# Rotating Wave Dynamics in Rings of Coupled Oscillators: Insights into Working Memory Models

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***Abstract*—This paper explores rotating waves, a captivating synchronization phenomenon observed in interconnected oscillators. These waves emerge from phase differences between neighboring oscillators, giving rise to stable periodic, quasiperiodic, or chaotic orbits. Rotating waves are observed in diverse nonlinear systems like electrical circuits, neural models, and various oscillators. The study employs techniques such as time series and phase-space analyses, bifurcation diagrams, power spectra, and basins of attraction to unveil the intricate dynamics. These methods reveal the route from stable equilibria to hyperchaos through a series of bifurcations with increasing coupling strength, including Andronov–Hopf, torus, and crisis bifurcations. Notably, the research identifies instances of multiple coexisting rotating waves under the same parameters. Special attention is given to neural oscillators due to their significance in brain neural rings associated with working memory. The study of rotating waves finds broad relevance in fields like lasers, chemical reactions, cardiorespiratory systems, and particularly neural networks and brain function.**

***Index Terms*—nonlinear dynamics, synchronization, multistability, neural networks**

## I. Introduction

The exploration of collective dynamics in coupled oscillators has captivated researchers across diverse scientific disciplines. Its importance has been greatly magnified by its applications in engineering, biomedicine, and communications [1]. Notably, the ring configuration stands out for its unique ability to generate rotating waves along interconnected nodes [2]. These rotating waves emerge due to phase differences among adjacent oscillators. Their initial observation occurred within reaction-diffusion systems [3], with subsequent experimental confirmation [4]. The genesis of rotating waves can be attributed to the Andronov–Hopf, torus, and crisis bifurcations as the coupling strength is increased [5]–[14]. Significantly, one-way communication plays a crucial role in neural networks leading to the appearance of rotating waves [15], [16]. Therefore, the characterization of rotating waves holds relevance within neuroscience and biomedicine [17]–[20].

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This paper provides a short review of findings on rotating waves in unidirectional rings of coupled Duffing, laser, and neural oscillators. In conclusion, we provide insights into network models of working memory.

## II. Rotating Waves in Duffing Oscillators

The dynamics of a cyclic ring comprising $N$ identical double-well damped Duffing oscillators can be mathematically expressed through the subsequent system of second-order differential equations:

$$\begin{array}{rcl} \ddot{x}_1 + a\dot{x}_1 + \omega_0^2 x_1 + \delta x_1^3 + \sigma(x_1 - x_N) & = & 0, \\ \ddot{x}_2 + a\dot{x}_2 + \omega_0^2 x_2 + \delta x_2^3 + \sigma(x_2 - x_1) & = & 0, \\ \vdots & & \vdots \\ \ddot{x}_N + a\dot{x}_N + \omega_0^2 x_N + \delta x_N^3 + \sigma(x_N - x_{N-1}) & = & 0, \end{array} \quad (1)$$

where $a = 0.4$, $\omega_0^2 = -0.25$, $\delta = 0.5$ are constants [8], [21], [22] and $\sigma$ is the coupling strength used as a control parameter.

In isolation, every oscillator resides within one of two stable fixed points. However, in the unidirectional ring coupling, the system showcases an array of intricate behaviors, spanning from coexisting stable fixed points to hyperchaos as the coupling strength $\sigma$ is increased.

Distinct coexisting attractors emerge and vanish with respect to the coupling strength and the number of oscillators in the ring. This phenomenon holds significant fascination [23]. Therefore, the coupling strength and the number of oscillators intricately regulate the system dynamics. For instance, it was found that in an $N = 11$ oscillator ring, very weak coupling ($\sigma < 0.06$) yields 32 coexisting stable fixed points. With stronger coupling ($0.06 < \sigma < 0.275$), 11 attractors coexist, encompassing two original fixed points and 9 periodic orbits from the Andronov-Hopf bifurcation at $\sigma \approx 0.6$. As $\sigma$ increases ($0.275 < \sigma < 0.34$), chaos emerges alongside the persisting original fixed points. With high coupling ($\sigma > 0.34$), solely the chaotic attractor endures.

The presence of multiple oscillatory attractors leads to the emergence of multiple circulating rotating waves within the ring.

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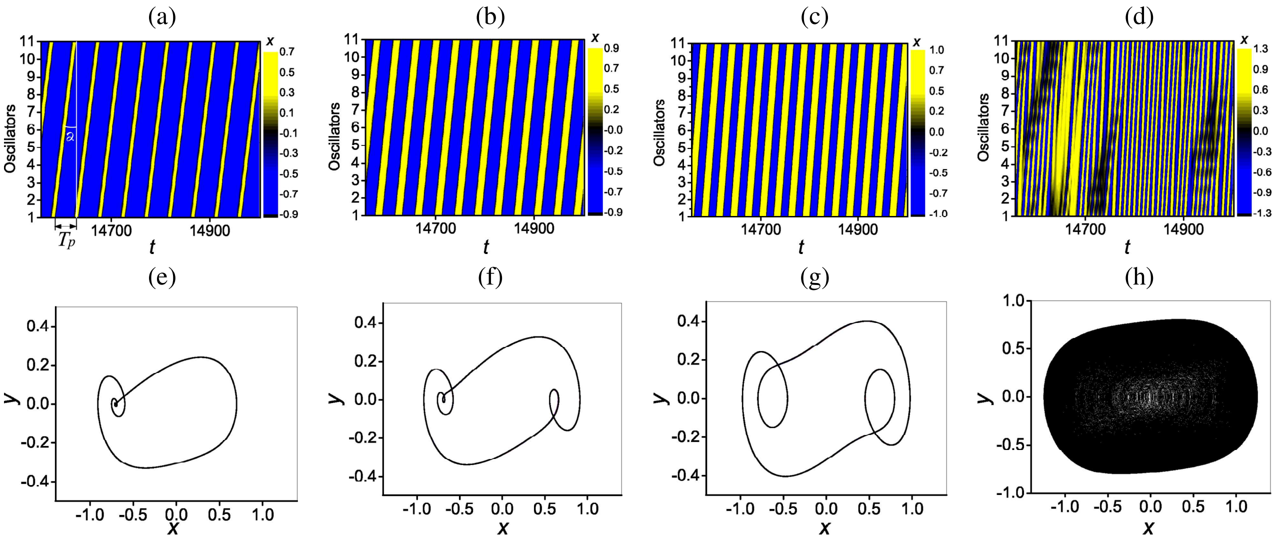


Fig. 1. (a–d) Rotating wave time series and (e–h) phase portraits of corresponding attractors in the ring of $N = 11$ oscillators for (a,e) $\sigma = 0.18$ (homoclinic orbit $Ho_2$), (b,f) $\sigma = 0.192$ (asymmetric heteroclinic orbit $HeA_2$), (c,g) $\sigma = 0.236$ (symmetric heteroclinic orbit HeS), and (d,h) $\sigma = 0.308$ (chaos). $T_p$ is the phase propagation time from node 1 to node 11, $\alpha$ is the inclination angle which defines wave speed $\mathcal{W}$ derived by Eq. 2.

Rotating waves are shown through time series and phase portraits in Fig. 1. This figure presents wave dynamics in the $N = 11$ Duffing oscillator ring for four coupling strength values: $\sigma = 0.18$, $\sigma = 0.192$, $\sigma = 0.236$, and $\sigma = 0.308$. The phase shift between the oscillators forms the propagating cyclic wave.

Figs. 1(a–d) depict time series patterns of all oscillators illustrating their behavior over time. The lower row displays corresponding attractor phase portraits in the $(x_i, y_i)$ projection $(i = 1, \dots, N)$. Oblique stripes in time series signify the rotating waves visually. The inclination angle $\alpha$, marked in Fig. 1(a), defines the phase shift velocity or wave speed $\mathcal{W}$. Simultaneously, wave frequency $f_w$ is discernible from the time series as the inverse period $T_w$ or through power spectra analysis. Note that phase portraits in Figs. 1(e–h) are consistent among identical oscillators. An intriguing feature is the growth of attractor size with increasing coupling strength.

The rotating wave speed can be computed as:

$$\mathcal{W} = \arctan \frac{T_p}{N-1}, \tag{2}$$

where $T_p$ represents the time taken for phase propagation from node 1 to node $N$. It is evident that as coupling strength rises, $\mathcal{W} = \alpha$ decreases.

Hence, rotating waves in the unidirectionally coupled double-well Duffing oscillator ring showcase intricate dynamics. Wave speed analysis involves studying the inclination angle of parallel stripes in time series. Notably, the wave velocity remains fairly constant across coexisting limit cycles, yet escalates exponentially with enhanced coupling strength. In contrast, wave frequency varies among coexisting attractors, exhibiting an almost linear growth tied to coupling strength.

## III. Rotating Waves in Lasers

Now, consider the emergence of rotating wave in the ring of three coupled erbium-doped fiber lasers (EDFLs). The dynamics of a single EDFL is governed by two differential equations representing laser intensity $x_j$ $(j = 1, 2, 3)$ and population inversion $y_j$ [14]:

$$\begin{aligned} \frac{dx_j}{d\theta} &= a x_j y_j - b x_j + c(y_j + 0.3075), \\ \frac{dy_j}{d\theta} &= d x_j y_j - (y_j + 0.3075) + P_{pmod_j}\Bigg\{1 - \\ &\quad - \exp\left[-18\left(1 - \frac{1-(y_j+0.3075)}{0.6150}\right)\right]\Bigg\} \end{aligned} \tag{3}$$

with pumping

$$P_{pmod_j} = 506\big[1 + k(x_{j-1} - x_j)\big], \tag{4}$$

Here, $k$ signifies the coupling coefficient, while $x_j$ and $y_j$ denote laser intensity and population inversion with parameters $a = 6.62 \times 10^7$, $b = 7.4151 \times 10^6$, $c = 0.0163$, and $d = 4.0763 \times 10^3$.

Rotating wave dynamics within the EDFL ring are depicted in Fig. 2 across four coupling strengths: $k = 2.58$, $k = 3.82$, $k = 5.49$, and $k = 5.84$. Oscillators differ only in initial phases, inducing phase shifts between nodes and creating rotating waves. The left column portrays two-dimensional time series patterns, revealing oblique stripes denoting rotating wave presence. Stripe inclination signifies wave propagation speed. The right column showcases identical phase portraits for corresponding attractors due to system symmetry. Notably, higher coupling strength results in larger attractors. Fig. 2 highlights various rotating wave types: a) periodic, b) quasiperiodic with two frequencies in the 2D torus, c) quasiperiodic with three frequencies in the 3D torus, and

d) chaotic. The figure's strip slope variation with coupling strength $k$ reveals decreasing wave speed. In chaotic scenarios, the wave transforms into a standing wave, ceasing propagation (vertical strips in Fig. 2(d)).

## IV. Rotating Waves in Neural Oscillators

Finally, consider the key contributions in the exploration of rotating waves within neural models. Unidirectional ring structures in neural networks are pivotal for modeling neural dynamics and investigating information processing in neuroscience and medicine. Such ring motifs are crucial for generating stable periodic motor commands via central pattern generators, responsible for rhythmic animal locomotion [24]. Additionally, ring-like topologies manifest in sequential cortico-striatal-basal ganglia-thalamo-cortical motor loop projections, crucial for understanding Parkinson's disease [25]. Consequently, scrutinizing rotating waves in neural models holds significant relevance.

Several studies have been devoted to rotating waves in unidirectional neural rings. The researchers used either a simple sigmoidal neuron model [15] or the more advanced FitzHugh–Nagumo (FHN) model [26]. Both approaches successfully identified rotating wave presence. Given the FHN model's precision, we focus on its outcomes in this review.

The dynamics of a ring comprising FHN neurons, coupled unidirectionally through chemical synapses, are described by the subsequent equations:

$$\begin{aligned} \dot{v}_j &= v_j - v_j^3/3 - w_j + I_j + C(V - v_j)sj + 1(t - \tau), \\ \dot{w}_j &= 0.08(v_j + 0.7 - 0.8w_j), \\ \dot{s}_j &= 0.5\frac{1 - s_j}{1 + \exp\left[-4(v_j - 1.5)\right]} - 0.6s_j, \end{aligned} \tag{5}$$

Here, $v_j$ and $w_j$ ($j = 1, 2, \ldots, N$) represent fast and slow variables linked respectively to the membrane potential and sodium channel reactivation and potassium channel deactivation of an individual neural cell $j$. $s_j$ symbolizes postsynaptic potential for synaptic coupling, $I_j$ regulates neural spiking dynamics as an external input, drawn from a Gaussian distribution around mean value $\bar{I} = 0.4$ with standard deviation $\sigma = 0.005$. $C$ stands for coupling strength, and $\tau$ denotes delay time. The reversal potential $V = 2$ models excitatory coupling. The unidirectional ring configuration is modeled as $sN + 1 = s_1$.

Fig. 3 portrays spatial dynamics of a 200-neuron ring with and without coupling delay ($\tau = 0.03$) as shown in Figs. 3(a) and (b) respectively. Rotating waves, with distinct speeds and frequencies contingent on initial conditions, manifest in synchronous states.

First, examine the pattern sans delay ($\tau = 0$) in Fig. 3(a). Uncoupled ($C = 0$) neurons fire asynchronously, yielding scattered black circles with no wave patterns. Conversely, elevating coupling strength generates multiple rotating waves. Notably, synchronized state frequency varies depending on initial conditions, signifying multistability in synchronized regimes.

Considering a coupling strength of $C = 5$ and tightly grouped initial conditions, neuron synchronization occurs at a frequency of $f_j = 18.2$, resulting in a stationary wave pattern (vertical red circles). In contrast, for widely dispersed initial conditions, synchronization happens at either $f_j = 18.8$ or $f_j = 17.7$, leading to the emergence of firing fronts depicted by green and blue circles, respectively. Within these synchronized states, dual firing fronts traverse the ring. Each neuron fires subsequent to its neighboring neuron's spike. Remarkably, these firing fronts can move in the direction of synaptic coupling or opposite to it. The slope of the lines in Fig. 3 determines the direction of wave propagation. A rightward slope implies opposing direction of coupling, while a leftward slope signifies coupling direction.

In addition to distinct frequencies, synchronized neurons exhibit diverse spatial configurations of spiking dynamics. Narrowly distributed initial conditions, as in $l = 0$, result in in-phase synchronized oscillators with simultaneous firing (Fig. 3(a), red circles). Conversely, broadly distributed initial conditions establish a rotating-wave regime with one or more firing fronts traversing the ring. Here, each neuron fires following its neighboring neuron's spike. Intriguingly, these firing fronts can travel clockwise or counterclockwise around the ring. For instance, $l = 2$ signifies two counterclockwise-propagating rotating waves (green circles), while $l = -2$ signifies two clockwise-propagating rotating waves (blue circles). Notably, uncoupled and desynchronized neurons lack distinguishable spatial structures, as shown by black circles in the figures.

## V. Insights into Working Memory Models

In this short review, we delved into the fascinating realm of rotating waves observed in rings of coupled identical oscillators. These stable periodic orbits, arising from phase differences between neighboring oscillators, have been observed across diverse nonlinear systems. The study of rotating waves has unveiled their intricate behavior influenced by coupling strength and oscillator count. Analyzing rotating wave dynamics required an array of techniques, including time series analysis, phase-space exploration, and bifurcation diagrams. These tools have unraveled the complex transitions from equilibria to hyperchaos, revealing sequences of Hopf, torus, and crisis bifurcations. This research highlighted the coexistence of multiple rotating waves under the same system parameters.

The insights gained from this thorough exploration hold significant implications across diverse scientific fields. They provide invaluable understanding of the mechanisms that underlie rotating waves in a multitude of real-world systems, ranging from lasers to neural circuits. This increased knowledge not only propels technological progress but also enriches our comprehension of fundamental dynamical principles that govern these intricate phenomena. Yet, amidst these achievements, crucial questions persist. Particularly challenging is unraveling the role of rotating waves within neural networks, potentially shedding light on the mechanisms of working

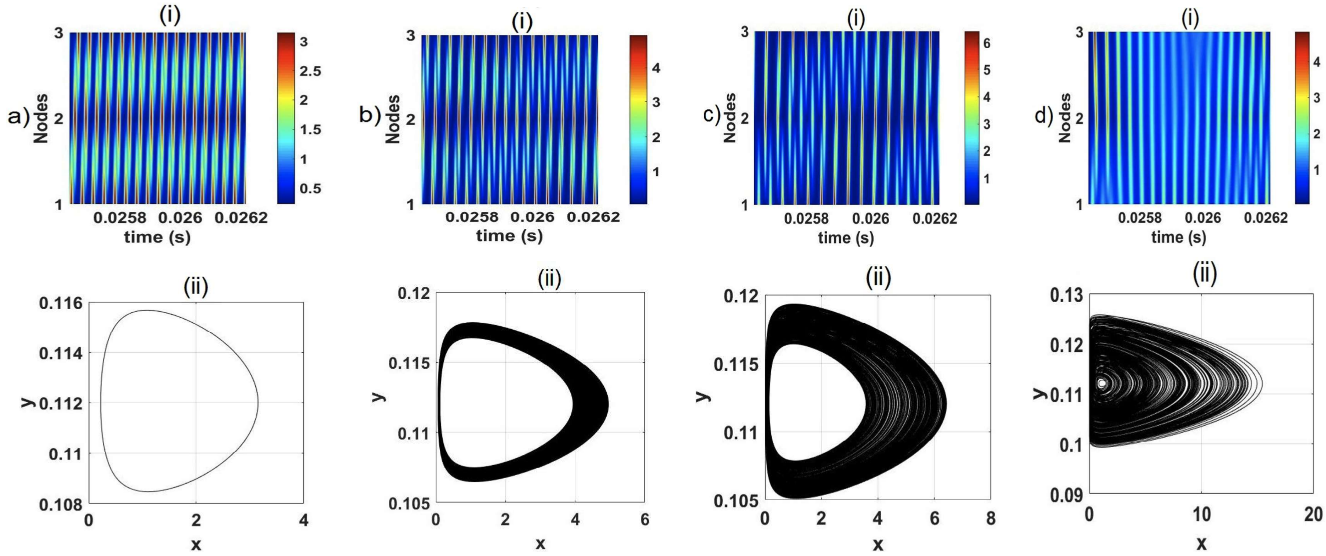


Fig. 2. (Left) Rotating wave and (right) phase portraits for a) $k = 2.58$ (periodic wave), b) $k = 3.82$ (quasiperiodic wave) (2D torus), c) $k = 5.49$ (quasiperiodic wave) (3D torus), and d) $k = 5.84$ (chaotic wave). The color in the left column indicates the laser intensity.

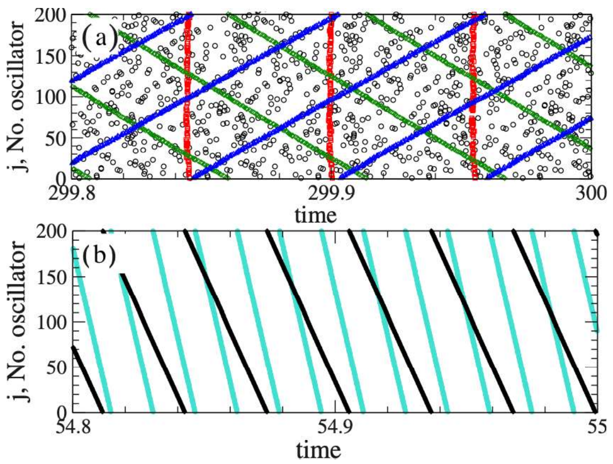


Fig. 3. Synchronized spatial arrangements within the ring of $N = 200$ unidirectionally synaptically coupled spiking neurons characterized by Eqs. (5). (a) Rotating waves in the ring without delay ($C = 0$). Non-uniformly distributed black circles – asynchronous firing, vertically aligned red circles – in-phase synchronization at $C = 5$ and $m = 0$, green circles in lines with negative slopes – two rotating waves propagating in the coupling direction ($C = 5$ and $m = 2$), and blue circles in lines with positive slopes – two rotating waves moving opposite to the coupling ($C = 5$ and $m = -2$). (b) Two coexisting rotating waves in neuronal firing, introducing a delay $\tau = 0.03$ and $C = 5$. Onsets of spikes for different initial conditions are marked by black and grey lines. Based on data from [26].

memory. Memory functions are intricately linked to transient amplifications of neuronal activity, with persistent activity observed even in the absence of external stimulation.

The utilization of network methods in controlled in vitro settings facilitates the examination of these phenomena, shedding light on the structures of working memory networks and the corresponding neural dynamics. The intriguing variability in wave propagation speeds within unidirectional neural rings prompts inquiries about their relationship with the number of neurons in the ring. Additionally, the participation of individual neurons in multiple memory loops underscores the intricate nature of neural networks. Analogous to individuals engaging in multiple social groups, neurons exhibit versatility within memory networks, and this intricate interplay among elementary memory loops contributes to the network's adaptable functionality.

Despite extensive research on rotating waves in various dynamical systems, numerous unresolved questions persist in this field. A particularly challenging endeavor is the exploration of rotating waves within neural networks of the brain, as it holds the potential to unveil the mechanisms underpinning working memory. It is worth noting that several scientists have suggested a close connection between the neural network structure in the brain and short-term (working) memory [27]–[31]. Working memory involves a transient boost in neuronal activity, lasting from milliseconds to seconds, for temporary information storage and cognitive tasks [32], [33].

The observation of sustained activity in local brain circuits, even without external stimulation, has been documented in various brain regions, including the prefrontal cortex and thalamus [34]–[36]. Researchers investigating phenomena like persistent activity and information transmission observed in vivo have employed the network geometry method in vitro. This approach offers controlled experimental conditions to explore such phenomena, shedding light on the network structure of working memory and the corresponding neural dynamics.

Given that connections between brain neurons mainly occur through chemical synapses, the resulting neural rings are usually unidirectional. This prompts a crucial question: How does the speed of wave propagation in neural rings correlate with the number of neurons within the ring?

Additionally, it is essential to recognize that a single neuron can participate in multiple memory loops. As an example, think about the experience of memorizing the name of a new person. This creates a memory loop specifically designed to retain the name, while simultaneously linking to other loops responsible for preserving memories associated with the individual's characteristics, such as their appearance, personality traits, the context of your meeting, and more. This multifaceted neural network enables the integration of diverse memory loops, aiding in the retrieval and association of related information. As a result, memory loops exhibit intricate interconnections, forming complex high-order networks.

Each neuron in such a network can engage in distinct elementary memory loops, contributing to its functional adaptability. This phenomenon mirrors social networks, where an individual might belong to various social groups, each involving different roles and interactions. This multi-dimensional engagement showcases the complexity and adaptability present in both neural and social systems.

In conclusion, this concise review has enriched our comprehension of rotating waves across various contexts, opening avenues for extended investigation and interdisciplinary applications. The captivating dynamics of rotating waves offer valuable insights into models of working memory, shedding light on the complexities of neural systems and their contributions to memory processes.

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