

Factor of border crossing limitation with an infection hub in the SIR covid model

Vladimir Khorev

Center for technologies in robotics and
mechatronics components
Innopolis University
Innopolis, Russia
Orcid: 0000-0001-6613-8940

Victor B. Kazantsev

Neurotechnology department
Lobachevsky University
Nizhniy Novgorod, Russia
Center for technologies in robotics and mechatronics components
Innopolis University
Innopolis, Russia
Center for Neurotechnology and Machine Learning
Immanuel Kant Baltic Federal University
Kaliningrad, Russia
Orcid: 0000-0002-2881-6648

Abstract—In this work we develop a new nonlinear net-based SIR epidemic problem modeling the spreading of coronavirus under the effect of a border crossing limits by the government measures to stop coronavirus spreading. We show that the emergence and disappearance of subsequent waves of infection on the degree of relationship with a hub is non-linear, and with a large degree of relationships, the number of waves can decrease.

Index Terms—COVID-19; forecasting; nonlinear SIR model; border closing

I. INTRODUCTION

The SARS-COV-2 coronavirus pandemic is currently increasing the interest of researchers to SIR models [1] and many works for different regions of the world [2]–[6] demonstrate their satisfactory performance, although there are other points of view [7]. Nevertheless, the question of the influence of quarantine on the flow of infection is still require further research [8]–[10]. There are many examples of network approach application to analysis of social, ecological and biological systems [8], [11]–[15]. Given the success demonstrated by the examples of using a network approach to model social systems, we decided to upgrade existing models in this way.

II. MODEL

For k interacting elements, in general, the system of differential equations will look like this:

$$\begin{aligned}\dot{S}_i &= -\beta \sum_j p_j S_i + \delta R_i - d H_i S_i + \mu_i \quad (i, j = 1..k); \\ \dot{I}_i &= \beta \sum_j p_j S_i + d H_i S_i - (\gamma + \varepsilon) I_i; \\ \dot{R}_i &= \gamma I_i - \delta R_i; \\ \dot{H}_i &= \alpha I_i - b H_i;\end{aligned}\tag{1}$$

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Here, $S_i(t)$ denotes the number of susceptible individuals of the i -th region, $I(t)$ — infected, $R(t)$ — recovered, $H(t)$ non-human carriers (viruses). The coefficient α denotes the number of carriers (viruses) emitted by an infected individual, β is the probability of infection in the case of contact of a susceptible subject with an infected subject, γ is the probability of recovery (usually, it is interpreted as the rate of recovery, since the average duration of the subject is in the state of infection is $1/\gamma$), δ is the probability of losing immunity and the appearance of the possibility of re-infection, ε is the probability of death, μ_i — the immigration of the population for the i -th region, p_j is the interaction coefficient between the j -th and i -th districts given from the communication matrix (P), Relative degree of openness of the borders, which is relevant for modeling quarantine events [16], b denotes the coefficient of natural loss of carriers, d is the degree of contagiousness of the carrier (virulence). It should be noted that in the case of $p = 0$ (complete closure of boundaries), the system is degenerated in case of infecting one subject in the absence of the remaining infection [17]. To check the effect of the presence of the central district (Hub), as well as the observed difference between the second and third areas, the communication matrix was as follows:

$$P = \begin{bmatrix} 1 & p & p \\ p & 0.5 & 0.05 \\ p & 0.05 & 0.1 \end{bmatrix}\tag{2}$$

III. RESULTS

The second and third districts are located in the outskirts and have weak interaction between themselves. The first area is considered as a hub, and the its level of exposure to other districts can vary. Typical curves for a small bond ($p = 0.1$) are shown in Fig. 1. The numerical simulation showed that despite the fact that the probability of internal infection in the third area is five times lower than in the second, the final number of infected differs by about twice. The graphs of the total infected over time for the degree of opening

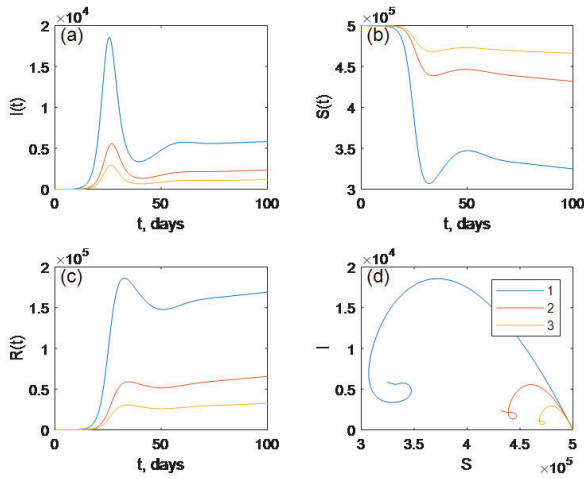


Fig. 1. The time series of infected (a), susceptible (b) and recovered (c), as well as the phase portrait for three districts (d), obtained for the values of the system parameters with the communication matrix 2: $n = 50000$, $\beta = 1.9$, $\gamma = 1.5$, $\delta = 0.05$, $b = 0.01$, $d = 10^{-7}$, $p = 0.1$.

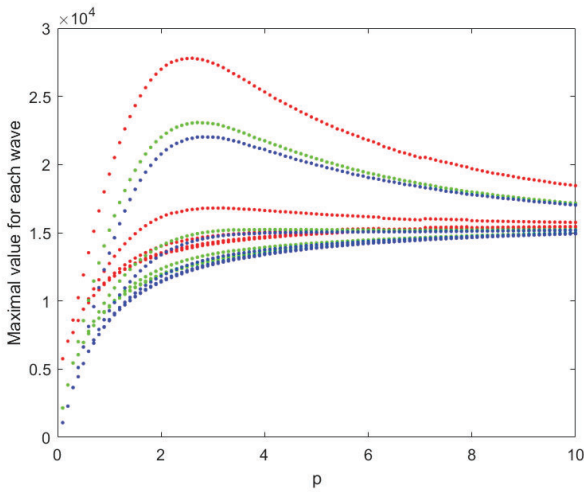


Fig. 2. The dependence of the maximum number of infected in the second and subsequent waves in areas (red – 1, green – 2, blue – 3) on the degree of openness of the boundaries between the hub and other districts (p).

of the borders for the districts are similar to the dynamics with the case of symmetric links, despite the difference in the probability of infection inside the areas and are primarily determined by the degree of relationship with a hub, which can be seen for the first wave, but the flow and final values of infected depend from the internal probabilities of infection with small communication values a hub. The final amount of infected in the area increases with a degree of connection with a hub. However, this dependence is close to a logarithmic form, and the maximum value (peak of the wave) number of infections demonstrates almost linear growth for the communication values with a hub of more than 0.5. Despite the difference in the likelihood of infection within the districts, infection curves

are primarily determined by the degree of relationship with a hub, which can be seen for the first wave, but the flow and final values of infected depend on the internal probabilities of infection with small connections with a hub. The occurrence and disappearance of subsequent waves of infection on the degree of relationship with a hub is non-linear, and with a large extent relationship the number of waves even decreases, and their amplitude is committed to a constant value as shown in Fig. 2.

IV. CONCLUSION

Thus we developed a new nonlinear net-based SIR epidemic problem modeling the spreading of coronavirus under the effect of a border crossing limits by the government measures to stop coronavirus spreading. We show that the emergence and disappearance of subsequent waves of infection on the degree of relationship with a hub is non-linear, and with a large degree of relationships, the number of waves can decrease.

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