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Analyzing the Structure of a Complex Network on the Basis of Its Macroscopic Characteristics

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Abstract—A method for analyzing structural changes and clusterization processes in complex networks by examining their macroscopic characteristics (a system's overall vector of state) using the continuous wavelet transform is proposed. It is demonstrated that the proposed method allows effective diagnostics of changes in network topology and the detection of structural clusters.

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INTRODUCTION

The study of synchronous regimes and processes of structure formation in networks with complex topologies is one of the most important problems currently facing the academic community. Modern scientific concepts of the world lean toward the idea of its network architecture [1]. Different network structures arise at all levels of organization in biological [2], technological [3], and social [4, 5] systems ranging from neuron ensembles [6, 7] to networks of cities and populations [8].

The presence of large numbers of elements in networks and inbound and outbound links that are nonuniformly distributed across them gives rise to several different phenomena in the collective dynamics of a network's structural components. Among these phenomena are the formation of subnetworks (clusters) [9] composed of tightly coupled elements and the emergence of synchronous regimes [10].

Synchronization is one of the most important phenomena observed in complex networks. For instance, synchronism in the behavior of interacting elements of adaptive networks [11] often turns out to be the primary factor influencing the temporal evolution of the network's topology.

Models of network structures that incorporate adaptive mechanisms are used more and more often nowadays to describe processes in social systems and neuron ensembles [12, 13]. Descriptions of the evolution of such networks' topology allows us to develop mathematical interpretations that closely approximating actual networks in which temporal changes in a system's links lead to the formation of different structural patterns.

However, it should be noted that studies of actual adaptive networks through the developing of mathematical models are not always efficient. The main problem in analyzing most systems with network structure is the lack of necessary data on the topology, the nature of links, and the dynamic state of individual elements and subnetworks. This renders the construction of a proper model that includes the links between interacting elements and their temporal evolution impossible, and we are usually forced to turn to macroscopic parameters (e.g., the signals from the electrical activity of the brain, produced by the interaction of individual neurons in a large neuron ensemble and obtained through electroencephalography) that characterize the processes in the network. The characteristics of such macroscopic network parameters vary over time, and it is believed that the patterns of their evolutionary dynamics can be ascribed to certain processes of network structure formation at the microscopic level. This would in turn allow us to analyze actual complex networks based on experimental macroscopic characteristics.

Studies of this possibility are of great interest to scientists dealing with research into complex network structures and working in different scientific fields. Acquiring data on the network topology and synchronous regimes of collective dynamics through the analysis macroscopic characteristics would allow us to gain further insight into the structural features and functioning of actual networks of different natures, which are difficult to study with current methods of analysis.

This work presents the results from studying the collective dynamics of network elements based on a network of interacting Kuramoto phase oscillators [14, 15] with a topology that varies over time in accordance with an adaptive mechanism [11]. We emphasized analyzing the structure of this network and studying clusterization processes with the help of a

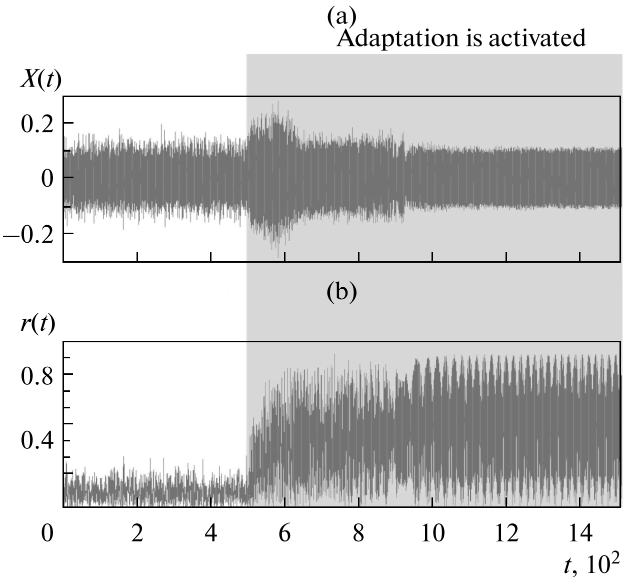

Fig. 1. Time variations in (a) the overall vector of state for network elements and (b) order parameter $r(t)$. Number of elements $N = 200$.

macroscopic characteristic (the signals produced by interacting oscillators).

The results from our macroscopic analysis are compared to characteristics calculated using the coupling matrices of the studied network and the vectors of state of individual elements. This comparison revealed a connection between the analyzed signals and the microscopic parameters characterizing the network topology.

SYSTEM AND MATHEMATICAL MODEL

The Kuramoto phase oscillators model is one of the most widely used network models. It was proposed in 1975 [16] as a mathematical interpretation of the collective dynamics of chemical and biological oscillators. Various modifications of this model of a phase oscillator network are now often used in analyzing clusterization and synchronization processes (including those found in social systems [11]).

The modified Kuramoto model proposed in [11] was used in this work. This modification is composed of $N = 200$ coupled oscillators. Each ith node of the analyzed network was characterized by its phase φ_i and interacted with all other $N - 1$ nodes. The behavior of each oscillator was described by the equation

$$\dot{\varphi}_i = \omega_i + \lambda \sum_j^N w_{ij} \sin(\varphi_j - \varphi_i), \quad (1)$$

where ω_i denotes natural frequencies set at random within the range of [1, 10]; w_{ij} is the weight of the link between nodes j and i; and λ is the link strength. The initial phases of interacting elements were set at random and distributed uniformly over the $[-\pi, \pi]$ interval, and the link weights were also set at random. Following a transient process, the adaptation mechanism in [11] was activated at point in time $t^* = 500$, and this affected the network dynamics. The results from studies of such networks with nonstationary links suggest that adaptation is the main reason for changes in network topology and the formation of structural clusters [11]. This makes such networks most interesting in terms of analyzing the processes of structure formation.

We used the signal produced by the network as a macroscopic parameter characterizing the dynamics of interacting oscillators (1):

$$X(t) = \sum_{i=1}^N x_i(t), \quad (2)$$

where $N = 200$ is the number of elements whose individual temporal behavior is described by the law $x_i(t) = A\cos(\varphi_i(t))$, where A is the unit amplitude.

Figure 1a shows the time dependence of state $X(t)$ of system (2). It was noted above that the adaptation mechanism in the network was activated at $t^* = 500$ (the region of adaptive dynamics region is shaded). The representation of (2) has the form of a random signal. It can be seen that the actuation of adaptation is followed by a complex transient process that is obviously related to network topology evolution and leads to a qualitative change in the oscillation regime. Figure 1b shows the time dependence of the $r(t)$ parameter, which reflects the system's regularity and represents the degree of oscillator coherence:

$$r(t) = \frac{1}{N} \sum_{i=1}^N e^{i\varphi_i(t)}. \quad (3)$$

It is easy to see that the $r(t)$ order parameter grows sharply when the adaptation mechanism is activated (Fig. 1b). This indicates the emergence of tightly coupled synchronous structures [11]. It should be noted that parameter $r(t)$ can be calculated only by using local characteristics of all the network elements (the vector of state of each element). However, its temporal evolution gives us only a general idea of the system dynamics in the evolution of its topology and provides no information on the total number and characteristics of structures formed within it.

APPLYING THE METHOD

It was noted above that only the behavior of the macroscopic parameters of the system is known in most cases of analyzing the dynamics of actual networks. The signals produced either by interacting elements of the entire analyzed network or by separate groups of elements included in it can serve as such parameters. It is obvious that changes in network topology lead to changes in the characteristics of these signals, so they can be used to study network structure's dynamics and the synchronization regimes observed in it.

The continuous wavelet transform technique is used most effectively to analyze complex signals with spectral compositions that vary over time. This technique ensures the highest time and frequency resolution and allows us to study the temporal evolution of a signal's spectral composition. Signal (2) corresponding to the macroscopic characteristic of the Kuramoto oscillator network was analyzed in this work using the wavelet transform

$$w(\omega, t) = \sqrt{\omega} \int_{t-\frac{4}{\omega}}^{t+\frac{4}{\omega}} X(t')\psi^*(\omega(t-t'))dt', \quad (4)$$

where ω corresponds to the frequencies in which the analyzed signal is expanded, $\psi^*(t - t')$ is the parent wavelet, and (*) denotes complex conjugation. The Morlet wavelet served as the parent function.

Figures 2a, b show the results from applying transform (4) to the signal shown in Fig. 1a. Figures 2c, 2d show visual representations of the structure of the network being analyzed. These representations were plotted using matrices of the couplings between the network elements.

Figure 2a presents the frequency distribution of the wavelet transform amplitude prior to the actuation of adaptation ($t_1 = 490$). The links between the elements were in this case set at random and did not change overtime. We see clearly that the energy was uniformly distributed over the frequencies. At the same time, certain isolated spectral components did persist, indicating the presence of groups of synchronized elements. However, it should be noted that the absence of link evolution prevented the elements from forming synchronous isolated clusters. This state of the system is illustrated in Fig. 2c, which shows a visual representation of the network topology at $t_1 = 490$.

Three isolated peaks corresponding to synchronous clusters formed in the network emerged in the frequency distribution of the wavelet transform amplitude (Fig. 2b) at time point $t_2 = 1300$, which corresponds to the regime established in the system following the transient adaptation process (see Fig. 1b). Figure 2c shows the network structure at this point in time. We clearly see that the elements are grouped into three isolated structural clusters.

It should be noted that the wavelet spectrum peaks (Fig. 2b) differ in power. This can be attributed to the difference between the number of elements in the synchronous clusters (Fig. 2d).

We thus demonstrated the possibility of analyzing the structural changes in a network and detecting emerging clusters by using the wavelet transform of the signal produced by all elements of this network.

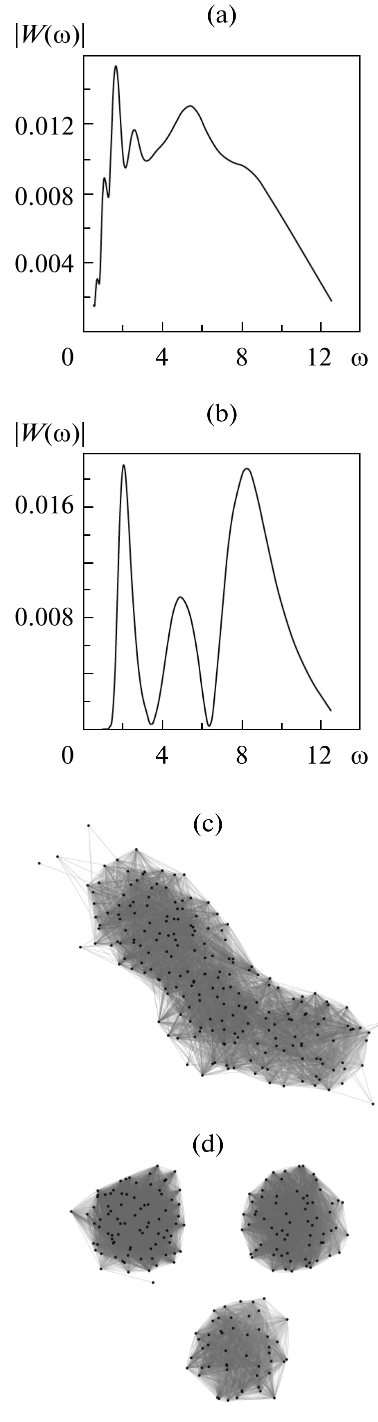

Fig. 2. Upper panels: amplitude wavelet spectra of the overall vector of state for elements of the network at (a) $t = 500$ and (b) $t = 1300$. Lower panels: graphic representations of this network at (c) $t = 500$ and (d) $t = 1300$.

CONCLUSIONS

The structure of a Kuramoto phase oscillator network was studied through wavelet analysis of the vector of state obtained from the nodes of this network. The results from applying this method fully characterized the network topology at different points in time. This was confirmed by comparing the results from our analysis and graphical representations of the network topology, plotted on the basis of matrices of the couplings of its elements.

We thus demonstrated the possibility of acquiring data on structural clusters emerging in a network by analyzing the macroscopic characteristics of an ensemble of interacting elements. Our results could find wide application in studies concerned with analyzing electroencephalograms and magnetoencephalograms, and help to detect regimes of the synchronization of neuron ensembles and reveal various forms of cognitive activity.

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