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Analyzing the Formation of Coherent Structures in a Helical Electron Flow with a Virtual Cathode

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Abstract—The results from numerical simulations of the nonstationary behavior of a helical electron flow in a system with a virtual cathode has been presented. The behavior of a beam with a virtual cathode and a beam in the squeezed state are analyzed using the Karhunen–Loève orthogonal expansion of space-time data.

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INTRODUCTION

Studying the complex nonstationary behavior of intense electron beams with virtual cathodes (VCs) is an important problem of modern radiophysics and electronics. The study of systems with VC is of fundamental interest, as they can exhibit a variety of chaotic behavior, turbulence, and the formation of dissipative electron structures [1–2]. Their importance for use as promising generators of powerful microwave radiation on the basis of VC (vircators) is also apparent [3, 4]. There have been numerous studies on the acceleration of charged particles using an oscillating VC [4] and creating sources of broadband noise-like radiation with different power levels [5].

The aim of this work was to conduct a numerical study of the nonstationary nonlinear processes of the formation and interaction between coherent electron structures in a low-voltage electron-wave system with a virtual cathode. Note that similar systems with VCs in bremsstrahlung electron flows can be of considerable interest as sources of noise-like broadband chaotic signals with average power levels in the microwave range [5, 6]. Their theoretical and experimental study is therefore important for practical application.

INVESTIGATED SYSTEM AND MATHEMATICAL MODEL

The nonstationary behavior of an electron flow with a virtual cathode was studied using the example of a low-frequency electron-wave system with a source of electrons in the form of a magnetron injector gun (MIG). The electron beam formed by a MIG has high levels of perveance and intrinsic noise that, as was shown by earlier studies, makes it an effective source of electrons for low-voltage systems with virtual cathodes [6–8].

Let us recall that the main elements of a MIG are [9] a cathode and accelerating electrode, executed in the form of coaxial cone-shaped electrodes inserted into one another. The cathode has an emitting belt that encircles the cathode around its axis. Due to the crossed steady-state electric and magnetic fields in the region of the cathode, the MIG forms a tubular helical electron flow. The VC in such a system is formed by introducing additional bremsstrahlung of the beam that facilitates an increase in the perveance of the beam.

Our model of the investigated system was a 2.5-dimensional self-consistent system of equations for the motion of charged particles and Poisson equations (the model was described in detail in [8, 10]). The motion of the particles of the flow was simulated using the method of large particles (PIC), i.e., the Poisson equation in combination with the standard nearest neighbor method (the five-point difference scheme). The system had two main governing parameters: α, the dimensionless current of the beam, and $\Delta\varphi$, the breamsstrahlung potential difference determined by the potential difference. The steady-state magnetic field was arrived at analytically and had the configuration of a magnetic trap. The system was thus described in the quasi–steady state approximation in dimensionless parameters, allowing us to extrapolate the results from simulations to cases of nonrelativistic to weakly relativistic systems with helical beams.

The orthogonal decomposition of space-time data using the Karhunen–Loève (KL) expansion [11–13] was used to describe the physical processes that occur in an electron beam with a VC. In using this method to analyze the complex behavior of a VC in a beam of charged particles, we encounter separate space-time structures in the electron flow that have characteristic spatial distributions and time scales in the electron beam whose interaction generally allows us to explain the features of the behavior of a beam with a VC.

Orthogonal decomposition is performed by solving an integral Fredholm equation of the second kind in the form

$$\int K(z, z^*)\Psi(z^*)dz^* = \lambda\Psi(z), \quad (1)$$

where $K(z, z^*)$ is the kernel of the equation

$$K(z, z^*) = \langle\xi(z,t)\xi(z^*,t)\rangle_T. \quad (2)$$

where $\langle\ldots\rangle_T$ denotes averaging over time. As function $\xi(z, t)$, we can select the space-time distribution of any physical quantity preliminarily reduced to zero average, on the basis of which we propose to analyze the behavior of the system. In this work, we analyze the space-time distribution of the spatial charge density of a beam, $\rho(z, t)$, averaged over the radius of a flow. The solution to initial integral equation (1) with kernel (2) formed on the basis of the space-time distribution of spatial charge density $\rho(z, t)$ is reduced to finding a set of eigen numbers λ_n and eigenvectors Ψ_n that determine the nth KL mode of the oscillatory process. The value of λ_n is proportional to the energy of the corresponding mode, which is conveniently considered in normalized form:

$$W_n = \frac{\lambda_n}{\sum_i \lambda_i} \times 100\%. \quad (3)$$

The temporal behavior of separate KL modes can be restored as

$$A_i(t) = \int\xi(z,t)\Psi(z)dz. \quad (4)$$

We then use Karhunen–Loève orthogonal decomposition to analyze the nonlinear behavior of a helical beam in the squeezed state.

RESULTS AND DISCUSSION

Let us discuss the results from modeling such a system. It was shown in [7, 8] that in the proposed geometry, a reflective VC (at low values of the bremsstrahlung potential difference $\Delta\varphi$) or the so-called squeezed state of a beam (SSB) [14-16] that is a VC distributed over space and characterized by high density and low kinetic energy of the electron flow [14], can be created in the system at selected values of the governing parameters. In the mode of SSB formation, complex near-chaotic behavior of the flow particles is observed in the beam.

Earlier studies [8] showed that low-frequency fluctuations of the potential of the drift space are visible in the squeezed state in the beam of particles. In this mode of charge fluctuations, no spatially localized bunches of electrons appear in the beam. The emergence of potential fluctuations in the system is due to the perturbations in charge density that appear in the beam acting as waves of the spatial charge.

KL analysis showed that the emergence of the squeezed state of the beam corresponds to the excitation of several interacting KL modes in the flow.

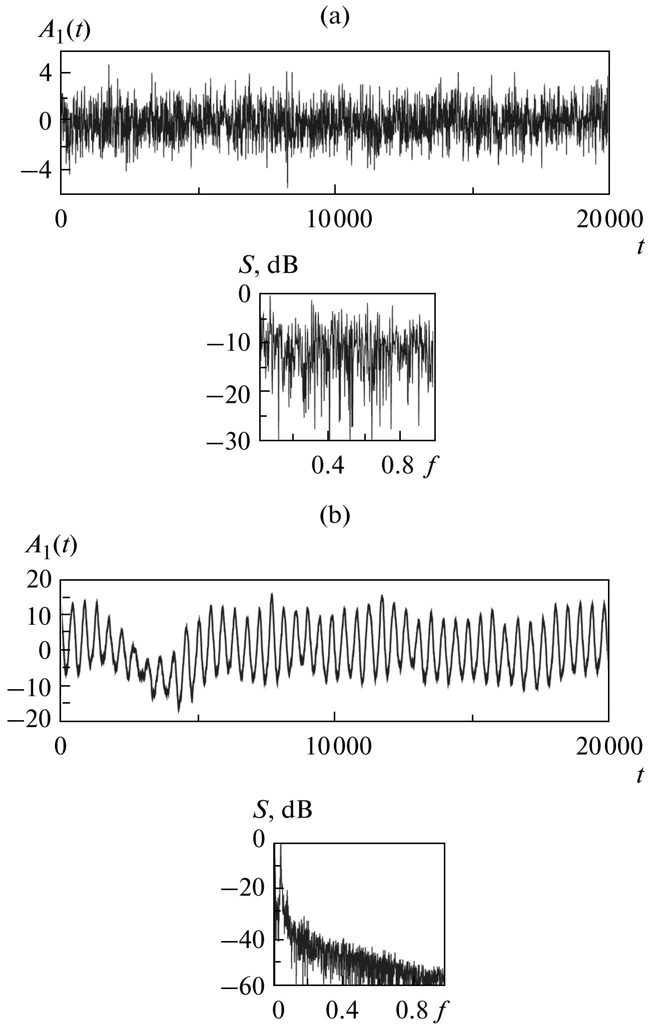

Fig. 1. Time implementations $A_1(t)$ (top) and power spectra of the fluctuations (bottom) of the first KL modes: (a) in the system with VC; (b) in the system with SSB.

Figure 1 shows the time implementations $A_1(t)$ of the first KL modes (the modes with the maximum energy of the fluctuations) and their power spectra for two different states of the beam: reflective VC (Fig. 1a) and SSB (Fig. 1b). Figure 1a shows that the time dependence of the fluctuations of the first KL mode for a beam with a reflective VC has relatively low amplitude, and its spectrum is extremely noisy. Analysis of the KL modes showed that in mode of the formation of a reflective VC in the beam, the energy of fluctuations is uniformly distributed over all modes. Figure 2 shows the dependence of KL mode energy W_i on mode number i (for the first six modes). Curve *1* corresponds to the formation of a reflective VC in the flow; curve *2*, to the squeezed state of the beam. It can be seen that the energy of the first (main) mode for the beam with the reflective VC (curve *1* in Fig. 2) does not exceed 1%, and the energies of the other modes are lower.

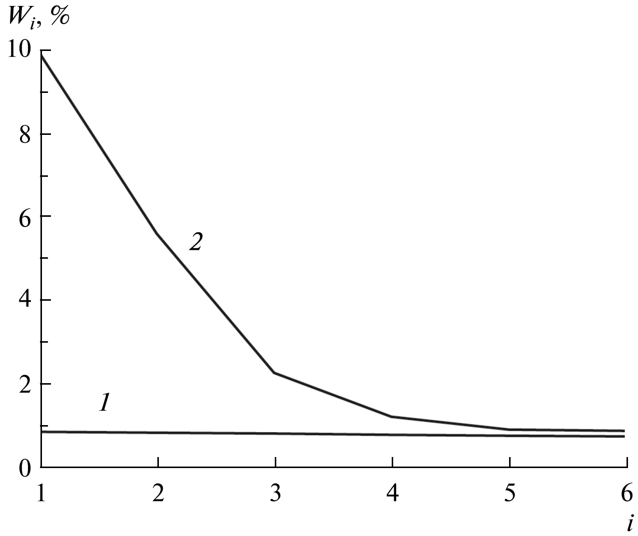

Fig. 2. Dependence of the energy of KL modes W_i on mode number i. Curve *1* corresponds to the formation of the reflective VC in the flow; curve *2*, to the squeezed state of the beam.

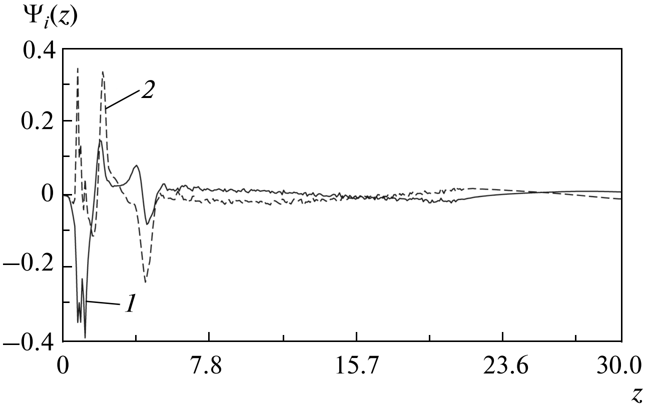

Fig. 3. Spatial distribution of eigenvectors $\vec{\Psi}_i(z)$ for the first (*1*) and second (*2*) KL modes.

Another pattern is observed for KL modes in the beam in the squeezed state. It can be seen from Fig. 1b that the first KL mode has near-periodic high-amplitude fluctuations. There is one prominent harmonic in the noise background of the power spectrum (Fig. 1a, on the right). This harmonic corresponds to the main frequency of the fluctuations in the potential of the spatial charge in the drift space of the system. The second and subsequent KL modes have lower amplitudes and different space-time behaviors. Figure 2 shows that the energy of the main KL modes for the beam in the squeezed state (curve *2*) is approximately 10%. The second mode provides approximately 3% of the total energy of the fluctuations. The level of the noise pedestal in this case rises and new harmonics appear in the spectrum of the signal. The energy of the subsequent modes falls, and their behavior is much noisier. When SSB is implemented in the beam, the behavior of the flow is thus completely determined by the two first KL modes, which describe the form of the electron structures in the beam: waves of the spatial charge. The other modes create the low-noise background for the fluctuations of the main structures.

Figure 3 shows the distribution of the two first KL modes in the space of eigenvectors $\vec{\Psi}_1(z)$ and $\vec{\Psi}_2(z)$ for a beam in the squeezed state. It can be seen that substantial differences between the eigenvectors are observed only at the onset of the drift space; in the rest of the drift space region, the distributions of the modes are similar to one another. We may conclude that the fluctuations of the modes in this region of the system's drift space determine the perturbations in the charge density of the beam. Appearing in the right-hand third of the system, these perturbations in the charge density then propagate along the space at the speed of the waves of the spatial charge. These perturbations are described by the distribution of KL modes in the space (Fig. 3). The fluctuations of these modes determine the spectral composition of the fluctuations in the potential of the drift space in the region of the beam.

CONCLUSIONS

Results from numerical simulations of the nonstationary nonlinear behavior of a helical electron flow in an electron—wave system with a virtual cathode were presented.

Analysis of the behavior of the beam using the Karhunen—Loève expansion showed that the behavior of the flow is completely determined by that of the first two KL modes in which the main fraction of the energy of the fluctuations of the beam charge is concentrated, depending on the values of the controlling parameters.

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