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# Recovery of hidden macroscopic signals in a Kuramoto phase oscillators network

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***Abstract*—We are solving the task of recovering the hidden signals in a model network. As a model we choose Kuramoto phase oscillator with adaptation of couplings. We divide the network for six parts and calculate macroscopic signals. We propose that knowing only a part of macroscopic signals allows us to recover the rest unknown ones. We show that using Reservoir computing successfully solves the problem.**

***Index Terms*—Reservoir computing, hidden signals, recovering, Kuramoto phase oscillator, complex network**

## I. Introduction

Currently, an actively developing area of scientific research is the recovery of hidden data in experimentally investigated systems, which can manifest itself in aspects such as recovering hidden features of the system to build an adequate model [1], recovering signals lost during the experiment [2], and recovering data that cannot be measured directly [3], [4].

An example from the field of neuroscience is the problem of whether EEG signals can be spatially and temporally extended based on a small number of experimentally recorded signals. In [5], a convolutional neural network model is proposed for generating new signals of brain electrical activity in order to increase the electrode spacing density. Comparing with standard spline interpolation methods, the authors show that the use of neural network allows to achieve better results. In another work [2], a new model based on a generative-adversarial network was developed to recover original EEG signals from noisy data in the presence of recording artifacts.

Nowadays, machine learning and artificial intelligence methods is now actively used for facing problems in complex networks and neuroscience [6]–[11]. Neural networks based on reservoir computing are a promising method for building such models [12]–[14]. Such models have demonstrated significant success in application to dynamical systems due to their efficient handling of time series due to the recurrent structure of communication within the hidden layer. A number of works [15], [16] have demonstrated the ability of neural networks on reservoir computing to recover the dynamics of the original system and predict bifurcation transitions when the control parameter changes, including in the presence of bistability in the system. Such networks are also actively used in the field of biomedicine: for monitoring electrocardiogram data and detecting abnormal states [17], for detecting microsleep states using data of brain electrical activity [18], for diagnosing neuromuscular diseases using electromyography data [19].

Here, we solve the task of recovering the hidden signals in a model network of Kuramoto phase oscillators with adaptation of couplings. We divide the network for six equal parts and calculate macroscopic signals. We propose that knowing only a part of macroscopic signals allows us to recover the rest unknown ones. We show that using Reservoir computing successfully solves the problem.

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## II. Methods

### A. Kuramoto phase oscillators network

We numerically simulate a network of $N = 300$ Kuramoto phase oscillators, analyzed in detail in Refs. [20], [21]. Each oscillator is described by the following equation:

$$\dot{\phi}_i(t) = \omega_i + \sum_{j\neq i} w_{ij}(t)\sin(\phi_j - \phi_i), \tag{1}$$

where $i = 1, ..., N$, $\{\omega_i\}$ is a set of randomly assigned natural frequencies distributed uniformly in $[-\pi, \pi]$, $w_{ij}$ is the weight of the connection between elements $i$ and $j$ and it is allowed to evolve in time according to the rule from [22]. For each oscillator $i$ and at each time $t$, the set of connection weights $\{w_{ij}\}$ satisfies the condition

$$\sum_{j\neq i}^{N} w_{ij} = 1. \tag{2}$$

The adaptive evolution of the weights $w_{ij}$ is governed by

$$\dot{w}_{ij}(t) = p_{ij}(t) - \left(\sum_{k\neq i} p_{ik}(t)\right) w_{ij}(t), \tag{3}$$

where the time dependent quantity $p_{ij}(t)$ is defined as

$$p_{ij}(t) = \frac{1}{T_m}\left|\int_{t-T_m}^{t} \exp^{i(\phi_i(t')-\phi_j(t'))}\,dt'\right|. \tag{4}$$

$p_{ij}(t)$ denotes, at time $t$, the average phase correlation between oscillators $i$ and $j$ over a characteristic memory time $T_m = 100$. The equations (2) and (3) describe homeostatic and homophilic processes respectively. So, this model describes the adaptive network of phase oscillators with the competition between homophily and homeostasis.

We consider a macroscopic signal averaged over all $N = 50$ phase oscillators:

$$X_{\text{avr}}(t) = \frac{1}{N}\sum_{i=1}^{N}\sin[\phi_i(t)]. \tag{5}$$

To solve the differential equations, we use the Runge-Kutta 4th order method with time step $\Delta t = 0.1$ s for $T = 7000$ s.

### B. Reservoir computing

We use a RC construct known as an echo state network, which uses a network of nodes as the internal reservoir [23], [24]. The network has the input, hidden (reservoir) and output layers. Every reservoir node has inputs drawn from other nodes in the reservoir or the input to the RC, and every input has an associated weight. Each reservoir node also has an output, described by the following equation:

$$\mathbf{h}_t = \tanh(\mathbf{W}_{h,i}\mathbf{o}_t + \mathbf{W}_{h,h}\mathbf{h}_{t-1}), \tag{6}$$

where $\mathbf{h}_t$ is the internal high-dimensional hidden state, that enables the encoding of temporal dependencies on the past state history; $\mathbf{W}_{h,i}$ is the input-to-hidden $d_h \times d_o$ couplings matrix, which values are uniformly sampled from $[-\sigma_{\text{in}}, \sigma_{\text{in}}]$, where $\sigma_{in}$ is the hyperparameter; $\mathbf{W}_{h,h}$ is the reservoir (hidden-to-hidden) $d_h \times d_h$ matrix which is set to a large low-degree matrix (node degree $D$ is the hyperparameter), scaled appropriately to possess a spectral radius (absolute value of the largest eigenvalue) $R$ whose value is also the hyperparameter; $\mathbf{o}_t$ is $d_h$ dimensional vector of the inputs. The output layer is described by

$$\hat{\mathbf{o}}_t = \mathbf{W}_{o,h}\tilde{\mathbf{h}}_t. \tag{7}$$

## III. Results

We simulate a network of $N = 300$ Kuramoto phase oscillators and divide it into $K = 6$ groups and calculate macroscopic signals as averaged over each group. Note, that each macrosignal contains unique oscillators, so, each oscillator is included only in one macroscopic signal, and there is no oscillator which is not included in any group.

Then, we choose $K - 1$ signals as inputs for RC and the rest one as a target signal. We train RC to restore the hidden signal and then test it by calculating RMSE between the target signal and the predicted one. Sorting through each macroscopic signal as a target on we test the capability of RC to restore any hidden signal. As a result we achieve mean RMSE $= 0.05$.

It is obvious that internal connectivity is highly influence on the recovering error: stronger connections between the different oscillators inside the network results in more similar and correlated macroscopic signals. The last one means more easier task of recovering a hidden signal.

Another factor that influences on the quality of recovering is intersecting of the groups of oscillators: more oscillators could be includen in more then one group, stronger connectivity between macroscopic signals.

Another one is the size of each group. We have found that increasing the number of macrosignals by decreasing the size of each group leads to decreasing the recovering accuracy because of decreasing the connectivity between the groups.

## IV. Conclusions

We were solving the task of recovering the hidden signals in a model network. As a model we have chosen Kuramoto phase oscillator with adaptation of couplings. We divided the network for six parts and calculated macroscopic signals. We proposed that knowing only a part of macroscopic signals allows us to recover the rest unknown ones. We have shown that using Reservoir computing successfully solves the problem.

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