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Wavelet Filtration of Noisy Images

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Abstract—Methods of noisy image filtration using wavelet transforms with real and complex basis sets have been compared. It is shown that the use of a complex wavelet transform provides more effective filtration and admits automatic optimization of the filter parameters. Optimized choice of the threshold level during filtration based on a complex wavelet transform significantly decreases the error of image reconstruction as compared to that achieved with a standard method of discrete wavelet transform employing basis sets of the Daubechies wavelet family.

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An important area in the development of communication systems is related to optimization of the process of digital filtration of noisy data transmitted via communication channels. In recent years, a significant progress has been achieved in upgrading the method of noisy signal filtration employing wavelet transforms [1–5]. The new approaches offer significant advantages compared to filtration based on the Fourier transform—in particular, by making possible effective elimination of local noises that cannot be effectively filtered by methods based on the Fourier transform employing the basis set of infinitely oscillating functions. A discrete wavelet transform (DWT) widely used in the framework of multiscale analysis [6–10] allows the signal to be separated into components corresponding to various scales. Then it is possible to perform correction of the expansion coefficients corresponding to small scales that are most subject to the influence of noise, after which the signal or image can be reconstructed by means of the inverse wavelet transform.

It should be noted that, since a simple variant (frequently used in practice) involving setting some coefficients to zero is not always effective, approaches based on variants with "soft" introduction of a threshold function during filtration have been proposed [3, 4]. According to this, the threshold function has no discontinuities and the values of all coefficients are corrected. In addition, various modifications of the method of expansion in wavelet functions can be used, in particular, the method of dual-tree complex wavelet transform (DTCWT) [11–15], which represents an extension of the classical DWT employing real basis set functions such as Daubechies wavelets [7].

Despite the extensive development of the methods of filtration employing wavelet transforms, their practical implementation still involves many open questions. For this reason, it is still topical to perform comparative analysis of various methods for selecting an approach capable of minimizing distortions introduced during reconstruction of a processed signal or image from its wavelet coefficients. The present work compares the results of wavelet filtration of noisy images with the aid of real and complex basis sets and gives recommendations on the choice of parameters of the wavelet filter.

In the framework of the standard DWT method, the analyzed signal $f(t)$ is expanded using the approximating and correcting functions called, respectively, scaling functions $\varphi(t)$ and wavelets $\psi(t)$:

$$f(t) = \sum_k c_k \varphi(t-k) + \sum_j \sum_k d_{j,k} 2^{j/2} \psi(2^j - k). \quad (1)$$

Here, expansion coefficients $d_{j,k}$ bear information on the structure of the signal, representing amplitude components on various scales at different moments of the time [6]. In the course of filtration, small wavelet coefficients, which are most subject to the influence of noise, have to be corrected. This is achieved predominantly by means of so-called "soft" setting of the threshold function in the following form [3]:

$$p(x) = \begin{cases} x - C, & x \geqslant C, \\ x + C, & x \leqslant -C, \\ 0, & |x| \leqslant C, \end{cases} \quad (2)$$

Fig. 1. Example of analyzed image (photograph of the main building of Saratov State University).

which eliminates the appearance of irregularities during reconstruction of the signal from wavelet coefficients. Despite the simplicity of the realization of wavelet filters based on a DWT (1), this approach has some disadvantages that influence the quality of filtration [11]—in particular, the oscillating character of coefficients $d_{j,k}$ in the vicinity of singularities and the lack of invariance with respect to a shift of the wavelet function. These drawbacks can be eliminated by using a method of filtration based on complex wavelets [12, 13], according to which real basis set functions $\varphi(t)$ and $\psi(t)$ are supplemented with imaginary parts constructed using the Hilbert transform. The passage to complex wavelets $\psi^c(t) = \psi^r(t) + j\varphi^i(t)$ implies the need for subsequently forming two orthonormalized basis sets of functions $\psi^r(t)$ and $\psi^i(t)$, which are treated in the framework of one-dimensional wavelet transform. The corresponding computational algorithm reduces to two pyramidal expansions of the one-dimensional signal. The approximately analytic complex scaling functions and wavelets are constructed using special methods such as described, e.g., in [11].

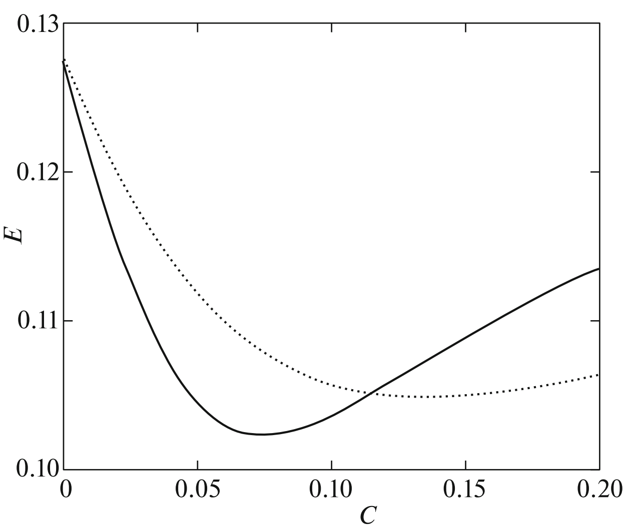

Fig. 2. Plots of mean-square error E of wavelet filtration vs. threshold level C for (dashed curve) DWT and (solid curve) complex wavelet transform. The variance of the normally distributed noise process added to the processed image was 0.1. DWT results were obtained with a D^8 Daubechies wavelet set, which ensured minimum filtration error as compared to other basis sets of this family.

Solving the problem of a two-dimensional (2D) complex wavelet transform is more complicated in comparison to the one-dimensional case. If $h_x + jh_x$ are conjugate filters for the first dimension (x) and $h_y + jh_y$ are those for the second dimension (y), the filters for 2D complex wavelet transform can be written in the following form:

$$(h_x + jh_x)(h_y + jh_y) = (h_x h_y - g_x g_y) + j(h_x g_y - g_x h_y). \tag{3}$$

In this case, the task reduces to calculating four "trees" representing expansions of the image constructed using filter sets (h_x, h_y), (g_x, g_y), (h_x, g_y), and (g_x, h_y) [11]. In what follows, we use the filters proposed in [12, 16].

In the present work, we have carried out the following investigation. The test image was a black-and-white photograph of the main building of Saratov State University (Fig. 1), which was rendered noisy by adding a random process with normal distribution at variable intensity. At every fixed noise level, the noisy image was wavelet-filtered using two approaches: (i) DWT with a real basis set (Daubechies wavelets) and (ii) DTCWT with a complex basis set [12, 16]. The wavelet coefficients were corrected using a “soft” variant of setting of the threshold function according to Eq. (2). Figure 2 shows typical plots of mean-square error E of filtration versus threshold level C. As can be seen from this figure, at optimum setting of the threshold (in this case, $C \approx 0.07$), the complex wavelet transform provides a minimum error of the wavelet filtration. Analogous results were obtained for various intensities of a noise admixture to the test image. Thus, the use of a complex wavelet transform reduces the error of image filtration as compared to that of a DWT and allows smaller C to be used that leads to lower distortions of the informative wavelet coefficients.

It should be noted that the optimum threshold C is not constant and depends on the signal to noise ratio (SNR): the higher the noise level, the greater number of wavelet coefficients is affected by noise and, hence, the threshold level must increase in order to provide correction of the wavelet coefficients in a broader range of scales. This circumstance leads to difficulties in automation of the wavelet filtration process. A key factor in this respect is estimation of the level of noise present in the image and optimum choice of threshold level C. If wavelet filtration is carried out for improving the quality of receiving video signal transmitted via a communication channel, the simplest variant of optimum adjustment of the wavelet filter parameters consists in preliminary transmission of a test (a priori known) image. In this case, one can readily estimate the level of noise in the communication channel and set the optimum threshold level C that will minimize the filtration error for the given noise level. The latter is determined by solving the problem of minimum in the dependence of the mean-square error on the threshold level, which can be performed automatically. This variant of adjusting wavelet filters verified in the framework of this investigation and proved to be effective for all (about ten) test images and various intensities of added noise. All tests led to unambiguous conclusion that image filtration based on complex wavelet transform is more effective and admits automatic adjustment of optimum parameters of the wavelet filter. After this adjustment, a decrease in the error of filtration in comparison to that based on the standard DWT was no less than 5% (and in some cases exceeded 10%).

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