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1 # Studying Noise-Induced Intermittency in Multistable Systems on the Basis of Reference Systems

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Abstract—The noise-induced intermittent behavior of multistable systems is studied using the example of a reference system (a Chua generator). It is shown that upon exposing the Chua generator to external noise, a
1 noise-induced intermittency is observed at certain values of the control parameters. The statistical characteristics obtained for this type of behavior are compared to the appropriate theoretical formulas.

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INTRODUCTION

Multistability is a fundamental physical phenomenon that is observed almost in all spheres of science and technology, including electronics engineering [1], optics [2], mechanics [3], and biology [4]. The term "multistability" was first used in a work devoted to problems of visual perception [5]. For dissipative systems, multistability means the simultaneous existence of several possible finite steady states (attractors) for a fixed set of system parameters. The steady state to which the system tends depends on the initial conditions of the system. In other words, its long-term dynamics corresponding to one of its steady states is determined by its initial conditions. The geometry of multistable systems' basins of attraction can be quite intricate and sometimes even fractal [6].

The domain of the coexistence of many steady states is critical, since a slight noise or any other external disturbance can cause the system to switch from one steady state to another; this in turn can trigger noise-induced intermittent behavior. Note that multistable states are quite characteristic of a wide range of real systems [5, 7]. Real systems are often exposed to noise and fluctuation influences that can be either inherent components of the considered system's dynamics, or caused by external impacts [8, 9]. The multistability observed in such systems in combination with fluctuations and noises can thus cause a multistable system to become metastable, since the noises can constantly switch it from one coexisting state to another. Such noise-induced switching between attractors triggers the intermittent behavior of real systems [10]; in addition, such behavior seems to be quite typical of multistable systems [11].

Despite the considerable interest in multistability and intermittency showed by many researchers, there 1
is still neither a deep understanding of the processes that occur in multistable systems exposed to noise, nor a theoretical description of their intermittent behavior. Many scientific works have been devoted to exploring this type of intermittency; some have shown that 1
noise-induced intermittency occurs in a wide variety 1
of systems (semiconductor lasers [12], erbium-doped fiber-optic laser [13], quantum point flicker [10], sensory neurons [14], Josephson junction [11], and others). Nevertheless, despite all studies of the impact of noise on multistable systems, and all efforts to describe noise-induced intermittency, there is still no general 1
theoretical model explaining this type of behavior. The problem of devising a general theory describing the intermittent behavior in multistable systems exposed to external noise is thus important from both the theoretical and practical points of view.

The aim of this work is to explore intermittent behavior induced by external noise by considering an example of reference multistable systems in continuous time. Statistical characteristics are obtained for this type of system behavior, and the results from numerical simulations are compared to theoretical formulas.

A BISTABLE CHUA GENERATOR

The model of a regulated bistable generator with chaotic dynamics proposed by L. Chua [15] was chosen as our reference system in continuous time. Note that while the canonical Chua generator circuit assumes a piecewise linear characteristic of its nonlinear element [15], the cubic approximation in [16] was

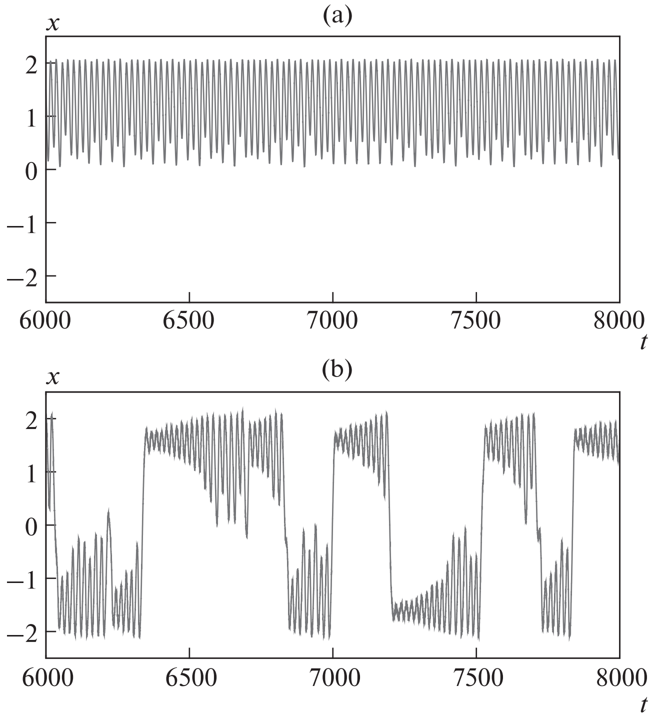

Fig. 1. Time dependence of variations in voltage in a nonlinear element of the Chua generator circuit: (a) noise amplitude $D = 0$; (b) noise amplitude $D = 1$.

used in this work. The equations describing the behavior of a bistable Chua generator with chaotic dynamics are written as

$$\begin{aligned}\frac{dx}{dt} &= y - x - h(x) + D\xi,\\ \frac{dy}{dt} &= \left(\frac{1}{\alpha}\right)(x - y + z),\\ \frac{dz}{dt} &= -\delta(y + \rho z),\end{aligned} \qquad (1)$$

where $\xi(t)$ is a Gaussian random process with a zero mean and unit dispersion; D is the intensity of noise action; variable x characterizes variations in voltage in the nonlinear elements of the circuit; variable y denotes the variations in voltage in the capacitors of oscillatory circuits; and variable z characterizes variations in the current in the inductance coils [16]. The constant coefficients, expressed in terms of the circuit parameters, are $\alpha = 9$, $\delta = 9$. Parameter ρ denotes dissipation; its value was set at $\rho = 0.01$, where autonomous oscillations are observed in one of two attraction basins of the considered system as a result of its initial conditions.

In our numerical study of system of Eqs. (1), the nonlinear element was written in the form of cubic approximation [16]:

$$h(x) = -1.25x + 0.1x^3. \qquad (2)$$

The fourth-order Runge–Kutta method, adapted for stochastic differential equations in [17] with timestep $\Delta t = 0.001$, was used to integrate the system of Eqs. (1). As noted above, the parameters of the considered system were chosen such that, depending on the initial conditions, the studied generator displayed autonomous oscillations in one of two attraction basins; i.e., there were two steady equilibrium positions in the explored system. Figure 1a shows a situation where the amplitude of external noise is zero, and the considered system stays in the vicinity of a certain steady equilibrium position for a certain period of time. When exposed to external noise with sufficiently high amplitude, the system displays sequential transits from one steady equilibrium position to another. In other words, noise-induced intermittent behavior is in this case detected in the Chua generator. This situation is illustrated in Fig. 1b, where the considered system is shown near one of two steady equilibrium positions.

EXPLORING NOISE-INDUCED INTERMITTENCY IN A BISTABLE CHUA GENERATOR 1

Theoretical formulas for statistical characteristics were obtained for this type of intermittent behavior by solving the Fokker–Planck equation, in analogy to [18]. The distribution of the times the system remained near one steady state or another was written as

$$p(t) = \frac{1}{T}\exp\left(-\frac{t}{T}\right), \qquad (3)$$

where T is the mean length of the system's stay near a given equilibrium position, and also the mean durations of the system's stay near one steady equilibrium state or another as a function of the external noise's amplitude, determined using the expression

$$T = \frac{\pi}{4k}\exp\left(\frac{1}{4D}\right)\left[I_{-1/4}\left(\frac{1}{4D}\right) + I_{1/4}\left(\frac{1}{4D}\right)\right], \qquad (4)$$

where $I_\alpha(x)$ is modified Bessel function of the first kind, $k = \text{const}$.

Statistical characteristics of this type of behavior were obtained in our numerical studies of noise-induced intermittency in a regulated bistable Chua 1
generator exposed to external noise. Figure 2 presents the distributions of the considered system's stays near each of the steady equilibrium states for fixed values of control parameters of the system. The results from our numerical simulations are shown by points, while approximating theoretical function (3) is shown by the solid line. In addition, the mean times of the considered system's stays near one steady equilibrium state or another are given in Fig. 3 as a function of the external noise's amplitude. As in the case mentioned above, the points show the results from numerical simulations, while the solid line shows theoretical dependence (4). We can see from Figs. 2 and 3 that our numerical results and the theory describing noise-

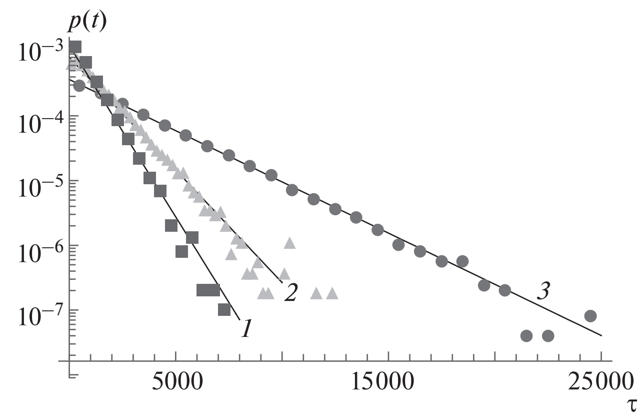

Fig. 2. Distributions of the times studied system (1) remained near the first steady equilibrium position and respective analytical dependence (3): *1* is noise amplitude $D = 0.26$, *2* is noise amplitude $D = 0.32$, *3* is noise amplitude $D = 0.37$. The theoretical curve is shown by solid line, and the data obtained numerically are shown by points. The ordinate is shown in the logarithmic scale.

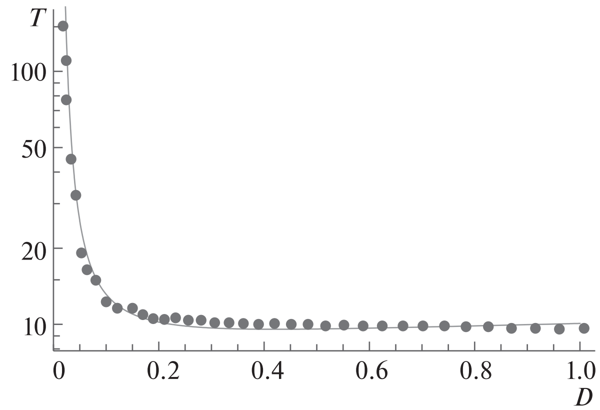

Fig. 3. Mean lengths of stay of studied system (1) near one steady state or another as a function of the external noise's amplitude, and respective analytical dependence (4), where $k = 0.318$. The theoretical curve is shown by solid line, and the data obtained numerically are shown by points. The ordinate is shown in the logarithmic scale.

1 induced intermittency in multistable systems are in good agreement.

CONCLUSIONS

1 A continuous-time study of noise-induced intermittency in multistable systems was performed using the example of a bistable Chua generator with chaotic dynamics exposed to external noise. Noise-induced intermittency was observed upon the impact of exter- 1
nal noise on the Chua generator at certain values of control parameters. We obtained statistical characteristics for such behavior: distributions of the lengths of the considered system's stays near each of the coexisting steady states, and the mean times it remained near one steady state or another as a function of the external noise's amplitude. Theoretical formulations of the statistical characteristics of this type of intermittency 1
were obtained by solving the Fokker–Planck equation. The proposed theoretical formulas agree well with the numerical results.

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REFERENCES

1. Maurer, J. and Libchaber, A., *J. Phys. Lett.*, 1980, vol. 41, p. 515.
2. Brun, E., et al., *J. Opt. Soc. Am. B*, 1985, vol. 2, p. 156.
3. Thompson, J.M.T. and Stewart, H.B., *Nonlinear Dynamics and Chaos*, Chichester: Wiley, 1986.
4. Foss, J., et al., *Phys. Rev. Lett.*, 1996, vol. 76, p. 708.
5. Atteneave, F., *Sci. Am.*, 1971, vol. 225, p. 63.
6. Grebogi, C., et al., *Phys. Rev. Lett.*, 1983, vol. 50, no. 13, p. 935.
7. Moon, F.C. and Li, G.-X., *Phys. Rev. Lett.*, 1985, vol. 55, no. 14, p. 1439.
8. Arecchi, F.T., et al., *Phys. Rev. Lett.*, 1982, vol. 49, p. 1217.
9. Neiman, A. and Russell, D.F., *Phys. Rev. Lett.*, 2002, vol. 88, p. 138103.
10. Kuno, M., et al., *J. Chem. Phys.*, 2000, vol. 112, p. 3117.
11. Siddiqi, I., et al., *Phys. Rev. Lett.*, 2005, vol. 94, p. 027005.
12. Pedaci, F., et al., *Phys. Rev. E*, 2005, vol. 71, p. 36125.
13. Pisarchik, A.N., *Phys. Rev. E*, 2012, vol. 86, p. 056219.
14. Longtin, A., Bulsara, A., and Moss, F., *Phys. Rev. Lett.*, 1991, vol. 67, p. 656.
15. Chua, L.O., et al., *IEEE Trans. Circuits Syst.*, 1986, vol. 33, p. 1072.
16. Kal'yanov, Er.V., *Tech. Phys. Lett.*, 2011, vol. 37, p. 1042.
17. Nikitin, N.N., et al., *Avtom. Telemekh.*, 1975, vol. 4, p. 133.
18. Runnova, A.E., et al., *Chaos, Solitons Fractals*, 2016, vol. 93, p. 201.

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SPELL: 1. intermittency