Study of pattern formation in multilayer adaptive network of phase oscillators in application to brain dynamics analysis

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ABSTRACT

In the report we study the mechanisms of phase synchronization in the model of adaptive network of Kuramoto phase oscillators and discuss the possibility of the further application of the obtained results for the analysis of the neural network of brain. In our theoretical study the model network represents itself as the multilayer structure, in which the links between the elements belonging to the different layers are arranged according to the competitive rule. In order to analyze the dynamical states of the multilyer network we calculate and compare the values of local and global order parameter, which describe the degree of coherence between the neighboring nodes and the elements over whole network, respectively. We find that the global synchronous dynamics takes place for the large values of the coupling strength and are characterized by the identical topology of the interacting layers and a homogeneous distribution of the link strength within each layer. We also show that the partial (or cluster) synchronization, occurs for the small values of the coupling strength, lead to the emergence of the scale-free topology, within the layers.

Keywords: Complex network, multilayer network, parallel computing, neural network of brain

1. INTRODUCTION

Study of the feedback between the dynamics of the individual elements, arranged in the network, and the evolution of the network topology represent itself as the key to the understanding of many processes, taken place in the real systems, referred to the networks of cities and populations,[1] electronic[2] and social systems,[3] biological networks,[4] and neural networks of brain.[5] In this context, along with the analysis of interaction between elements within the single network, study of the interaction between the adaptive networks, which can be arranged according to co-operative and competitive mechanisms, is an important task, which is associated with a more holistic view on the processes occurring in real systems including neuronal network of the brain.

The dynamics of the complex dynamical system, which consist of the number of the element, arranged in the set of interaction subnetworks, can be described in the framework of the multilayered network model. In the simplest case the different layers contain an identical set of the nodes, but have different topologies of the links between them. The more complicated models deal with the nonidentical nodes, which properties, as well, as the topology, can vary both within the single layer and within the different layers according to the specific features of the task. In the analysis of the neural network of the brain the multilayered models can be used for the study of the local synchronization between the neurons, located in the different parts of the brain, and the global synchronization, associated with the interaction between the remote regions of the brain. Moreover, the multilayered models can be effectively used for the analysis of the networks, where the elements interacts with each other via the several links simultaneously. In this case, the each layer can be associated with the interaction between the nodes through the each type of the links.

In this paper we consider the fundamental multi-layer model of adaptive network of phase oscillators, where the topology of the links between elements is controlled by the feedback with the dynamic of the elements in

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Saratov Fall Meeting 2016: Laser Physics and Photonics XVII; and Computational Biophysics and Analysis of Biomedical Data III, edited by Vladimir L. Derbov, Dmitry E. Postnov, Proc. of SPIE Vol. 10337, 103370Z · © 2017 SPIE · CCC code: 0277-786X/17/$18 · doi: 10.1117/12.2267651

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the accordance with the additivity principles[3] and homeostasis.[6] The interaction between the layers of the considered network model is based on the principles of competition, leading to the dynamical redistribution of the link strength within each layer.

2. MATHEMATICAL MODEL UNDER STUDY

The model under study is a multilayer adaptive network of Kuramoto oscillators, which is a recognized tool used for studies of various forms of collective dynamics.[7–9] In our study, we created a model based on the adaptive model proposed by S. of Assenza et al.[10] This model reflects the two key of features of natural networks, namely, scale-free distribution of the weight of bonds and formation of mesoscale structures. Such phenomena are caused by the above mentioned mechanisms: the homophily associated with the strengthening of ties between synchronized nodes, and the homeostasis — the mechanism of competition, by which increasing a connection from one network element is balanced by the weakening of other bonds of the same node in the network, implemented by holding the condition:

$$\sum_{j\neq i}^{N} \omega_{ij}^{l} = 1 \tag{1}$$

which means that the sum of all the weights within the node holds at all times; i.e., the sum of the weights of all incoming connections at each node is conserved. ω_{ij}^{l}–coefficient that determines the strength of the link connecting the nodes j and i on the layer l of the network.

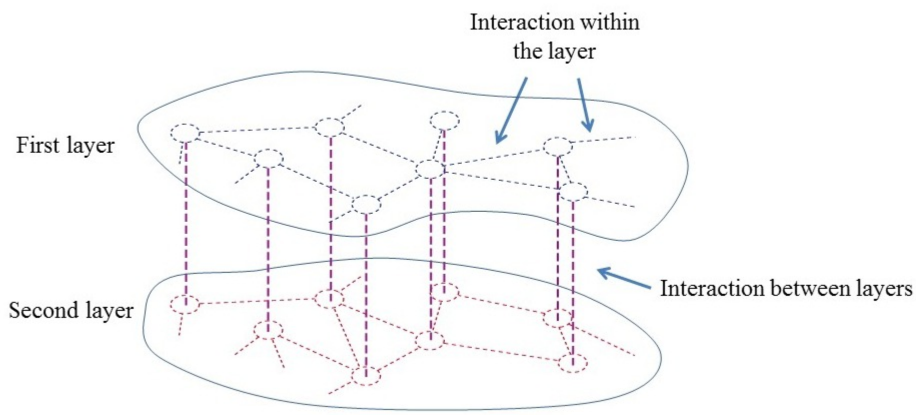

Figure 1. Schematic representation of the model under study

The model (see Fig. 1) consists of M layers, each one containing N generators. In each layer $l = 1, \ldots, M$ each node $i = 1, \ldots, N$ interacts with all other nodes, that form the set:

$$\dot{\varphi}_i^l = \delta_i + \lambda \sum_{j=1}^{N} \omega_{ij}^{l} \sin(\varphi_i^l - \varphi_j^l), \tag{2}$$

where φ_i^l is the phase of i–th Kuramoto oscillator on the layer l, δ_i is the randomly selected frequency circular frequency phase Kuramoto oscillators, ω_{ij}^{l} is the weight of the connection between nodes j and i within layer l as stated above and λ is a coupling strength.

The value ω_{ij}^{l} changes over time according to the law:

$$\dot{\omega}_{ij}^{l}(t) = p_{ij}^{l}(t) - \left(\sum_{k\in N^l} p_{ik}^{l}(t)\right)\omega_{ij}^{l}(t) - \left(\sum_{k\neq l} p_{ij}^{k}(t)\right)\omega_{ij}^{l}(t), \tag{3}$$

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where

$$p_{ij}^{l}(t) = \left| \frac{1}{T} \int_{t-T}^{t} e^{\sqrt{-1}[\varphi_j(\tau) - \varphi_i(\tau)]} d\tau \right|. \tag{4}$$

is the degree of coherence between the local oscillator i and j are averaged over time interval $[t-T,t]$. Thus, larger values p_{ij}^{l} increase the weight of the connection between the corresponding nodes. Together with the value (4) the second term of equation (3) describes the adaptive interaction between elements within the layers, while the third is a competition of layers for the optimal topology.

3. RESULTS OF NUMERICAL SIMULATION OF MULTILAYER ADAPTIVE NETWORK

Let us consider the results of numerical simulation of the dynamics of multilayer adaptive network of coupled phase oscillators (1). In our study we investigate the network, which contains two layers ($M = 2$), ($N = 300$) oscillators in each layer, the value of angular frequencies of the Kuramoto oscillators are assigned randomly in the range $[-\pi, \pi]$. Initially, the phases are also selected randomly in the interval $[-\pi; \pi]$, all weights are set to $\frac{1}{N}$. The coupling parameter between the oscillators is being changed between $0 \leq \lambda \leq 6$ with the constant step $\Delta\lambda = 0.1$, and the characteristic time T – in the interval $0 \leq T \leq 100$ with a constant step $\Delta T = 5$

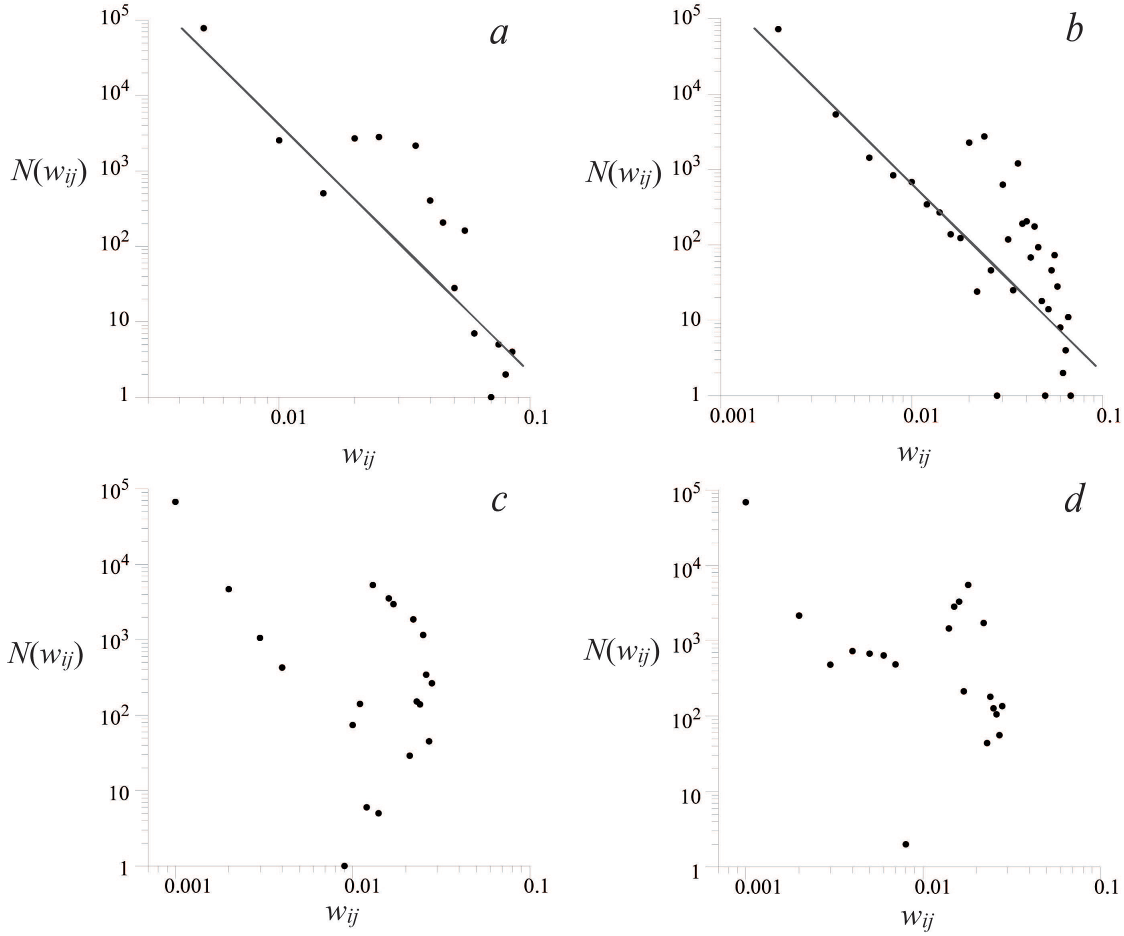

Figure 2. Power-law distribution weights of connections in different situations: two layers of the network(a,b) with $\lambda = 0.5$, $T = 50$, and two layers (c,d) of this system at another moment when $\lambda = 1$, $T = 50$

First, the phases of elements were calculated at a certain period of time, greater than the characteristic time interval T. Then, the adaptive links were taken into consideration and the equation, described the change in

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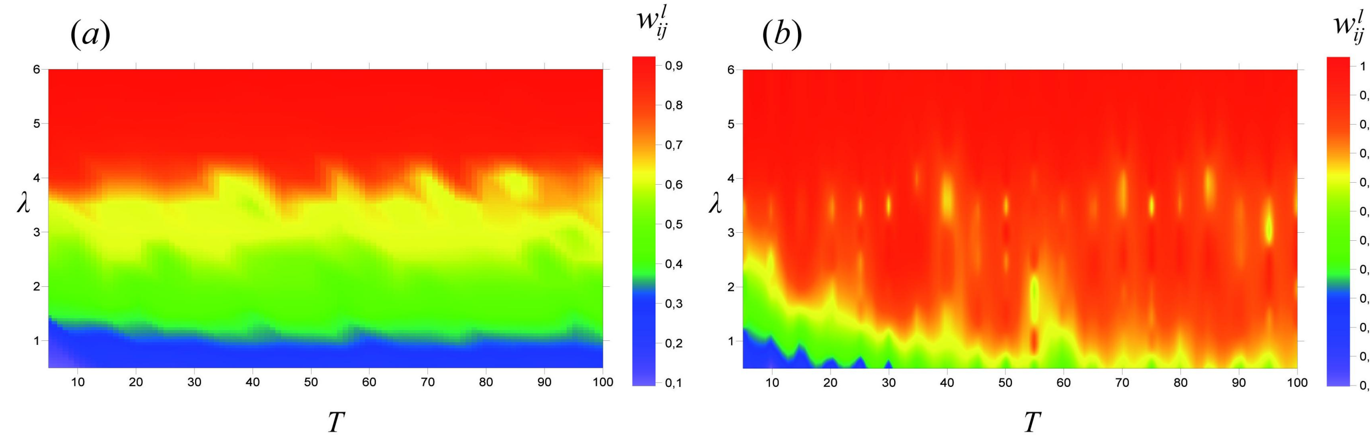

Figure 3. Two-parametric dependence on the coupling strength λ of the characteristic adaptation time T: layers order parameter $r_{layer}(a)$ and the local order parameter $r_{local}(b)$

the weights of connections began to be integrated together with the equations of the Kuramoto model. For the selection of the stationary structures we have calculated parameter, which represents the total change of the matrix relations related to the one step of integration:

$$\gamma = \frac{1}{M}\sum_{l=1}^{M}\sqrt{\sum_{i,j}^{N}\left[\omega_{ij}^{l}(t) - \omega_{ij}^{l}(t-1)\right]^2} \tag{5}$$

Used the distribution of the weights of connections we obtained the stationary structures and found that for the low value of the coupling parameter the layers of network started to demonstrate scale-free structure[1] with different layers. It is a network whose degree distribution follows a power law, at least asymptotically, as evidenced by the type of distributions on fig. $2(a,\ b)$. In such network structures, the distribution interactions of nodes get uneven. Along with the presence of several elements with lots of interactions, these nodes are called Hubs, there is a large number of nodes with few links. When we increased the strength of the connection power-law distribution breaks down. Scale-free network structure is destroyed, because it formed a large area where we can see that increasing the number of nodes with a large number of connections, which are linked and form a tightly coupled homogeneous clusters, fig. $2(c,\ d)$

Using the developed model were number of calculated dynamics of the competition process in a multilayer network of oscillators with changing control parameters of the adaptation time, T, and the coupling strength, λ.

To obtain a complete picture of the evolution of the network topology, we determined the order parameter, which characterizes the degree of synchronization of any selected pair of elements averaged for all pairs of the whole network as:

$$r_{local}(t) = \frac{1}{MN}\sum_{l=1}^{M}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\omega_{ij}^{l}(e^{\sqrt{-1}(\varphi_i^l - \varphi_j^l)}\right) \tag{6}$$

and the second order parameter, which characterizes the degree of synchronization of nodes within the single level, averaged over all levels of the network:

$$r_{layer}(t) = \frac{1}{MN}\sum_{l=1}^{M}\left|\sum_{i=1}^{N} e^{\sqrt{-1}\varphi_i^l}\right| \tag{7}$$

Was built several two-parameter dependencies, and it was found that at coupling strength raises the area where r_{layer} - the synchronization inside the network layers is very small, represented on fig. $3(a)$, whereas the local order parameter r_{local} is high, illustrated on fig. $3(b)$. This shows the emergence of homogeneous clusters that

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are loosely connected. Found that modes global synchronous dynamics are characterized by the occurrence of identical topology of the interacting layers and a homogeneous distribution of connections within each layer.

We also calculated parameter:

$$w_{ij}^d = \sum_{i=1}^{N}\sum_{j=1}^{N} |w_{ij}^1 - w_{ij}^2| \tag{8}$$

which is the total difference between the weights of the connections between two points in time (fig. 4). Presented dependence shows that the small value of the force connection λ leads to the formation of different topologies on different levels of the network. This illustration has a irregularity character, because the state of the system is highly dependent on the initial conditions.

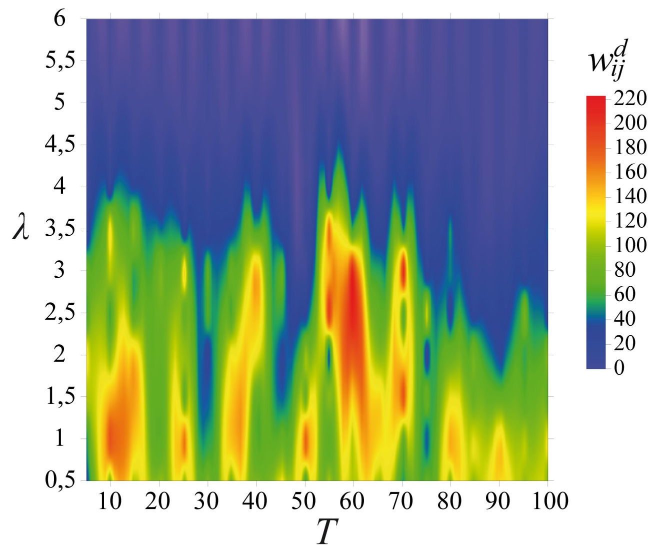

Figure 4. The total difference between the weights of the connections between two points in time.

4. CONCLUSION

We considered the fundamental multi-layer model of the adaptive network of phase oscillators, where the topology of the links between elements is controlled by the feedback with the dynamic of the elements in the accordance with the additivity principles[3] and homeostasis.[6] Using the developed model we numerically studied the processes of competition and synchronization between the coupled nodes, belonging to the different layers depending on the value of control parameters, represented themselves as the adaptation time, T, and the intra-layer coupling strength, λ.

We found that the regimes of the global synchronous dynamics occurred for the larger values of the coupling strength and were characterized by the identical topology of the interacting layers and a homogeneous distribution of the link strength within each layer. We also shown that the partial or cluster synchronization, occurred for the small values of the coupling strength, led to the emergence of the scale-free topology, within the layers.

The obtained results can be considered as the universal phenomena which can take place in the wide range of the real system, including the neural networks of the brain.[11] In particular, the observed dynamical state, which is characterized by the global synchronization between the layers can by associated with the hyper-synchronuous dynamics of the neural network,[12–15] which take place during the onset of the epileptic seizure due to the high degree of the interaction between the cortex and thalamus.[16,17] The regime of the cluster synchronization can be vise-versa associated with the background brain activity, where the different areas of the brain interact weakly, while take part in the different form of the cognitive activity and precursor activity of epileptic seizure.[18]

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It should be noted that despite the simplicity of the the considered network model, it demonstrates the effects, which reflects the key properties of the real networks, in which the nature of the nodes and links is much more complicated. We believe that the proposed model of adaptive network can be used as the base model for the development of the more specific and realistic structures, according to the key features of the concrete objects, which provide the possibility to study and reveal new phenomena in the real-world networks including brain neuronal network.

5. ACKNOWLEDGMENTS

This work has been supported by the Russian Foundation for Basic Research (Grant No. 15-02-00624) and the Ministry of Education and Science of Russian Federation (Grant No. 3.861.2017/PCh).

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