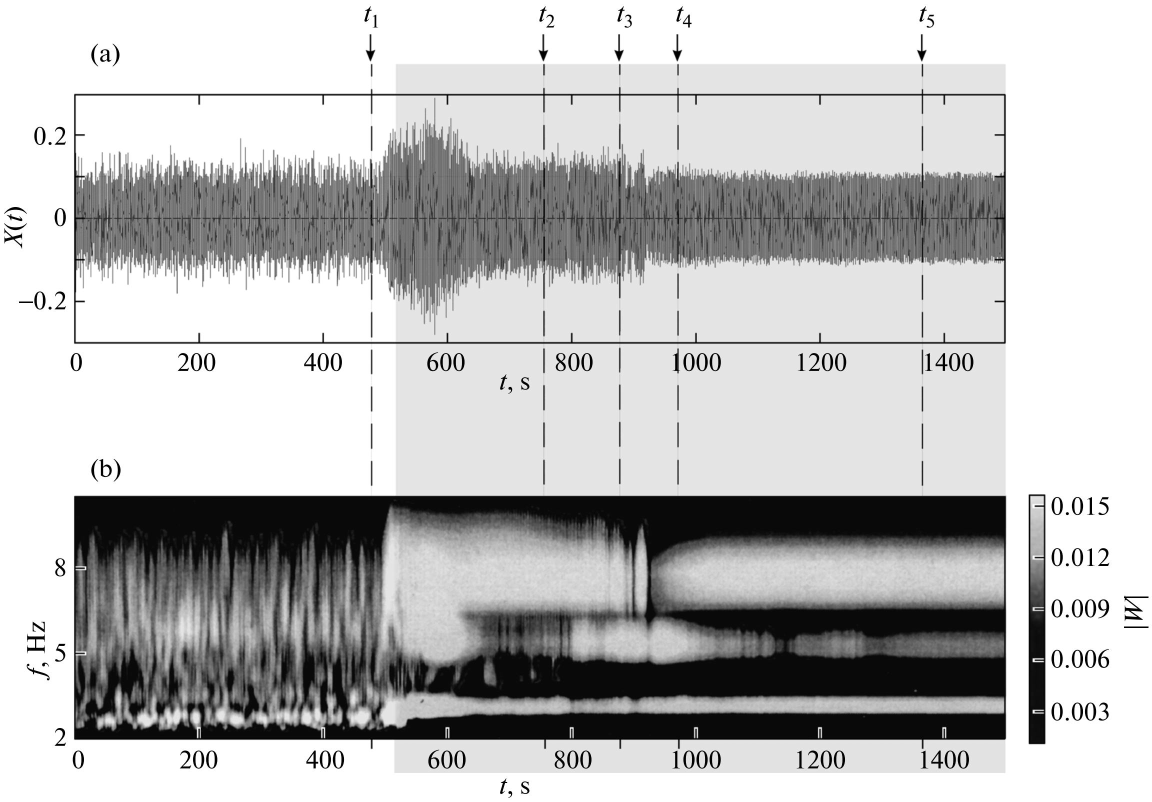

Fig. 1. (a) Total signal of the interacting Kuramoto phase oscillators. (b) Amplitude of the wavelet transform of the total signal. The region of adaptive dynamics is shaded.

ments are set at random and distributed uniformly over the $[-\pi, \pi]$ interval; here, connection weights $w_{ij}(t_0)$ are also set at random.

A specific feature of the given model is transient temporal dynamics of coefficients w_{ij}, described by the adaptive law

$$w_{ij}(t) = w_{ij}(t)\left[s_i p_{ij}^T(t) - \sum_{l=1}^{N} w_{ij}(t) p_{ij}^T(t)\right], \qquad (2)$$

where $s_i = \sum_{j=1}^{N} w_{ij}$ is the sum of the inbound links of the ith element and quantity $p_{ij}^T(t)$ defines the degree of synchronization of elements i and j, averaged over time interval $T = 100$ s [4],

$$p_{ij}^T(t) = \frac{1}{T}\left|\int_{t-T}^{t} e^{i[\varphi_j(\tau)\varphi_l(\tau)]} d\tau\right|. \qquad (3)$$

Adaptive law (2), (3) considered here ensures the feedback between the dynamics of elements and the network structure and is the basic mechanism governing the change in its topology. It follows from Eq. (2) that the value of derivative $w_{ij}(t)$ is determined by the degree of synchronization between the corresponding elements and attains the highest value in the case of strong synchronization.

As noted above, the actual networks are often analyzed on the basis of integral characteristics. For the model network considered here, for such a parameter we used the total signal of interacting phase oscillators:

$$X(t) = \sum_{j=1}^{N} A\cos(\varphi_j). \qquad (4)$$

In this case, $N = 200$ is the number of the network elements and $A = 1$ is the dimensionless amplitude of the signal received from each node. This dependence is shown in Fig. 1a. It can be seen from the figure that activation of adaptive mechanism (2), (3) (instant $t = 500$ s) leads to sharp qualitative changes in the signal, which is obviously connected with changes in the network topology. In this case, such a transient process lasts about 400 s, after which the network attains a steady state characterized by time-independent characteristics of the macroscopic parameter.

Taking into account the relation of the adaptive processes with the establishment of the regimes of synchronous dynamics between interacting oscillators, we can assume with a high degree of confidence that the change in the spectral composition of the total signal considered here corresponds to a change in the network topology under investigation. To verify this assumption, we analyzed the network integral characteristic using the wavelet transform, which is the most suitable method for studying signals with a transient spectral composition [8].

Let us consider wavelet transform (4) of the signal, which serves as the network integral characteristic. In terms of linear frequencies $f = \omega/2\pi$, the transform has the form

$$W(f, t) = \sqrt{f} \int_{t-4/f}^{t+4/f} X(t')\psi^*(f(t'-t))dt', \qquad (5)$$

where f corresponds to frequency range [1–10 Hz], in which the signal is expanded, $\psi^*(t - t')$ is the Morlet parent wavelet [8], and symbol (*) denotes complex conjugation.

The resulting transform is shown in Fig. 1b. It can be seen that at the initial instant, energy $|W(f, t)|$ is uniformly distributed over the frequency range under investigation (instant t_1) due to the initial spread of the oscillator frequencies and phases. Analyzing the results of the wavelet transform for $t > 500$ s (when the adaptation is activated), we see that synchronization between individual groups of elements begins to increase. In this case, their connection with other elements becomes weaker under the action of adaptive mechanisms (instants t_2 and t_3), which leads to the network partition into groups of tightly coupled elements being in the phase synchronization regime, each of which corresponds to a peak of the wavelet surface.

To illustrate the clusterization process described above, Fig. 2 shows visualizations of the network structure at instants t_1, ..., t_5 marked by arrows in Fig. 1b (the direction of time progress is indicated in Fig. 2 on the right of the presented visualizations). The visualizations are plotted on the basis of the values of coupling factors w_{ij} using the Cytoscape software [9]. Figure 2a corresponds to the network dynamics before the activation of the adaptive mechanism. It can be seen that the network is disordered in this case. Its structure is determined by links w_{ij} between the elements, which are set at random. When adaptation is activated, the network structure begins to evolve (Figs. 2b–2d), which ultimately causes its clustering (Fig. 2e). Comparing the results of the presented visualization with the results of the wavelet analysis of the network integral characteristics (see Fig. 1b), we note that the structural changes in the network and the formation of clusters can be successfully detected using the wavelet analysis of the network integral characteristic.

Thus, we have demonstrated the possibility of detecting emerging clusters in an adaptive network by using the wavelet analysis of a macroscopic characteristic (total signal of the interacting elements). The proposed method for network analysis can be used for detecting the clusterization processes in the brain neuron nets, where the record of EEG and MEG can serve as a macroscopic characteristic.

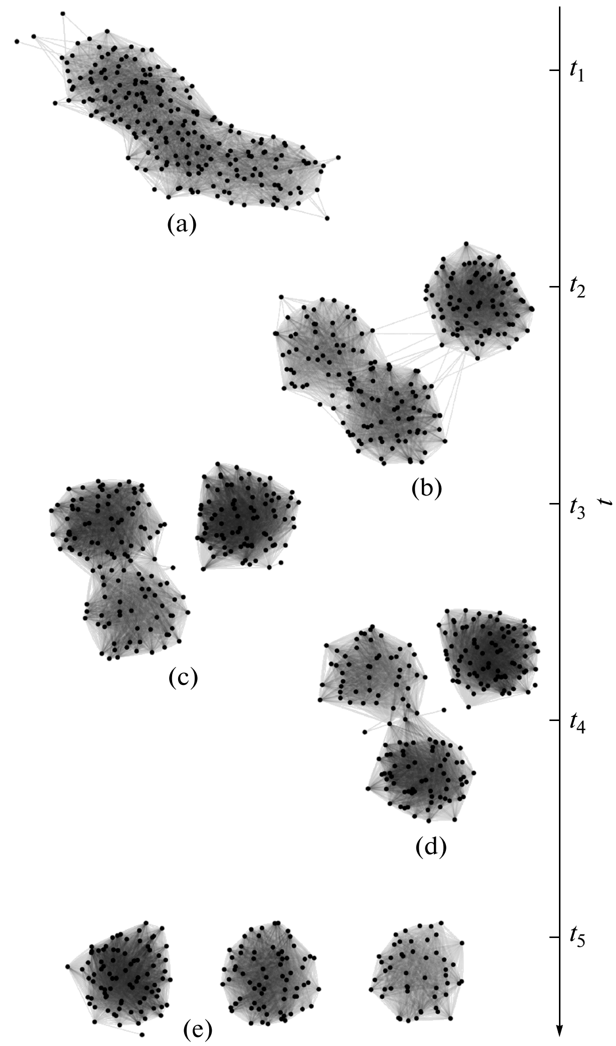

Fig. 2. (a) Visualization of the network structure of the Kuramoto phase oscillators considered here at instant $t_1 =$ 450 s preceding the activation of the adaptive mechanism, and at instants (b) $t_2 = 730$ s, (c) $t_3 = 880$ s, (d) $t_4 = 960$ s, (e) and $t_5 = 1360$ s corresponding to the adaptive dynamics.

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