

# Intermittent phase synchronization in human epileptic brain

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## ABSTRACT

We found the intermittent phase synchronization in human epileptic brain. We show that the phases of the synchronous behavior are observed both during the epileptic seizures and in the fields of the background activity of the brain. We estimate the degree of intermittent phase synchronization in both considered cases and found that the epileptic seizures are characterized by the higher degree of synchronization in comparison with the fields of background activity. For estimation of synchronization degree the modification of the method for estimation of zero conditional Lyapunov exponent from time series proposed in [PRE 92 (2015) 012913] has been used.

**Keywords:** intermittent phase synchronization, epileptic brain, human, epileptic seizure, time series, Lyapunov exponent, background activity

## 1. INTRODUCTION

One of the most interesting types of the synchronous behavior observed in physiological systems is the phase synchronization regime.<sup>1,2</sup> It is the generalization of classical synchronization of periodical oscillations on the case of non-autonomous or coupled chaotic systems and means the presence of the phase locking of interacting systems in the absence of any correlations of their amplitudes.<sup>3,4</sup> Near the boundary of the phase synchronization the intermittent behavior is observed.<sup>5,6</sup> In such case the phase locking condition is satisfied only in certain time intervals called by the laminar phases, which are persistently interrupted by the phase slips called by the turbulent phases. Such regime is called by the intermittent phase synchronization. Intermittent phase synchronization is a generic type of the synchronous behavior observed both in physical, biological and physiological systems, for example, during the epileptic activity of animals and humans.<sup>2,7</sup>

The phenomena of “synchronization” and “phase locking” are closely connected with the behavior of zero conditional Lyapunov exponent.<sup>8</sup> In particular, it is known that the transition of zero conditional Lyapunov exponent in the field of the negative values in the non-autonomous systems (demonstrating periodic dynamics) in the presence of noise and coupled chaotic systems indicates the phase synchronization onset. At the same time, the difference between the critical values of the coupling parameter corresponding to the threshold of the phase-locking and moment of transition of Lyapunov exponent in the negative field, can be a high enough. In other words, the zero conditional Lyapunov exponent become negative earlier then the phase synchronization regime onset, and therefore, its value can be considered as the degree of intermittent phase synchronization.<sup>8,9</sup>

It is easy to find the value of zero Lyapunov exponent in the case when the evolution operator is known explicitly.<sup>10</sup> There also several methods allowing to estimate the value of Lyapunov exponent in the case of unknown evolution operator. But all of them have some artifacts, especially high sensitivity to noise and errors, that makes evident the necessity of new algorithm development.<sup>11–15</sup>

In the present paper we propose a new effective method for calculation of the value of zero conditional Lyapunov exponent of interacting systems from time series and apply it for estimation of the degree of intermittent phase synchronization regime in real neurophysiological systems.

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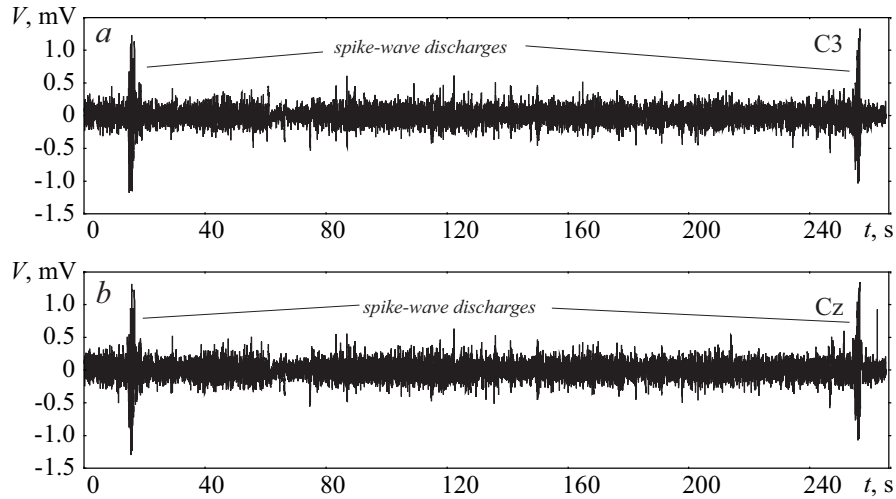


Figure 1. EEG signals taken from channel C3 (a) and Cz (b) of the human brain

## 2. DATA AND METHOD

As the signals under study we have selected real experimental neurophysiological data being the signals of electroencephalograms (EEG) taken from different regions of human brain suffering from epilepsy (see Figure 1). Epileptic EEG are intermittent time series containing the epileptic seizures and the fields of background brain activity. It is known that the epileptic seizures are characterized by a high degree of synchronism.<sup>7,16</sup> At the same time, the fields of background activity are also known to demonstrate the existence of phases with the synchronous behavior.

Figure 2 illustrates the dependencies of the phase difference between the signals taken from channels C3 and Cz of the human brain (the location of the electrodes is shown in<sup>17</sup> in Figure 24). The figures show only the phases of the synchronous behavior. Figure 2,a corresponds to the epileptic seizures, whereas Figure 2,b refers to the synchronous fields of background brain activity. The phases of signals have been introduced into consideration by means of the continuous wavelet transform with the Morlet mother wavelet function.<sup>7,18</sup>

For the synchronous fields of each type of the dynamics a new method for estimation of the value of conditional zero Lyapunov exponent has been applied. By analogy with<sup>9,19</sup> Lyapunov exponent has been calculated by the formula:

$$\Lambda = \int_{x_1}^{x_2} \rho(x) \ln |1 + 2\Omega x| dx \quad (1)$$

where  $\rho(x)$  is the invariant probability density of the  $x$  variable. Probability density  $\rho(x)$  has been calculated numerically and approximated by the analytical regularity

$$\rho(x) = A \exp \left[ -\frac{2}{D} \left( \varepsilon x - \frac{\Omega x^3}{3} \right) \right], \quad (2)$$

where  $A$  is the normalization factor defined from condition

$$\int_{x_1}^{x_2} \rho(x) dx = 1, \quad (3)$$

$D$  is the noise variance,  $\varepsilon$  and  $\Omega$  are the control parameters. The probability density (2) is applicable to the supercritical region of the control parameter  $\varepsilon$  corresponding to the synchronization regime of the flow systems.<sup>8,9</sup>

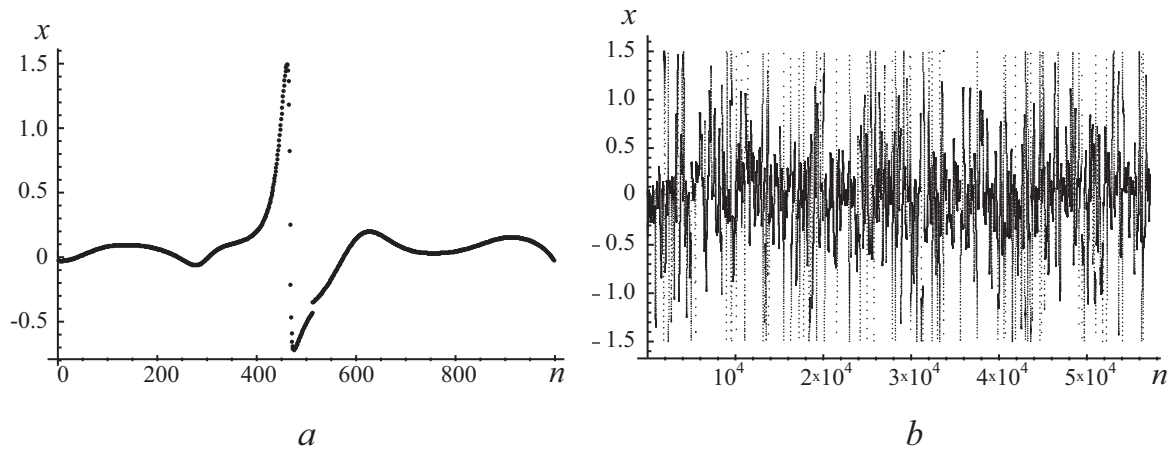


Figure 2. Time dependencies of the phase difference between the signals taken from channel C3 and Cz of the human brain during the epileptic seizures (a) and the synchronous fields of the background brain activity (b)

### 3. RESULTS

The first step is the search of the approximation parameters. To find the parameter  $D$  we have assumed that the distribution of the probability density  $\rho(x)$  is visually similar to the Gaussian probability density distribution which in a simplified form can be written as

$$\rho_G(x) = A_G \exp [-2B(x - K)^2], \quad (4)$$

where  $K$  and  $B$  are analogues of the ensemble average and variance,  $A_G$  is the normalization factor. Parameter  $K$  corresponds to the maximum value of the distribution (4) and can be expressed from the control parameters  $\varepsilon$  and  $\Omega$  of the distribution (2) in the following way:

$$K = -\sqrt{\frac{\varepsilon}{\Omega}}. \quad (5)$$

Expanding the right-hand sides of (2) and (4) in a Taylor series in point (5), limiting the terms of the second infinitesimal order and comparing the coefficients from equal powers one can find the relationship between the parameters  $B$  and  $D$ :

$$D = \frac{\sqrt{\varepsilon\Omega}}{B}. \quad (6)$$

The parameters  $A_G$  and  $K$  can be found by comparison of the maximum values of numerically obtained probability density distribution and its approximation by the regularity (4). Parameter  $B$  can be estimated by the least square method. The relationship between parameters  $D$ ,  $\varepsilon$  and  $\Omega$  is defined by the relationship (6), where the parameter  $B$  is known. This expression makes possible to search the rest of the approximation parameters: the relation between  $A$  and  $\varepsilon$  can be found by the comparison of the maximum of obtained numerically and analytically (2) probability distributions, parameter  $\Omega$  should be estimated by the least square method. The parameters  $x_1$  and  $x_2$  in the formula (1) can be determined empirically from the kind of  $\rho(x)$ .

The proposed method is applied to the epileptic seizures and to the synchronous fields of the background activity. Obtained distributions of the probability density of the phase difference and their approximations by the expression (2) are shown in Figure 3,a (the epileptic seizures) and Figure 3,b (the synchronous fields of the background activity), respectively. The approximation parameters for the fields of epileptic seizures are the following:  $B \approx 56.70$ ,  $D \approx 0.182$ ,  $\varepsilon \approx 0.468$ ,  $A \approx 1.917 \times 10^{-38}$ ,  $\Omega \approx 0.4$ . The same parameters for the fields

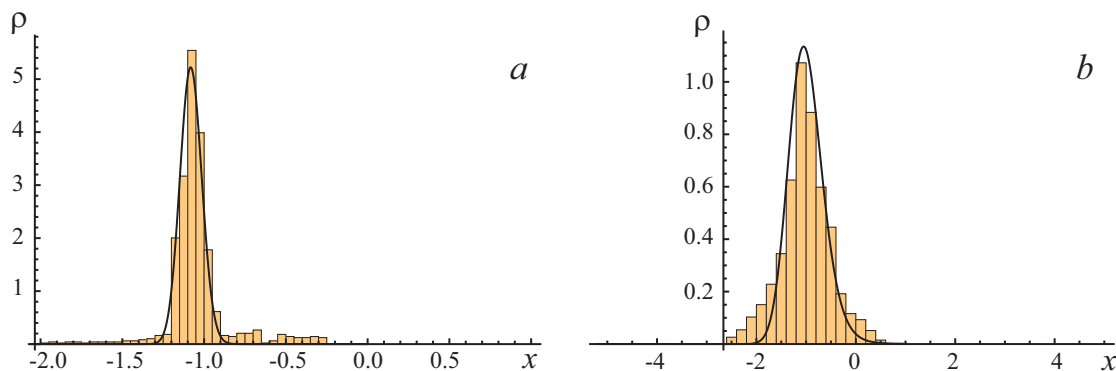


Figure 3. Distributions of the probability density  $\rho(x)$  obtained for the epileptic seizures (a) and the synchronous fields of the background activity (b) (histograms) and their approximations by the regularity (2)

of background activity are obtained as:  $B \approx 2.30$ ,  $D \approx 0.182$ ,  $\varepsilon \approx 0.441$ ,  $A \approx 0.384$ ,  $\Omega \approx 0.4$ . The values of Lyapunov exponent in both cases are negative, and their relation with each other is equal to  $\Lambda_1/\Lambda_2 \approx 1.12$ , that means the higher degree of synchronization of epileptic seizures in comparison with the fields of the background human brain activity.

#### 4. CONCLUSIONS

In conclusion, the method for estimation of zero conditional Lyapunov exponent from time series has been proposed. We have shown that the phases of the synchronous behavior are observed both during the epileptic seizures and in the fields of the background activity of brain. We have estimated the degree of intermittent phase synchronization in both considered cases and found that the epileptic seizures are characterized by the higher degree of synchronization in comparison with the fields of the background activity.

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#### REFERENCES

- [1] Janson, N. B., Balanov, A. G., Anishchenko, V. S., and McClintock, P. V. E., "Phase synchronization between several interacting processes from univariate data," *Phys. Rev. Lett.* **86**(9), 1749–1752 (2001).
- [2] Bob, P., Palus, M., Susta, M., and Glaslova, K., "Eeg phase synchronization in patients with paranoid schizophrenia," *Neuroscience Letters* **447**, 73–77 (2008).
- [3] Rosenblum, M. G., Pikovsky, A. S., and Kurths, J., "Phase synchronization of chaotic oscillators," *Phys. Rev. Lett.* **76**(11), 1804–1807 (1996).
- [4] Postnov, D. E., Balanov, A. G., Janson, N. B., and Mosekilde, E., "Homoclinic bifurcation as a mechanism of chaotic phase synchronization," *Phys. Rev. Lett.* **83**, 1942–1945 (September 1999).
- [5] Pikovsky, A. S., Osipov, G. V., Rosenblum, M. G., Zaks, M., and Kurths, J., "Attractor–repeller collision and eyelet intermittency at the transition to phase synchronization," *Phys. Rev. Lett.* **79**(1), 47–50 (1997).
- [6] Hramov, A. E., Koronovskii, A. A., Kurovskaya, M. K., and Boccaletti, S., "Ring intermittency in coupled chaotic oscillators at the boundary of phase synchronization," *Phys. Rev. Lett.* **97**, 114101 (2006).
- [7] Hramov, A. E., Koronovskii, A. A., Midzyanovskaya, I. S., Sitnikova, E., and Rijn, C. M., "On-off intermittency in time series of spontaneous paroxysmal activity in rats with genetic absence epilepsy," *Chaos* **16**, 043111 (2006).

- [8] Hramov, A. E., Koronovskii, A. A., and Kurovskaya, M. K., “Zero Lyapunov exponent in the vicinity of the saddle-node bifurcation point in the presence of noise,” *Phys. Rev. E* **78**, 036212 (2008).
- [9] Moskalenko, O. I., Koronovskii, A. A., and Hramov, A. E., “Lyapunov exponent corresponding to enslaved phase dynamics: Estimation from time series,” *Phys. Rev. E* **92**, 012913 (2015).
- [10] Benettin, G., Galgani, L., Giorgilli, A., and Strelcyn, J. M., “Lyapunov characteristic exponents for smooth dynamical systems and for Hamiltonian systems: A method for computing all of them. P. I. Theory. P. II. Numerical application,” *Meccanica* **15**, 9–30 (1980).
- [11] Eckmann, J. P., Kamphorst, S. O., Ruelle, D., and Ciliberto, S., “Lyapunov exponents from a time series,” *Phys. Rev. A* **34**(6), 4971–4979 (1986).
- [12] Wolf, A., Swift, J., Swinney, H. L., and Vastano, J., “Determining lyapunov exponents from a time series,” *Physica D* **16**, 285 (1985).
- [13] Bryant, P., Brown, R., and Abarbanel, H. D. I., “Lyapunov exponents from observed time series,” *Physical Review Letters* **65**(13), 1523–1526 (1990).
- [14] Brown, R., Bryant, P., and Abarbanel, H. D. I., “Computing the lyapunov spectrum of a dynamical system from an observed time series,” *Phys. Rev. A* **43**(6), 2787–2806 (1991).
- [15] Porcher, R. and Thomas, G., “Estimating lyapunov exponents in biomedical time series,” *Phys. Rev. E* **64**(1), 010902(R) (2001).
- [16] Hramov, A. E., Koronovskii, A. A., Makarov, V. A., Pavlov, A. N., and Sitnikova, E., [*Wavelets in Neuroscience*], Springer Series in Synergetics, Springer, Heidelberg, New York, Dordrecht, London (2015).
- [17] Pavlov, A. N., Hramov, A. E., Koronovskii, A. A., Sitnikova, Y., Makarov, V. A., and Ovchinnikov, A. A., “Wavelet analysis in neurodynamics,” *Physics-Uspekhi* **55**(9), 845–875 (2012).
- [18] Hramov, A. E. and Koronovskii, A. A., “An approach to chaotic synchronization,” *Chaos* **14**(3), 603–610 (2004).
- [19] Moskalenko, O. I., Koronovskii, A. A., Hramov, A. E., and Zhuravlev, M. O., “Estimate of the degree of synchronization in the intermittent phase synchronization regime from a time series (model systems and neurophysiological data),” *JETP LETTERS* **103**(8), 539–543 (2016).