Study of correlation between macroscopic and microscopic characteristics of adaptive networks with application to analysis of neural ensembles

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ABSTRACT

This paper is devoted to the analysis of topological changes in complex networks that are reflected in the macroscopic characteristics. We consider a model of the complex network with the adaptive links, in which the synchronous dynamics leads to the appearance of clusters of strongly coupled elements and show that structural changes significantly affect the macroscopic dynamics. As the result, we demonstrate a high possibility of cluster formation in the network that can be analyzed via the consideration of macroscopic characteristics. We also discuss a prospective application for the detection of structural features of neural networks.

Keywords: Adaptive network, cluster, continuous wavelet transform, neuronal ensemble, synchronization

1. INTRODUCTION

A study of synchronous modes and pattern formation in complex networks is one of the most important challenges for the global scientific community. Various structures of networks arise at all levels of organization of biological,[1] technological[2,3] and social systems[4–6] starting from neural networks[7,8] to networks of cities and populations.[9] In complex networks, interactions between elements lead to inhomogeneous distribution of the ingoing and the outgoing links that causes a number of different phenomena in collective dynamics including formation of structural patterns[10] and the emergence of the synchronous modes.[11–13]

Synchronization is a phenomenon of the adjustment of dynamical states of interacting elements for some time period. In natural systems, such as social structures or biological systems, synchronization between networking elements is a determinant factor for the evolution of links between them. Synchronization between nodes leads to the strengthening of their interaction (that represents itself the homophily mechanism), although every node can have only a limited number of strong links (known in sociology as a Dunbar's number[14]).

Analysis of natural networks is a complicated task which is often associated with the lack of information about the evolution of dynamical states of individual elements. Generally, these microscopic parameters are experimentally unavailable, and the macroscopic characteristics become the only source of our knowledge about the evolution of network dynamics and structure. Such situation is widely observed, e.g., at the analysis of cognitive processes, where the experimental EEG[15] or MEG[16] signals reflect the cooperative dynamics of large neuron ensembles in the brain.[17,18] In such cases, the study of structural changes in the network topology and the analysis of cluster formation are of a high importance.

In this paper, based on the continuous wavelet transform[19,20] we show how the time-dependent macroscopic characteristics are connected with the network dynamics, topology evolution and pattern formation. In Sec. 2 we briefly describe a model of adaptive network and discuss the relevance of this model and its association with natural objects. In Sec. 3, a simple case is analyzed, namely, a small network of randomly coupled phase oscillators. We show how the microscopic dynamics of the nodes correlates with the evolution of macroscopic parameters. In Sec. 4, we consider large networks demonstrating complex dynamical regimes. In Conclusion we discuss the obtained results and promising applications for the analysis of natural systems.

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2. THE MODEL

When studying the evolution of network structure, the so-known adaptive networks are widely considered. In the adaptive network, dynamics of networking elements is the determinant factor for evolution of links between them. Such feedback between structure and dynamics leading to time evolution of the topology that makes such model very useful.

In our study we considered an adaptive model recently proposed by S. Assenza *et al.*[21] This model reflects two main features of natural networks, namely, the scale-free distribution of link weights and the formation of mesoscale structures. Such phenomena are caused by the above mentioned mechanisms: the homophily associated with the strengthening of links between synchronized nodes, and the homeostasis, implemented by holding the condition

$$\sum_{j \neq i}^{N} w_{ij} = 1 \tag{1}$$

at all time moments, i.e. a sum of weights of the incoming links for each node should be constant.

The considered model represents a network, where each node has its own frequency ω_i and phase ϕ_i evolving in time according to the Kuramoto equations[22]

$$\frac{d\phi_i}{dt} = \omega_i + \lambda \sum_{j=1}^{N} w_{ij} \sin(\phi_j - \phi_i). \tag{2}$$

Here, λ is the coupling strength, and $w_{ij}(t)$ are non-negative quantities characterizing the weight of the link from the node j to the node i at time moment t. The value of $w_{ij}(t)$ varies in time as:

$$\frac{dw_{ij}(t)}{dt} = w_{ij}(t) \left[s_i p_{ij}^T(t) - \sum_{l=1}^{N} w_{il}(t) p_{il}^T(t) \right], \tag{3}$$

where $s_i = \sum_{j=1}^{N} w_{ij}$ is the total incoming strength of the element i, $p_{ij}^T(t)$ is the degree of local synchronization between elements i and j, averaged over time in the interval $[t-T, t]$, which is defined through the equation

$$p_{ij}^T(t) = \left| \frac{1}{T} \int\limits_{t-T}^{t} e^{\mathrm{i}[\phi_j(\tau) - \phi_i(\tau)]} d\tau \right|. \tag{4}$$

Here, $\mathrm{i} = \sqrt{-1}$, and T is the control parameter, which is chosen as $T = 100$ for all estimations according to the recent work.[21]

3. MICROSCOPIC AND MACROSCOPIC NETWORK DYNAMICS

Traditionally, the key quantity used for understanding the system dynamics is the order parameter, which characterizes the extent of synchronization of N oscillators[23]

$$r(t) = \frac{1}{N} \sum_{j=1}^{N} e^{\mathrm{i}\phi_j(t)}, \tag{5}$$

with $r(t) = 1$ corresponding to the perfectly synchronized state.[23] This parameter accounts for the phases of all elements of the network, therefore, it can be used only in the case when the evolution of microscopic characteristics ($\phi_j(t)$) is determined.

When studying such systems as, e.g., neuron ensembles or groups of networking microwave generators, researcher deal only with summary signals that are integrated over all elements. In our case the corresponding macroscopic signal is given by the expression

$$X(t) = \sum_{i=1}^{N} x_i(t), \tag{6}$$

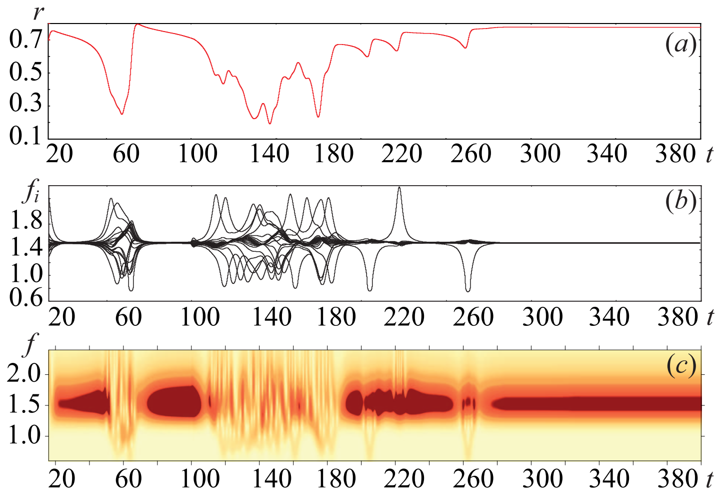

Figure 1. The order parameter $r(t)$ (a), effective frequencies of nodes $f_i(t)$ (b) and the spatial distribution of the wavelet energy $|W(f,t)|$ of the macroscopic network characteristics (c), obtained for the network of twenty Kuramoto phase oscillators for different time moments

with $x_i(t)$ being a time-dependent signal produced by each node

$$x_i(t) = A_i \cos(\phi_i(t)), \tag{7}$$

where A_i is the amplitude and $\phi_i(t)$ is the phase of oscillations generated by the corresponding node i. For simplicity, we consider here $A_i = 1$.

In this work we propose to use the continuous wavelet transform to study the dynamics of individual elements and to consider the macroscopic dynamics of groups of nodes (6). The use of wavelets is caused by their efficiency at the analysis of non-stationary signals. Thus, this mathematical tool is well suited for the analysis of networks, where effective frequencies of coupled oscillators evolve in time.

The continuous wavelet transform of a time-dependent function $F(t)$ is written as

$$W(f,t) = \sqrt{f} \int\limits_{t-4/f}^{t+4/f} F(t')\Phi^*((t-t')f)dt', \tag{8}$$

where f is the frequency, $\Phi(\eta)$ is the mother wavelet function, and "$*$" denotes the complex conjugation. Here we use the Morlet wavelet

$$\Phi(\eta) = \pi^{-1/4} e^{\mathrm{i}\omega_0 \eta} e^{-\eta^2/2} \tag{9}$$

with the central frequency $\omega_0 = 2\pi$.

Within the performed study we considered $N = 20$ Kuramoto phase oscillators (2) which are involved into the global synchronous mode. The frequencies ω_i were uniformly distributed over the range $[0.5, 2.5]$, and the phases ϕ_i were selected randomly in the range $[-2\pi : 2\pi]$. The coupling matrix was assembled randomly accounting interconnections between all nodes and the satisfying of the condition (1).

The model (2) was integrated during the time interval t=100 without evolution of link weights ($w_{ij}(t) = const$). After this period, the adaptation mechanism was activated by considering the corresponding evolution, i.e., by integrating Eqs. (2) – (3).

In Figure 1, a time-dependent behavior of the network is illustrated. Considering the order parameter (Fig. 1,a) one can see that the system twice reaches the state of full synchronization before the adaptation is ON, however this state is unsteady. After accounting the adaptation, a transient process occurs caused by

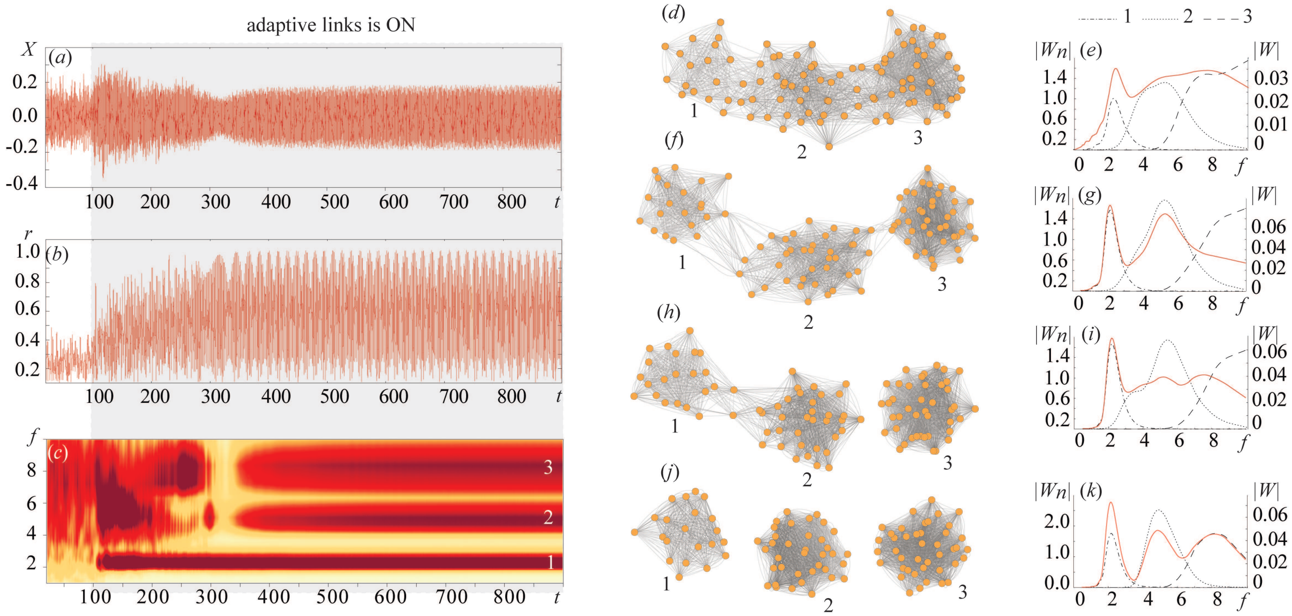

Figure 2. (a) Time dependence of the macroscopic characteristic (6) of the network of $N = 300$ Kuramoto oscillators, (b) the order parameter (5) estimated for this network, (c) the wavelet energy surface, obtained for the macroscopic characteristic (6) using the transform (8), (d, f, h, j) the visualizations of the network structure at the time moments: $t_1 = 100$, $t_2 = 200$, $t_3 = 220$, $t_4 = 400$, (e, g, i, k) the instantaneous distributions of the wavelet energy of the macroscopic parameters, obtained for all elements (solid curve) and for each cluster W_n (curves 1, 2, 3, respectively)

redistribution of link weights, and then the order parameter $r(t)$ increases the system reaches the steady state of full synchronization.

To describe the dynamics of individual elements, the evolution of their frequencies is shown in Figure 1,b. The values f_i being the frequencies associated with the “ridges” of the wavelet-transform (8) are estimated after searching for local maxima of $|W(f,t)|$ at each time moment, where $F(t) = x_i(t)$ (7).

One can see that the evolution of the obtained values, f_i, strongly correlates with the order parameter (Fig. 1,a). When order parameter takes large values (e.g., for $t = 30$ and $t = 70$), all frequencies become close, reflecting the phase-locking of all systems. When adaptation is ON, a redistribution of weights causes a decreasing of the order parameter. Effective frequencies of individual units are quite different until $t \approx 140$, although small clusters of about 3-5 nodes occur. The latter represent groups of strongly coupled nodes (structural clusters) formed by the adaptation process. For $t > 140$, these clusters adjust their frequencies resulting in the growth of the order parameter and in the transition to the state of full synchronization characterized by single effective frequency in Figure 1,b ($t > 280$).

In order to study macroscopic characteristics (6), we performed the wavelet transform (8) with $F(t) = X(t)$. The obtained result is shown in Figure 1,c, where the module of the wavelet transform is shown. It is clearly seen that areas of global synchronous mode can easily be detected in the wavelet energy as a high-amplitude isolated pattern in the corresponded frequency area. Areas of partial phase-locking regimes are also well distinguished.

Thus, we observed strong correlation between time-dependent microscopic and macroscopic characteristics of complex adaptive network. Such correlation persists for different dynamical regimes including an asynchronous behavior, a partial phase-locking and the perfectly synchronized state.

4. ANALYSIS OF THE COMPLEX NETWORK STRUCTURE

Aiming to consider the case of a more complicated dynamics, the number of oscillators was increased up to $N = 100$ with the frequencies uniformly distributed over the range $[0.5, 10]$. The phases ϕ_i and the coupling matrix was assembled as in the previous case.

In Figure 2,a, the macroscopic signal (6) obtained from all nodes is shown. Some transient process leading to the stationary regime can be observed, however, the signal is too complicated and does not contain information about network structure. The order parameter (Fig. 2,b) has a pronounced frequency after the transient process ($t > 350$), that reflects the presence of large mesoscale structures.[21]

To analyze the evolution of network structure, we performed the wavelet transform of the network macroscopic characteristic (6). The result is presented in Figure 2,c. It is clearly seen that frequencies are distributed rather uniformly before the adaptive mechanism starts to operate, however, some synchronous modes are observable. The network structure for this case is schematically illustrated in Figure 2,d. Here, the links between nodes are distributed homogeneously and any structural patterns are absent. At $t_1 = 100$, the network topology starts to evolve. One can see from the presented wavelet energy surface that the isolated pattern immediately appears in the area of low frequencies. The distribution of the instantaneous wavelet energy for the moment $t_1 = 100$ is shown in Figure 2,e by a solid curve. There is a peak at $f \approx 2.5$ related to a cluster formation. Further consideration of the network dynamics using the wavelet energy (Fig. 2,d) shows that the occurred high-frequency cluster of a large ensemble of nodes is splitted into two smaller clusters. In Figure 2,f and Figure 2,g, visualization of the network structure and the instantaneous wavelet energy are shown for the time moment $t_2 = 200$. One can see that the network begin splitting into three clusters, connected with each other (Fig. 2,d). Further, the adaptive mechanisms lead to the isolation of clusters. At $t_3 = 220$, the network structure (Fig. 2,h) and the distribution of the instantaneous wavelet energy (Fig. 2,i) evidence that the strength of links between the clusters decreases and in the stationary regime $t_4 = 400$ the network structure is represented by three isolated clusters of strongly coupled elements (Fig. 2,j,k).

To provide the correspondence between the isolated high-amplitude patterns in the wavelet energy surface and the occurred structural patterns, let us consider the macroscopic signals

$$X_n(t) = \sum_{j \in R_n} x_j(t), \tag{10}$$

where $n = \{1, 2, 3\}$ is the number of cluster and R_n is the set of integer numbers corresponding to elements related to this cluster. In Figure 2,e,g,i,k, the distribution of the instantaneous wavelet energy W_1 (dash-dotted line), W_2 (dotted line), W_3 (dashed line) are shown for the considered time moments t_1, t_2, t_3, t_4 with the wavelet energy, corresponding to the macroscopic signal 6 (solid line). One can see correlation between peaks on the wavelet surface obtained from entire macroscopic signal and its components from separate structural clusters, that verifies our assumption about strong correlation between time-dependent macroscopic signal and structure features of the system under study.

5. CONCLUSION

In this paper we have shown the correspondence between the dynamics of elements of a complex network and the macroscopic network characteristics represented by the summary signal of all elements. Using the continuous wavelet transform we considered how the frequencies of interacting oscillators evolve in time under the influence of adaptive mechanism and compared this microscopic dynamics with the evolution of the macroscopic signal. As a result, a possibility of detection of structural changes in the network topology via the macroscopic analysis was provided. The proposed approach was successfully applied for the analysis of large network of the interacting Kuramoto phase oscillators with adaptive links pocesing a time-dependent structure. The performed wavelet analysis of the macroscopic network characteristics allowed not only detection of the structural clusters, but also identification of their frequencies and following their time evolution. This macroscopic analysis was compared with the visualization of the network structure, performed in accordance with the strength of links between the nodes, and significant correspondence between the observed patterns in wavelet energy surface and the structural clusters was obtained.

Practical importance of the presented results is associated with the analysis of real objects consisting from a large number of networking elements, where the experimental data is generally limited by the macroscopic characteristics. Examples include a study of neural network by means of electroencephalography, analysis of large ensembles of microwave generators used in the processes of data encryption and consideration of many other real networks described by mathematical models where the elements are identified by their frequencies.

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