Space-charge-limited current in an external magnetic field

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***Abstract*—This work is devoted to the study of one of the fundamental properties of the beam, namely, space-charge-limited current in a cylindrical drift tube located in an external magnetic field of various intensities. The value of the external magnetic field is determined for different beam energies, at which a sharp change in the critical current and beam dynamics occurs.**

***Index Terms*—nonlinear dynamics, electron beams, virtual cathode**

I. Introduction

One of the fundamental issues of electronics and plasma physics is the analysis of the nonlinear dynamics of the various systems [1]–[6]. The processes of interaction of electromagnetic fields with electron beams in the VC formation mode have a unique combination of fundamental and applied significance for high-power relativistic electronics, radiophysics, and plasma physics [7]–[13]. Indeed, the collective effects of space charge leading to the formation of VCs are characteristic of various types of vacuum electronics devices (vircators, relativistic klystrons, etc.) [14]–[20].

At the same time, beam-plasma system with VC is a typical active distributed electron-wave medium capable of demonstrating various nonlinear effects such as the formation and interaction of electronic structures, turbulence, etc. [21]–[29], [29]–[31]. Powerful electromagnetic radiation generated by devices with VC can be used in various practical applications [1], [2]. This work is devoted to the study of one of the fundamental properties of the beam, namely, space-charge-limited current in a cylindrical drift tube located in an external magnetic field of varying intensity.

II. Main Results

It is well known that when an electron beam is injected into the equipotential drift space, the potential sags due to the action of space charge forces. In this case, the potential sagging in the system increases with an increase in the injected current. As a result, in the region between the injection plane and the potential minimum, the electron beam is decelerated until it stops completely and turns back to the injection plane at a current equal to the critical one. Such processes lead to the accumulation of negative charge in the region of the potential minimum, which contributes to its further sagging.

Let's consider the results of analysis of the critical current. Note, that this current is an important value for understanding the physical processes in VC-based generators from both fundamental and applied points of view since it determines the starting conditions for their generation. The critical current in a vacuum for a completely magnetized electron beam is determined by the following expression:

$$I_{cr} = \frac{mc^3}{e}\frac{(\gamma_0^{3/2}-1)^{3/2}}{G}, \qquad (1)$$

where γ_0 is a relativistic factor, G is a geometric factor equal $G = \frac{d}{r_b} + 2ln\frac{R_w}{r_b}$ for a tubular electron beam, and $G = 1 + 2ln\frac{R_w}{r_b}$ – for a continuous one, r_b is the average radius of the beam, R_w is the radius of the drift tube, d is the thickness of the beam.

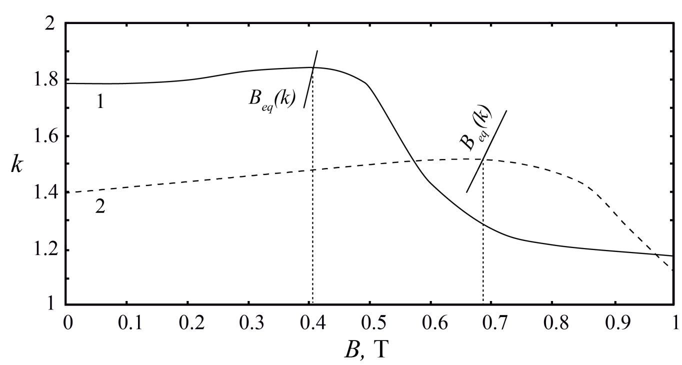

Fig. 1. Dependence of the normalized critical current on the external longitudinal magnetic field B for a beam in a cylindrical drift chamber of radius $R_w = 10$ mm. Curve 1 – critical current for the beam energy $W = 400$ keV; Curve 2 – critical current for the beam energy $W = 800$ keV; $B_{eq}(k)$ – dependence of the magnetic field on the injected current at which the expanding beam reaches the radius of the drift tube.

Figure 1 shows the dependences of the critical current normalized to the current determined by the formula (1) on the

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external longitudinal magnetic field B for two values of the beam energy (400 and 800 keV). One can see that the critical current smoothly increases with an increase in the magnetic field to a certain characteristic value B_{eq}, upon overcoming which the critical current in the system falls.

Consider the motion of electrons in a magnetic field. We will assume that the external magnetic field is strong enough that the intrinsic magnetic fields of the beam can be neglected. Let the electron flow with current I_0 have radius R_b in the injection region, and radius R in the VC region. When electrons move in a constant magnetic field between points with radii R_b and R, they acquire an angular momentum, which is proportional to the difference between the induction flows through the REB cross-sections at points with radii R_b and R [32]:

$$R^2\frac{d\theta}{dt} = \frac{\eta B_0}{2\gamma_0}\left(R^2 - R_b^2\right), \quad (2)$$

where $d\theta/dt$ is the azimuthal velocity of the electrons. The motion of electrons is determined by the action of the centrifugal force $F_c = \gamma_0 m_e r\left(d\theta/dt\right)^2$, the Coulomb repulsive force $F_k = -eE_r$ and the Lorentz force $F_L = -er(d\theta/dt)B_0$ (where e and m_e are the charge and mass of the electron, respectively, r is the radial coordinate of the electron, E_r is the radial component of the space charge field strength). Considering the above, the relation (2) and the equation $d^2r/dt^2 = (2\eta V_0/\gamma_0)d^2r/dz^2$, one can write down the equation of motion for the boundary electron of the beam:

$$\frac{d^2r}{dz^2} + \frac{\eta B_0^2}{8V_0\gamma_0}R\left[1 - (\frac{R_b}{R})^4\right] - \frac{I_0\sqrt{\gamma_0}}{4\pi\varepsilon_0\sqrt{2\eta}V_0^{3/2}R} = 0, \quad (3)$$

where V_0 is the accelerating voltage.

From the equation (3) it follows that there is a characteristic value of the beam current I_{ch} for a fixed external magnetic field B_0 at which the relativistic electron beam (REB) maintains a constant radius R. Indeed, we can determine it if we put $d^2r/dz^2 = 0$ in the equation (3), which means there is no acceleration in the radial direction. Let us fix $R = R_w$ and find the dependence of the equilibrium value of the magnetic field on the value of the injected current:

$$B_{eq}(k) = R_w\sqrt{\frac{\sqrt{2}kI_{cr}\gamma_0^{3/2}}{\pi\varepsilon_0\eta^{3/2}\sqrt{V_0}(R_w^4 - R_b^4)}} \quad (4)$$

where B_{eq} is the equilibrium value of the magnetic field, r_b is the initial radius of the considered electron, I_{cr} is the critical current in a vacuum for a completely magnetized electron beam, k is the ratio of the injected current to I_{cr}.

To understand the physical mechanisms responsible for the sharp drop in the critical current, it is necessary to consider the dynamics of the relativistic electron beam in a certain neighborhood of B_{eq}. Let us describe the beam dynamics for three characteristic values of the magnetic field.

- At $B < B_{eq}$, the injected electron beam is rapidly expanded under the action of space charge forces and is deposited on the walls of the waveguide. In this case, the potential sagging in the beam is observed in the region between the injection plane and the nearest point of beam settling on the waveguide walls. In this case, the critical current is relatively large due to the fact that expanding of the beam significantly reduces the space charge density.
- At $B \approx B_{eq}$, the external magnetic field compensates the effect of the Coulomb forces. When propagating in the drift chamber due to the Coulomb forces of the space charge, the electron beam acquires a radial velocity directed to the walls of the waveguide and begins to twist in the external magnetic field. Upon reaching a radius of the order of the drift tube, the beam is compressed to the radius approximately equal to the injection radius. Note that when it enters the drift space, the longitudinal beam velocity decreases, while the radial velocity, on the contrary, increases. When approaching the walls of the drift chamber, the REB electric field lines are closed on the walls of the drift chamber, that leads to an increase in the critical current in this region of space, despite the fact that the longitudinal beam velocity has decreased. Further, when the beam reaches the equilibrium radius, the radial velocity changes its sign, the beam begins to compress and the reverse process begins. The field lines become less closed on the walls and the critical current for this region of space falls. When it becomes less than the beam current, a VC forms. Note that a VC forms much farther from the injection plane, in comparison with the previous case.
- At $B > B_{eq}$ the equilibrium beam radius becomes less than the drift tube radius. The beam does not settle on the walls. At the same time, this leads to an increase in the space charge density and a greater deceleration of the bunch in the longitudinal direction when entering the drift tube. A decrease in the equilibrium radius also leads to a smaller closure of the electric field lines on the walls. Thus, the combination of these factors leads to a sharp decrease in the critical current and its approach to the theoretical value for a completely magnetized beam.

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REFERENCES

[1] J. Benford, J. A. Swegle, and E. Schamiloglu, *High Power Microwaves*, third edition ed., ser. Series in Plasma Physics. CRC Press, Taylor and Francis Group, 2016.

[2] V. D. Selemir, A. E. Dubinov, V. V. Voronin, and V. S. Zhdanov, "Key ideas and main milestones of research and development of microwave generators with virtual cathode in rfnc-vniief," *IEEE Transactions on Plasma Science*, 2020.

[3] V. V. Makarov, D. Kirsanov, M. Goremyko, A. Andreev, and A. E. Hramov, "Nonlinear dynamics of the complex multi-scale network," in *Saratov Fall Meeting 2017: Laser Physics and Photonics XVIII; and Computational Biophysics and Analysis of Biomedical Data IV*, vol. 10717. International Society for Optics and Photonics, 2018, p. 1071729.

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[4] A. V. Sadovnikov, A. Grachev, E. Beginin, S. Odintsov, S. Sheshukova, Y. P. Sharaevskii, and S. A. Nikitov, "Spatial dynamics of hybrid electromagnetic spin waves in a lateral multiferroic microwaveguide," *JETP Letters*, vol. 105, no. 6, pp. 364–369, 2017.

[5] A. Andreev, O. Moskalenko, A. Koronovskii, and A. Hramov, "Chaos and its suppression in a system of two coupled rydberg atoms," *Bulletin of the Russian Academy of Sciences: Physics*, vol. 79, no. 12, pp. 1432–1434, 2015.

[6] A. V. Sadovnikov, A. A. Grachev, S. A. Odintsov, S. E. Sheshukova, Y. P. Sharaevskii, and S. A. Nikitov, "Spin-wave transport along in-plane magnetized laterally coupled magnonic stripes," *IEEE Magnetics Letters*, vol. 8, pp. 1–4, 2017.

[7] S. Kurkin and A. Hramov, "Virtual cathode formation in annular electron beam in an external magnetic field," *Technical Physics Letters*, vol. 35, no. 1, pp. 23–25, 2009.

[8] S. Kurkin, A. Badarin, A. Koronovskii, and A. Hramov, "The development and interaction of instabilities in intense relativistic electron beams," *Physics of Plasmas*, vol. 22, no. 12, p. 122110, 2015.

[9] S. Kurkin, A. Koronovskii, and A. Hramov, "Output microwave radiation power of low-voltage vircator with external inhomogeneous magnetic field," *Technical Physics Letters*, vol. 37, no. 4, pp. 356–359, 2011.

[10] S. Kurkin, A. Hramov, and A. Koronovskii, "Nonlinear dynamics and chaotization of virtual cathode oscillations in annular electron beam in uniform magnetic field," *Plasma Phys. Rep*, vol. 35, no. 8, pp. 628–642, 2009.

[11] A. Hramov, A. Koronovskiy, S. Kurkin, and I. Rempen, "Chaotic oscillations in electron beam with virtual cathode in external magnetic field," *International journal of electronics*, vol. 98, no. 11, pp. 1549–1564, 2011.

[12] A. E. Dubinov, A. G. Petrik, S. A. Kurkin, N. S. Frolov, A. A. Koronovskii, and A. E. Hramov, "Beam-plasma instability in charged plasma in the absence of ions," *Physics of Plasmas*, vol. 23, no. 4, p. 042105, 2016.

[13] A. Hramov, S. Kurkin, A. Koronovskii, and A. Filatova, "Effect of self-magnetic fields on the nonlinear dynamics of relativistic electron beam with virtual cathode," *Physics of Plasmas*, vol. 19, no. 11, p. 112101, 2012.

[14] S. Kurkin, N. Frolov, A. Rak, A. Koronovskii, A. Kuraev, and A. Hramov, "High-power microwave amplifier based on overcritical relativistic electron beam without external magnetic field," *Applied Physics Letters*, vol. 106, no. 15, p. 153503, 2015.

[15] E. Egorov, A. Koronovskii, S. Kurkin, and A. Hramov, "Formation and nonlinear dynamics of the squeezed state of a helical electron beam with additional deceleration," *Plasma Physics Reports*, vol. 39, no. 11, pp. 925–935, 2013.

[16] S. Kurkin, A. Koronovskii, and A. Hramov, "Formation and dynamics of a virtual cathode in a tubular electron beam placed in a magnetic field," *Technical Physics*, vol. 54, no. 10, p. 1520, 2009.

[17] A. E. Dubinov, A. G. Petrik, S. A. Kurkin, N. S. Frolov, A. A. Koronovskii, and A. E. Hramov, "Virpertron: A novel approach for a virtual cathode oscillator design," *Physics of Plasmas*, vol. 24, no. 7, p. 073102, 2017.

[18] S. Kurkin, A. Koronovskii, and A. Hramov, "Effect of the electron beam modulation on the sub-thz generation in the vircator with the field-emission cathode," *Journal of Plasma Physics*, vol. 81, no. 3, 2015.

[19] N. S. Frolov, S. A. Kurkin, A. A. Koronovskii, and A. E. Hramov, "Nonlinear dynamics and bifurcation mechanisms in intense electron beam with virtual cathode," *Physics Letters A*, vol. 381, no. 28, pp. 2250–2255, 2017.

[20] N. S. Frolov, S. A. Kurkin, A. A. Koronovskii, A. E. Hramov, and A. O. Rak, "High-efficiency virtual cathode oscillator with photonic crystal," *Applied Physics Letters*, vol. 113, no. 2, p. 023503, 2018.

[21] Y. A. Kalinin, A. Starodubov, and A. Fokin, "Hybrid vircator microwave oscillator with a nonlaminar electron beam and an electrodynamic section," *Plasma Physics Reports*, vol. 45, no. 8, pp. 770–776, 2019.

[22] S. Mumtaz, J. S. Lim, B. Ghimire, S. W. Lee, J. J. Choi, and E. H. Choi, "Enhancing the power of high power microwaves by using zone plate and investigations for the position of virtual cathode inside the drift tube," *Physics of Plasmas*, vol. 25, no. 10, p. 103113, 2018.

[23] S. Mumtaz, P. Lamichhane, J. S. Lim, S. H. Yoon, J. H. Jang, S. W. Lee, J. J. Choi, and E. H. Choi, "Enhancement in the power of microwaves by the interference with a cone-shaped reflector in an axial vircator," *Results in Physics*, p. 102611, 2019.

[24] S. Mumtaz, L. N. Nguyen, H. Uhm, P. Lamichhane, S. W. Lee, and E. H. Choi, "A novel approach to form second virtual cathode by installing a floating zone plate inside the drift tube," *Results in Physics*, p. 103052, 2020.

[25] A. A. Badarin, S. A. Kurkin, A. A. Koronovskii, A. E. Hramov, and A. O. Rak, "Processes of virtual cathodes interaction in multibeam system," *Physics of Plasmas*, vol. 25, no. 8, p. 083110, 2018.

[26] A. A. Badarin, S. A. Kurkin, A. V. Andreev, A. A. Koronovskii, N. S. Frolov, and A. E. Hramov, "Virtual cathode oscillator with elliptical resonator," in *2017 Eighteenth International Vacuum Electronics Conference (IVEC)*. IEEE, 2017, pp. 1–2.

[27] A. A. Badarin, S. A. Kurkin, A. A. Koronovskii, A. O. Rak, and A. E. Hramov, "Simulation of the development and interaction of instabilities in a relativistic electron beam under variation of the beam wall thickness," *Plasma Physics Reports*, vol. 43, no. 3, pp. 346–353, 2017.

[28] A. Dubinov and V. Tarakanov, "Simulation of a magnetically isolated vircator with an under-limit electron beam," *PLASMA PHYSICS REPORTS*, vol. 46, no. 5, pp. 570–573, 2020.

[29] A. Badarin, S. Kurkin, and A. Hramov, "Multistability in a relativistic electron beam with an overcritical current," *Bulletin of the Russian Academy of Sciences: Physics*, vol. 79, no. 12, pp. 1439–1442, 2015.

[30] N. S. Frolov, S. A. Kurkin, and M. V. Khramova, "Perspective sub-thz powerful microwave generator "nanovircator" for t-rays biomedical diagnostics," *Proc. SPIE.*, no. 9917, p. 991721, 2016.

[31] S. Kurkin, A. Badarin, A. Koronovskii, N. Frolov, and A. Hramov, "Modeling instabilities in relativistic electronic beams in the cst particle studio environment," *Mathematical Models and Computer Simulations*, vol. 10, no. 1, pp. 59–68, 2018.

[32] S. E. Tsimring, *Electron beams and microwave vacuum electronics*. John Wiley and Sons, Inc., Hoboken, New Jersey, 2007.

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