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Development of an Algorithm for Detecting Saccadic Eye Movements Based on Model Approximation

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Abstract—We present an algorithm for detecting saccadic (fast) eye movements from electrooculogram data based on approximation using a parametric saccade model. The algorithm is based on a sliding window approximation on two coordinates using a parametric saccade model.

Keywords: saccades, approximation, detection of fast eye movements, saccade model, algorithm

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INTRODUCTION

The study of eye movements serves as a crucial tool for investigating visual perception processes, analyzing visual information, and diagnosing various neurological disorders. This method offers unique opportunities for assessing fatigue levels and the degree of individual engagement in tasks related to visual perception. It enables the evaluation of shifts in visual attention, the identification of moments of reduced concentration, and the assessment of cognitive load. The analysis of oculomotor movements finds broad application in areas such as vehicle operation, high-stress work environments, and education, contributing to the development of strategies aimed at enhancing efficiency and productivity in these fields. This method is also widely used in neuromarketing for accurately tracking consumer gaze direction, which enhances content quality and optimizes its placement. Eye movement analysis plays a key role in understanding cognitive processes, including visual analysis strategies for various stimuli [1]. Precise identification of oculomotor signal components helps detect various indicators of an individual's psychophysiological state, such as fatigue [2]. Currently, numerous methods exist for recording oculomotor activity; however, the most widely used are video-based analysis (eye tracking) and electrooculography (EOG) [3]. Each method has distinct advantages and limitations. For example, an eye tracker enables highly accurate measurement of pupil position and size, but in most cases, it requires head fixation, which can cause discomfort and potentially affect research outcomes. Notably, many wearable eye-tracking systems are now available that do not require head fixation and can be worn like regular glasses [3]. These eye trackers are highly applicable in practical tasks due to their mobility and ease of use. However, measurement accuracy heavily depends on their stability on the participant's head, requiring recalibration if displacement occurs. In contrast, EOG systems are firmly attached to the participant's skin, preventing abrupt and unpredictable changes in recorded values. However, these systems also have certain drawbacks, such as noise and low-frequency drift caused by the skin-galvanic response, as well as artifacts associated with muscle movements [4]. However, these drawbacks can be effectively mitigated through preprocessing and filtering techniques. Additionally, recent studies [5] have demonstrated that time-series decomposition methods enable the extraction of eye movement information from electroencephalograms (EEG), thereby enhancing analytical capabilities when analyzing data from previously conducted neurophysiological experiments.

Based on the above, there is a clear need to develop precise and noise-resistant methods for detecting and characterizing individual components of the oculomotor signal, such as saccades and fixations. These components play a crucial role in attentional mechanisms, as they are directly linked to shifts in visual focus between objects. Identifying the onset and offset of saccades, excluding post- and pre-saccadic oscillations, enables the precise determination of fixation periods. This, in turn, facilitates a comprehensive analysis of the relationships between attention, perception, and behavioral responses. This study focuses on developing an algorithm for detecting saccadic (rapid) eye movements to characterize accurately saccades and fixations.

ALGORITHM FOR SACCADE DETECTION

The developed algorithm is based on a sliding two-coordinate window approximation of oculographic data using a parametric saccade model proposed by Dai et al. [6]. The selection of this model for saccade detection is justified by its unique ability to represent accurately saccadic waveforms with varying amplitudes and durations. The model accounts for an essential physiological characteristic known as the "main sequence" [7], which describes the relationship between peak angular velocity and saccade amplitude. The model also exhibits high accuracy in reproducing large saccades, which are particularly challenging to model due to peak velocity saturation as amplitude increases. Study [6] emphasizes that conventional methods, such as sigmoid or Gumbel functions, fail to accurately approximate these characteristics, whereas the proposed model achieves a high degree of precision. The fundamental equation describing the saccadic eye movement model is

$$s(t;\eta,c,\tau) = cf\left(\frac{\eta t}{c}\right) - cf\left(\frac{(\eta(t-\tau))}{c}\right). \tag{1}$$

In the general case, the model includes three free parameters: η, c, and τ, which define the key characteristics of a saccade. The saccade shape is determined by a piecewise function $f(t)$, given in the equation

$$f(t) = \begin{cases} t + 0.25e^{-2t}, & t \geq 0, \\ 0.25e^{2t}, & t \leq 0. \end{cases} \tag{2}$$

The parameters η, c, and τ presented in Eq. (1) do not reflect the actual physiological characteristics of saccades, such as amplitude, duration, and velocity. Therefore, through analytical calculations, the parameter η was derived from the amplitude a and duration d. The resulting equation for calculating the parameter η is

$$\eta = c\frac{\ln\dfrac{0.5\left(e^{\frac{a}{c}}+1\right)}{p}}{d}. \tag{3}$$

Since the model is based on an exponential function, where the derivative of the saccade velocity asymptotically approaches zero, it is crucial to determine accurately the time boundaries for the onset and offset of the saccade. In this study, a threshold of 1% of the maximum saccade velocity was selected as the boundary, which is defined by the parameter p in Eq. (3). The selection of threshold is based on the fact that most algorithms detect the onset and offset boundaries of saccades at points where the derivative exceeds 10% of the peak velocity [8]. This high threshold is due to the presence of significant noise in the electrooculogram signal. Therefore, a 1% threshold for the model is sufficiently relevant for saccade boundary detection tasks. The model parameter τ from Eq. (1) can be defined as the ratio of amplitude a to the parameter η, as given in the following equation:

$$\tau = \frac{a}{\eta}. \tag{4}$$

Based on the equations presented above, the following set of parameters for the saccade model was obtained: a is the amplitude, d is duration, and c is a free parameter. To fit the parameters of the saccade model, the trust region reflective (TRF) method was used, implemented in the curve_fit function from the SciPy library. This approach ensures optimal approximation results with a minimal number of measurements, which enhances the algorithm's performance. The initial data for approximation included a time series of coordinates x and y (horizontal and vertical EOG) within a 200-ms window, as well as timestamps t calculated based on the sampling frequency. The data were approximated using the TRF model and algorithm, which allows for the incorporation of specified boundary conditions for the parameters. Amplitude and duration parameters can vary significantly from person to person and depend on visual stimuli and external conditions. However, in this study, we used commonly accepted ranges derived from numerous studies involving different participants and equipment. The duration of saccades typically ranges from 10 to 200 ms [9]. The average duration of saccades in adults usually falls between 30 and 50 ms for short saccades (up to 10°) and can reach 100–150 ms for longer saccades.

Thus, the minimum and maximum boundaries for the duration parameter d were selected within the range of values [10; 150]. The boundaries for the duration parameter d were then set to these values. For determining the boundaries of the amplitude parameter a, values from the literature were chosen. Saccade amplitude is typically measured in degrees and ranges from 1° to 50° [9]. The average amplitude of saccades in adults is often around 15°–20°. Based on this, the boundaries for the amplitude parameter a were selected within the range [1; 50]. The dimensionless parameter c was chosen over a broad range of values [1; 1000] to allow for more precise fitting and increased variability. Since the employed saccade model does not describe movement along the two axes, x and y, an additional transformation must be introduced, as shown in the following equation, where the parameter α defines the projection angle on the axis and is set within the range $[-\pi; \pi]$.

$$\begin{cases} sx(t) = s(t)\cos(\alpha), \\ sy(t) = s(t)\sin(\alpha). \end{cases} \tag{5}$$

This transformation allows for the simultaneous approximation of both vertical and horizontal electrooculogram data using the employed saccade model. The algorithm consists of two sequential stages (steps), and the block diagram is presented in Fig. 1. In the

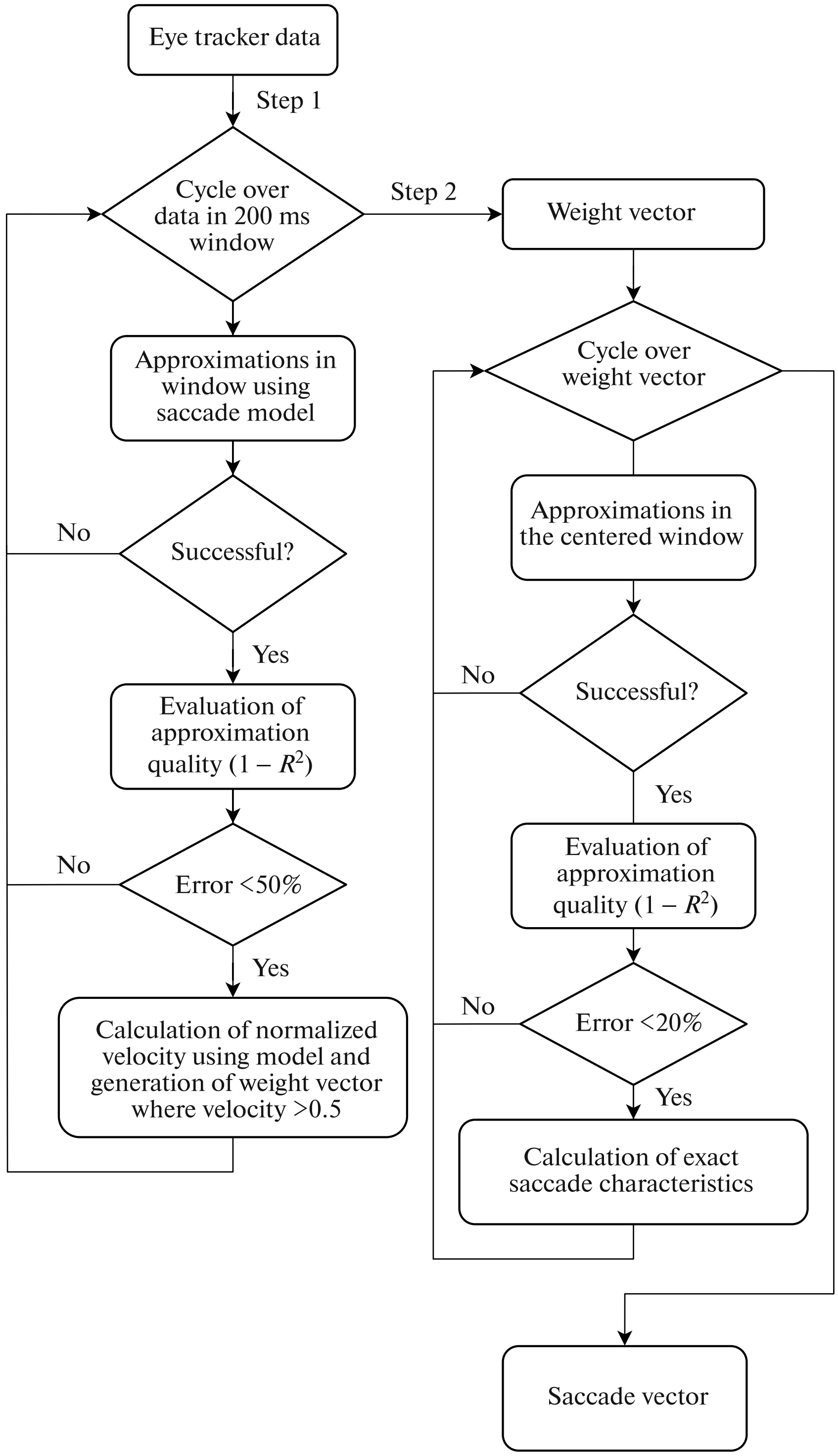

Fig. 1. Block diagram of the algorithm's operation, where Error = $1 - R^2$; R^2 is the coefficient of determination.

first stage, the algorithm scans the entire time series of oculographic data along the x and y axes using a sliding window of 200 ms with a step size of 20 ms. The window size is determined by the maximum saccade duration of 150 ms and a margin for accurate approximation of the longest saccades (in this case, 50 ms), which can vary depending on the task. For a more accurate approximation, it is preferable to use a smaller sliding window step. However, decreasing the step size leads to slower algorithm performance, which complicates the processing of lengthy experimental data. Therefore, an optimal step size of 20 ms was

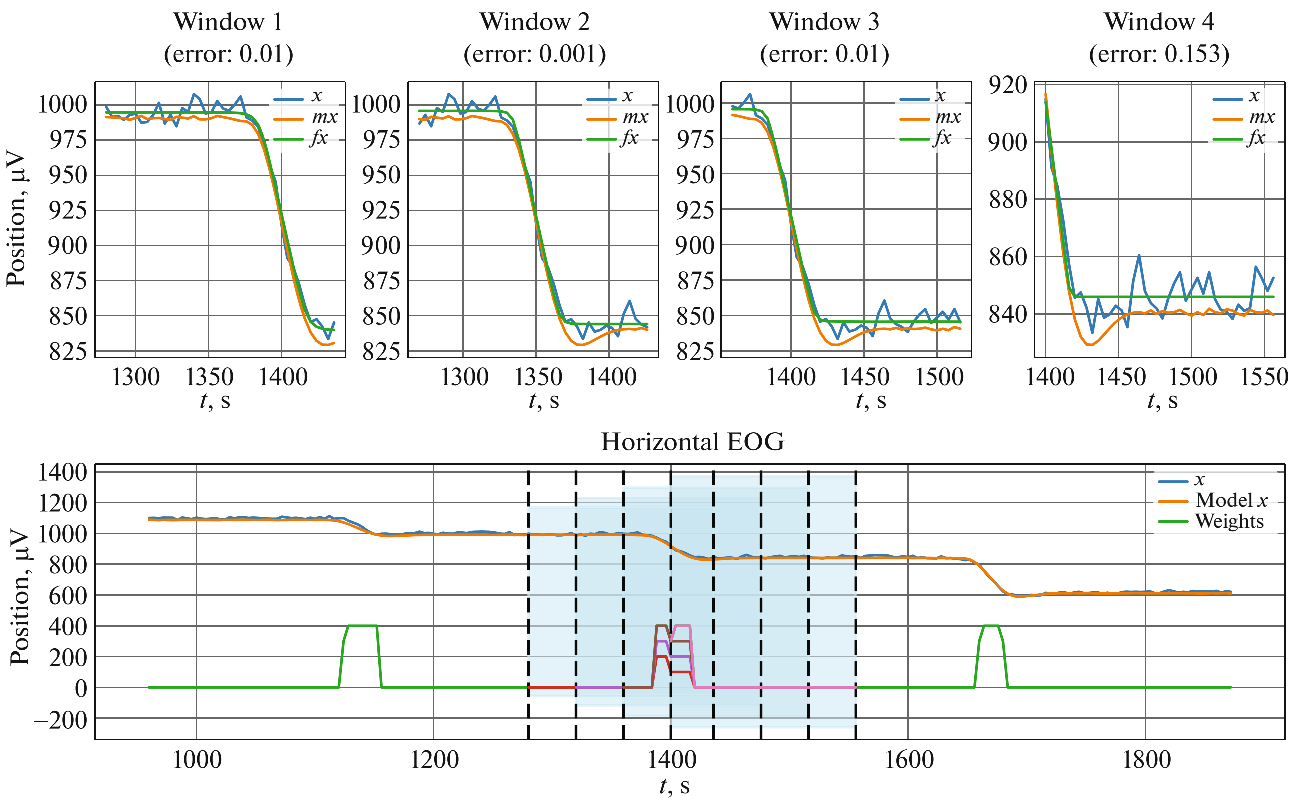

Fig. 2. Window-based approximation of the eye-tracker data for the horizontal electrooculogram: Windows 1–4 are approximation windows; error = $1 - R^2$; *mx* (model *x*) is the model data for the *x*-axis; *fx* is the approximating curve, weights are the weight vector.

empirically selected, balancing both acceptable accuracy and processing speed. At each iteration (in each time window), the algorithm approximates a 200 ms data set using the parametric saccade model. In the case of successful approximation, the quality of the fitted model to the data along the x and y axes is assessed. This is done by calculating the error as $1 - R^2$, where R^2 is the coefficient of determination. If the error for at least one axis does not exceed 50%, it indicates that the model sufficiently describes the data, and the algorithm can proceed to the next stage. In the next stage, the velocity is calculated based on the obtained saccade model, and a weight vector is formed. The weight vector is initially initialized with zero values and has a dimension equal to the length of the data set. The formation of this weight vector occurs as follows: the obtained velocity is normalized to its maximum value, then the indices of points where the value exceeds 0.5 are saved. After this, each point at the corresponding index in the weight vector is incremented by one. In this way, a weight coefficient array is formed, which, in the case of a subsequent occurrence of a saccade within the approximation window, will guaranteed increase the weight of the potential saccade, as shown in Fig. 2.

In the second stage, the algorithm processes the previously obtained weight vector for final selection and approximation of potential saccades. The approximation window is set within the range of −100 to 100 ms relative to the center of the found potential saccades in the weight vector. After successful approximation, and if the error ($1 - R^2$) along one of the axes is less than 20%, the final calculation of the precise saccade characteristics is performed based on the model parameters. If the error exceeds 20%, the saccade is excluded from further consideration.

RESULTS AND DISCUSSION

The developed algorithm was tested on a simulated oculographic signal. Considering that manual classification of EOG data does not always give reliable results due to the influence of various factors [10], the algorithm was evaluated using a simulated signal with the model proposed by Richard Schweitzer et al. [11], incorporating different noise levels. The selection of the model proposed by Schweitzer for simulating the horizontal and vertical components of the EOG signal is due to its ability to generate post-saccadic oscillations, making the data more relevant to real signals. Additionally, the eye movement activity simulation

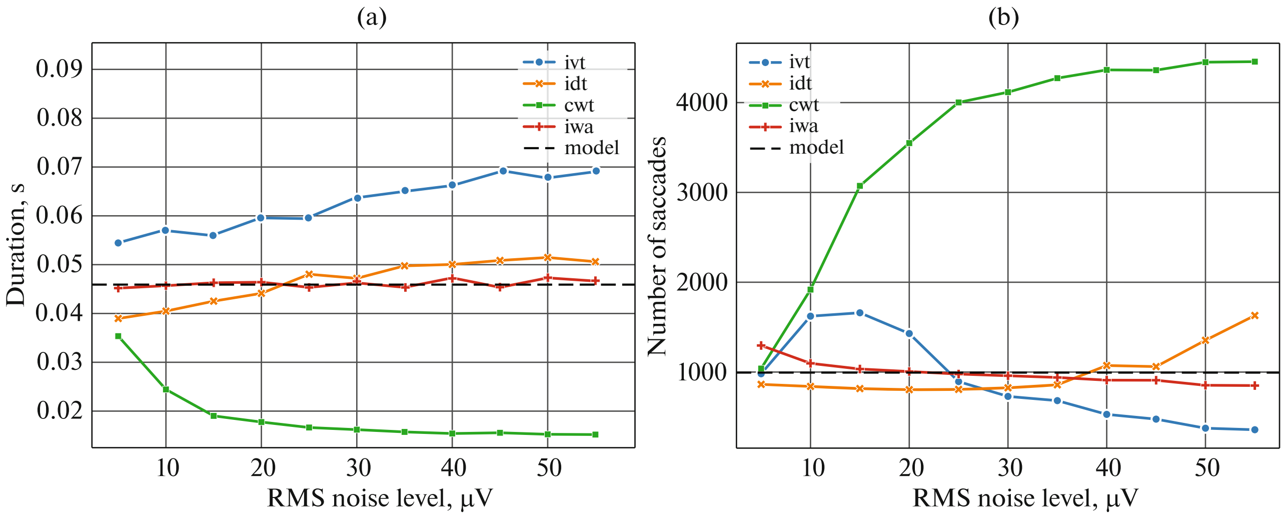

Fig. 3. (a) Dependence of the saccade duration on the RMS noise level. (b) Number of detected saccades vs. RMS noise level for the algorithms ivt (velocity-threshold identification); idt (dispersion-threshold identification); cwt (continuous wavelet transform–saccade detection); iwa (the proposed algorithm) "model" is the specified values for saccade duration (a) and number of saccades (b) in the simulated electrooculogram signal.

model used allows for the modeling of fixation drift (tremor, micromovements of the eyes), further bringing the simulated signal closer to actual oculographic data [12]. The noise applied was $1/f$ (pink) RMS noise [13].

The developed algorithm demonstrated high efficiency in determining the total number of saccades (Fig. 3) compared to popular algorithms such as ivt, velocity-threshold identification [14]; idt, dispersion-threshold identification [15]; and cwt, a method using continuous wavelet transform for saccade detection [8].

Figure 3b shows that the developed algorithm detects a total number of saccades approximately equal to 1000, which matches the value set in the signal simulation parameters. The accuracy of determining saccade duration is equally crucial. Figure 2a presents a graph showing the relationship between saccade duration and varying levels of RMS noise. For the developed algorithm, an increase in noise level does not significantly affect the accuracy of saccade duration determination (Fig. 2a). It can also be observed that the other algorithms tested were less robust to noise, with the accuracy of saccade duration determination decreasing as the noise level increased. This indicates that the proposed algorithm is more noise-resistant and holds promise for application in various practical and research tasks.

CONCLUSIONS

Thus, we have developed and implemented an algorithm for detecting saccadic eye movements based on electrooculogram (EOG) data. The developed algorithm accurately determines the boundaries of the start and end of saccades without accounting for distortions associated with post- and pre-saccadic oscillations. The algorithm demonstrated high robustness to increased noise levels in the data. The developed algorithm does not account for cases where more than one saccade falls within each window sequentially. This limitation arises because the model used in this work only considers a single saccade at a time.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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