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Elena Pitsik, Vladimir Makarov, Stefano Boccaletti, "Pattern formation in spatially distributed networks via spatially correlated preferential attachment," Proc. SPIE 11067, Saratov Fall Meeting 2018: Computations and Data Analysis: from Nanoscale Tools to Brain Functions, 110670U (3 June 2019); doi: 10.1117/12.2527748

Event: International Symposium on Optics and Biophotonics VI: Saratov Fall Meeting 2018, 2018, Saratov, Russian Federation

Pattern formation in spatially distributed networks via spatially correlated preferential attachment

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ABSTRACT

In this paper we propose a model of the spatially distributed network based on the spatially correlated preferential attachments. Nodes in the spatially distributed networks of the real word, such as various urban or biological networks, aren't establishing randomly: the probability of emergence of new nodes is higher in the area of already existing ones. In this work we unite two principles of the real network modeling: the correlated percolation model and preferential attachment. To regulate spatial limitations of the network, we use density gradient, which determines the decrease of the probability of the connection emergence between two nodes with increase of the distance between them. We also consider the consistency of our results in the context of the real-world system modeling.

Keywords: Spatially distributed network, scale-free network, complex network, preferential attachment

1. INTRODUCTION

Spatial limitations of the world is one of the main factors influencing the formation of many networks topologies. Interaction in biological systems, such as populations of animal species, is highly dependent on their habitats;[1] topologies of city networks are also strongly associated with the nature of their occurrence;[2] even social networks demonstrate the dependency on their spatial configuration.[3] There is a fairly clear idea of the principles of a spatial growth of such systems since they are widely studied in general,[4] however, considering such biological systems as neural cultures, the features of the spatial distribution influence on topology remains insufficiently studied. The ability of such systems to maintain the optimality of their connections is unique,[5] different layers of the brain neural networks organization show different topological scales and, accordingly, have a different degree of connectivity(for example see[6,7]). The existence of synoptic connections[8] that unite different parts of the brain is well known, but studies also show that, in addition to this type of connection, there is also a slow neural interaction at short distances,[9] an important role in the formation of which is played by astrocyte cells.

To our knowledge, the approach to study real biological networks by modeling evolutionary processes and self-organization in complex networks proved itself as productive. Indeed, there are evidence that the biological neural ensembles produces activity similar to chimera states in complex networks based on Kuramoto-Sakaguchi model and Hindmarsh–Rose neuron model.[10,11] A number of studies are dedicated to various types of behaviour observed in complex networks which have analogues in biological brain networks: adaptive coupling in the network based on randomly couple Rulkov maps,[12] inter-layer competition[13] and synchronization[14] in Kuramoto model. Results obtained in such researches can shed light on the nature of the processes occurring in the brain, including pathological ones.

The facts mentioned above reveal great importance of detection of topological features of spatially-distributed networks along with the understanding of their formation principles. In recent studies, a lot of attention has been focused on the features of spatially-distributed scale-free networks.[15,16] Is has been recently revealed that spatial-distributed networks aren't necessary scale-invariant,[17,18] e. g., social networks, along with the airport networks of various countries, demonstrate the features of a small world with the Poisson degree distribution. In the present paper we investigate how the transition from connectivity at short distances to a scale-free structure affects the main topological properties of a spatially distributed network.

Saratov Fall Meeting 2018: Computations and Data Analysis: from Nanoscale Tools to Brain Functions,
edited by Dmitry E. Postnov, Proc. of SPIE Vol. 11067, 110670U · © 2019 SPIE
CCC code: 0277-786X/19/$18 · doi: 10.1117/12.2527748

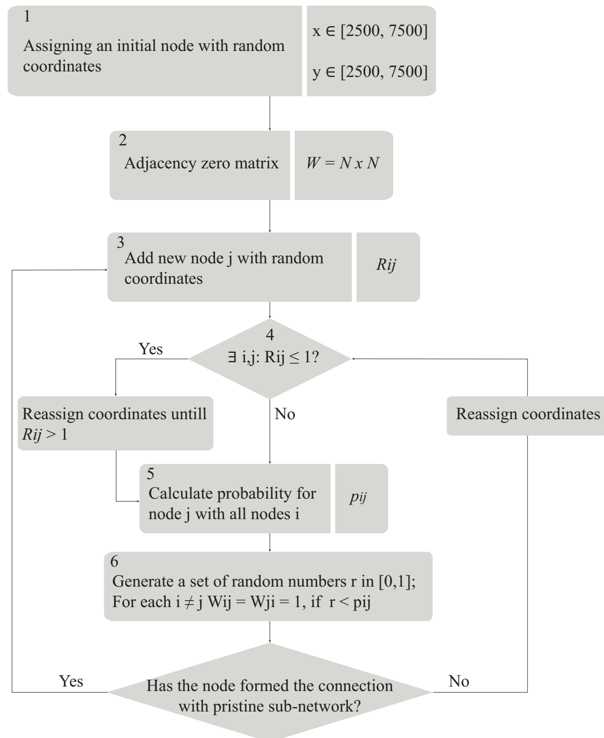

Figure 1. The method of network generation

2. METHODS

A numerical model has been developed using principles of spatial growth and preferential attachment. In real spatially distributed networks such as urban systems and brain neural networks the coordinates of the nodes are not random, because the probability of emergence of a new node is higher in the vicinity of existing one. To implement this condition, we use modified correlated percolation model[4] which assumes that the probability of a new node j depends on how densely its surroundings is. In particular, the probability p_{ij} is:

$$p_{ij} = e^{-\lambda R_{ij} \cdot d_i^{-\beta}}, \tag{1}$$

where R_{ij} is Euclidian distance between nodes i and j, the density gradient λ defines the decrease of the probability p_{ij} with increasing R_{ij}, β stands for the degree factor which allows to reduce the impact of λ when the degree of the node i becomes sufficiently large.

To generate the research compliant network we used an algorithm presented on the Fig. 1. At the first step, we select random coordinates x and y of an initial node with $x, y \in [2500 : 7500]$. Each next no de added in the network gets randomly assigned coordinates that are satisfying the following conditions:

1. Euclidian distance $R_{ij} > 1$ for all nodes i.
2. The node must establish connection with the pristine sub-graph. If such connection is formed, the process of the network growth continues; otherwise, the coordinates are reassigned and the algorithm starts from step 3 of Fig 1.

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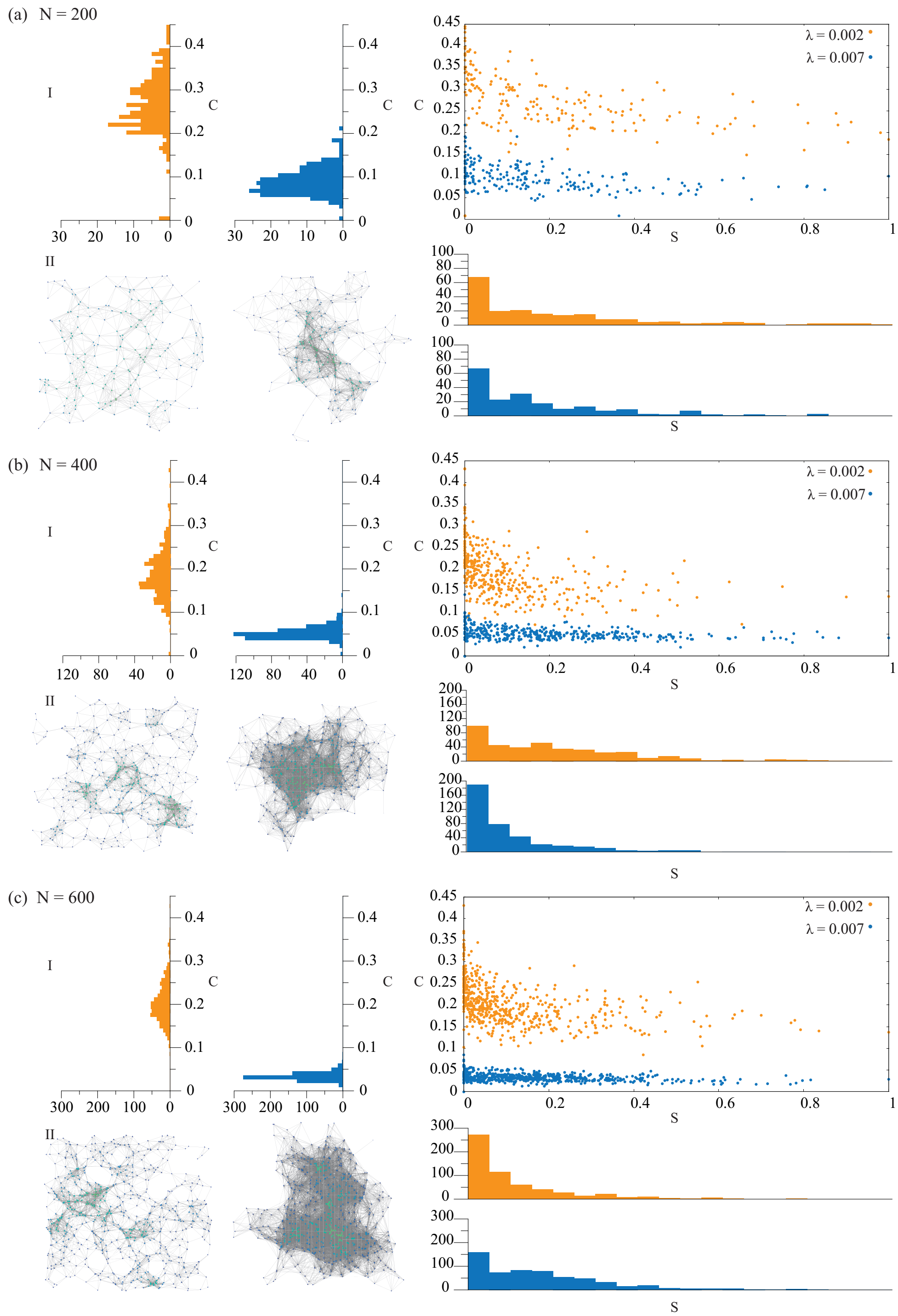

Figure 2. Obtained correlations for networks with varying density gradient λ and number of nodes N: (a) $N = 200$, (b) $N = 400$, (c) $N = 600$. For each part of the figure, the left part of panel I represents the clustering coefficient C distribution and the right part is correlation between clustering coefficient C and the number of shortest paths S. Accordingly, the left part of panel II is visualization of the network, and the right part is the shortest paths S distribution.

New connections in the network are emerging as follows: the probability p_{ij} is calculated with Eq. 1. According to the 6^{th} step of the scheme on Fig. 1, we generate a set of random numbers with a uniform probability distribution, normalized in $[0, 1]$. The smaller than p_{ij} value of generated number corresponds to established connection between nodes, thus we set $W_{ij} = W_{ji} = 1$ in adjacency matrix.

Such method of network generation provides the emergence of spatially inhomogeneous structures, emergence of hubs and long-distance connection, which brings the model closer to the real system.

Then, we estimate a few network parameters that reflect its structural and topological features. In particular, we consider a correlation between global clustering coefficient C averaged over all network, which is characteristic of the degree of attachment of a node to tightly bounded group, and the number of the shortest paths in the network S.

3. RESULTS

We consider two networks with different values of the density gradient, $\lambda = 0.002$ and $\lambda = 0.007$. As follows from the Eq. 1, the density gradient determines the spatial constraints of network evolution, reducing or increasing the probability of establishing a connection between nodes depending on the Euclidian distance. Fig 2 illustrates the structural features of these networks using correlation between clustering coefficient and the number of shortest paths. As can be seen from these correlation and network visualization presented for $N = 200$, $N = 400$, and $N = 600$ nodes, different values of λ result in completely different topology.

As follows from Fig. 2(a), the network with $\lambda = 0.002$ has rather homogeneous topology close to random network. Indeed, low value of λ allows to maintain the radius of the nodes efficiency, which means the prevalence of short-range connections in the network. Correlation between clustering coefficient and the shortest paths presented on the right part of panel I confirms this conclusions, showing scattered character of nodes location.

On the contrary, increasing value of λ causes more pronounced inhomogeneity in network structure. As shown in Fig. 2, the large density gradient enhances the dependence of probability of the node emergence on Euclidian distance from other nodes of the network, resulting in appearance of new nodes only in close proximity to other nodes. The clustering coefficient distribution on the left part of the panel I shows that the increase of λ also leads to disintegration of clusters and emergence of nodes capable to maintain long-distance connections.

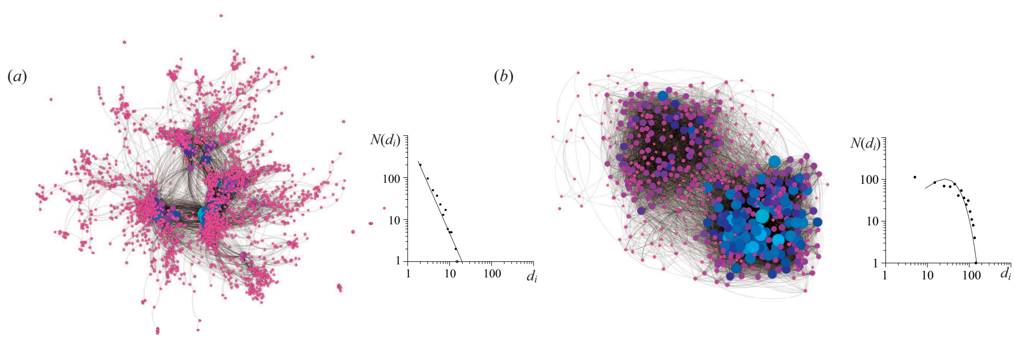

Figure 3. Visualizations of (a) the global airline network (Openflight.org) and (b) the resting-state fMRI network of human brain with corresponding degree distributions.

Fig. 2(b) and Fig. 2(c) present the same correlations with growing of the network capacity, $N = 400$ and $N = 600$, respectively. As can be seen from the results, a picture for both values of N is rather the same: the network size growth enhances the effects described above. In case with $\lambda = 0.002$, although the presence of hub-like nodes are observed, the whole structure of the network remains rather homogeneous, the nodes establish mostly short-distance connections. However, with $\lambda = 0.007$, the hubs gaining an ability to connect with nodes at longer distances, which leads to the prevalence of the preferential attachment in the network topology.

Fig. 3 provides the insight on the accordance of obtained results to the way of the real-system organization. The network of resting-state fMRI of the human brain[19] (Fig. 3(b)) represents the structure which topological features are dictated by the spatial limitations. At the same time, the global airline network[20] on the Fig. 3(a) is a classic example of spatially distributed scale-free network with power law degree distribution, which is determined by a sufficient amount of long-range connections. One can see that preferential attachment is the common property that is shared by both proposed model and this example of real network.

4. CONCLUSION

In our paper the focus of attention was on spatially distributed networks whose structural and topological properties are changing in the process of evolution, and on the preferential attachment as on the key concept of complex networks research. Proposed model provides an insight into the mechanisms of real-world network emergence: we observed formation of inhomogeneous structures when the process of networks evolution was accompanied by strict spatial limitations, which resulted in scale-free topology and long-distance connections with hubs.

Obtained results are consistent with several observations made in other investigations of preferential attachments. The same pattern has been observed in the networks of scientific collaboration and citation,[21,22] protein network[23] and the Internet indexation.[24]

5. ACKNOWLEDGMENTS

This work has been supported by the Russian Science Foundation (project 17-72-30003).

REFERENCES

[1] Fortuna, M. A., Gómez-Rodríguez, C., and Bascompte, J., "Spatial network structure and amphibian persistence in stochastic environments," *Proceedings of the Royal Society of London B: Biological Sciences* **273**(1592), 1429–1434 (2006).

[2] Makarov, V. V., Hramov, A. E., Kirsanov, D. V., Maksimenko, V. A., Goremyko, M. V., Ivanov, A. V., Yashkov, I. A., and Boccaletti, S., "Interplay between geo-population factors and hierarchy of cities in multilayer urban networks," *Scientific reports* **7**(1), 17246 (2017).

[3] Sailer, K. and McCulloh, I., "Social networks and spatial configuration—how office layouts drive social interaction," *Social networks* **34**(1), 47–58 (2012).

[4] Makse, H. A., Andrade, J. S., Batty, M., Havlin, S., Stanley, H. E., et al., "Modeling urban growth patterns with correlated percolation," *Physical Review E* **58**(6), 7054 (1998).

[5] de Santos-Sierra, D., Sendina-Nadal, I., Leyva, I., Almendral, J. A., Anava, S., Adeli, A., Papo, D., and Boccaletti, S., "Emergence of small-world anatomical networks in self-organizing clustered neuronal cultures," *PLoS One* **9**(1), e85828 (2014).

[6] Makarov, V. V., Zhuravlev, M. O., Runnova, A. E., Protasov, P., Maksimenko, V. A., Frolov, N. S., Pisarchik, A. N., and Hramov, A. E., "Betweenness centrality in multiplex brain network during mental task evaluation," *Physical Review E* **98**(6), 062413 (2018).

[7] Maksimenko, V. A., Runnova, A. E., Frolov, N. S., Makarov, V. V., Nedaivozov, V. O., Koronovskii, A. A., Pisarchik, A. N., and Hramov, A. E., "Multiscale neural connectivity during human sensory processing in the brain," *PHYSICAL REVIEW E* **97** (May 2018).

[8] Bullmore, E. and Sporns, O., "Complex brain networks: graph theoretical analysis of structural and functional systems," *Nature Reviews Neuroscience* **10**(3), 186 (2009).

[9] Makovkin, S., Kumar, A., Zaikin, A., Jalan, S., and Ivanchenko, M., "Multiplexing topologies and time scales: The gains and losses of synchrony," *Physical Review E* **96**(5), 052214 (2017).

[10] Makarov, V. V., Kundu, S., Kirsanov, D. V., Frolov, N. S., Maksimenko, V. A., Ghosh, D., Dana, S. K., and Hramov, A. E., "Multiscale interaction promotes chimera states in complex networks," *Communications in Nonlinear Science and Numerical Simulation* **71**, 118–129 (2019).

[11] Frolov, N. S., Maksimenko, V. A., Makarov, V. V., Kirsanov, D. V., Hramov, A. E., and Kurths, J., "Macroscopic chimeralike behavior in a multiplex network," *Physical Review E* **98**(2), 022320 (2018).

[12] Andreev, A., Pitsik, E., Makarov, V., Pisarchik, A., and Hramov, A., "Dynamics of map-based neuronal network with modified spike-timing-dependent plasticity," *The European Physical Journal Special Topics* **227**(10-11), 1029–1038 (2018).

[13] Pitsik, E., Makarov, V., Kirsanov, D., Frolov, N., Goremyko, M., Li, X., Wang, Z., Hramov, A. E., and Boccaletti, S., "Inter-layer competition in adaptive multiplex network," *New Journal of Physics* **20**(7), 075004 (2018).

[14] Makarov, V. V., Koronovskii, A. A., Maksimenko, V. A., Hramov, A. E., Moskalenko, O. I., Buldu, J. M., and Boccaletti, S., "Emergence of a multilayer structure in adaptive networks of phase oscillators," *Chaos, Solitons & Fractals* **84**, 23–30 (2016).

[15] Makarov, V. V., Kirsanov, D. V., Frolov, N. S., Maksimenko, V. A., Li, X., Wang, Z., Hramov, A. E., and Boccaletti, S., "Assortative mixing in spatially-extended networks," *Scientific reports* **8**(1), 13825 (2018).

[16] Barthelemy, M., "Crossover from scale-free to spatial networks," *EPL (Europhysics Letters)* **63**(6), 915 (2003).

[17] Jamakovic, A. and Uhlig, S., "On the relationships between topological measures in real-world networks," *Networks and Heterogeneous Media* **3**(2), 345 (2008).

[18] Herrmann, C., Barthelemy, M., and Provero, P., "Connectivity distribution of spatial networks," *Physical Review E* **68**(2), 026128 (2003).

[19] Crossley, N. A., Micheli, A., Vértes, P. E., Winton-Brown, T. T., Patel, A. X., Ginestet, C. E., McGuire, P., and Bullmore, E. T., "Cognitive relevance of the community structure of the human brain functional coactivation network," *Proceedings of the National Academy of Sciences* **110**(28), 11583–11588 (2013).

[20] Verma, T., Araújo, N. A., and Herrmann, H. J., "Revealing the structure of the world airline network," *Scientific reports* **4**, 5638 (2014).

[21] Newman, M. E., "Clustering and preferential attachment in growing networks," *Physical review E* **64**(2), 025102 (2001).

[22] Jeong, H., Néda, Z., and Barabási, A.-L., "Measuring preferential attachment in evolving networks," *EPL (Europhysics Letters)* **61**(4), 567 (2003).

[23] Eisenberg, E. and Levanon, E. Y., "Preferential attachment in the protein network evolution," *Physical review letters* **91**(13), 138701 (2003).

[24] Capocci, A., Servedio, V. D., Colaiori, F., Buriol, L. S., Donato, D., Leonardi, S., and Caldarelli, G., "Preferential attachment in the growth of social networks: The internet encyclopedia wikipedia," *Physical review E* **74**(3), 036116 (2006).