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A Method of Distinguishing Between the Characteristic Phases of Behavior in Complex Networks in the Intermittent Generalized Synchronization Regime

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Abstract—A method of identification of the phases of synchronous and asynchronous intervals in the time realizations of interacting chaotic systems representing elements of a network with complex coupling topology in the state of transition to the generalized synchronization regime is proposed. The method allows determining the duration of the phases of synchronous and asynchronous dynamics, which potentially allows analyzing the statistical characteristics of the intermittent behavior of the discussed systems.

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The generalized synchronization regime is one of the most interesting known types of chaotic synchronization [1], such as phase synchronization, lag synchronization, and complete synchronization [2–4]. Originally, the concept of generalized synchronization was introduced for two unidirectionally coupled oscillators [2], and it was extended later to include mutually coupled oscillators and complex networks [5–7]. Different dynamic systems can play the role of interacting oscillators. In so doing, the interacting oscillators can have different dimensions of the phase space [4].

Currently, the phenomenon of generalized synchronization is being investigated in detail for a wide range of interacting systems, such as unidirectionally and mutually coupled discrete time systems [8, 9], along with unidirectionally [10, 11] and mutually [5] coupled flow systems (including spatially distributed systems [12]). Analysis of networks of nonlinear elements exhibiting complex coupling topology [13] represented the next step in the investigation of the effect of generalized chaotic synchronization. In particular, the process of setting up the generalized synchronization regime upon transition from an asynchronous dynamics to a synchronous one in a small network of logistic maps has been studied in [14].

The processes of setting up a synchronous regime with increasing intensity of coupling between the interacting systems are important for understanding the nature of chaotic synchronization. It is well known that the transition from the asynchronous behavior to the synchronous regime in two coupled chaotic oscillators is accompanied by an intermittence wherein the intervals of synchronous dynamics (laminar phases) are interrupted by intervals of asynchronous behavior (turbulent phases) in time realizations of the discussed systems at a fixed value of the coupling parameter. In so doing, specific types of intermittence correspond to different kinds of the chaotic synchronization [15–18]. It is known that, in the case of two unidirectionally coupled oscillators, the transition to the generalized chaotic synchronization regime is accompanied by "on–off" intermittence [17]. At the same time, it is obvious that setting of the generalized chaotic synchronization regime in a substantially more complex system, such as a network of coupled oscillators, can have a different scenario or, at least, different characteristics of the intermittent behavior, although setting up the generalized chaotic synchronization regime in networks with complex coupling topology through "on–off" intermittence cannot be ruled out. As of today, this question remains completely open.

Determination of the type of the intermittent behavior is largely based on the analysis of the statistical characteristics, such as the dependence of the average length of the laminar phases on the supercriticality parameter or the distribution of the length of the laminar behavior intervals at fixed values of control parameters [19]. However, it turns out to be impossible to distinguish between the intervals of synchronous and asynchronous behavior by means of the traditional method [17] based on using an auxiliary system [4] in the case of setting up the generalized chaotic synchronization regime in the networks with complex coupling topology, because this method cannot be

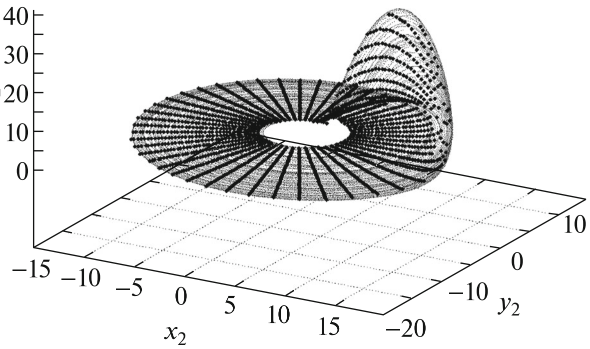

Fig. 1. Coverage of the chaotic attractor of the second oscillator by control points $\{\mathbf{x}_l^2\}_{l=1,\ldots,M}$.

used when the system contains mutually coupled oscillators [5]. Correspondingly, the key problem in the investigation of transition to the generalized synchronization regime in the network of nonlinear oscillators consists in creation and approbation of the method of identifying the characteristic intervals of synchronous and asynchronous dynamics (laminar and turbulent phases) in the time realizations of the interacting systems.

Thus, this work aims at developing a method of distinguishing between the characteristic phases of behavior in complex networks in the vicinity of the generalized synchronization regime boundary.

The approach proposed for identifying the laminar and turbulent phases of behavior of the network of interacting oscillators is based on the nearest-neighbor method [2, 20]. Let us analyze two oscillators of the network under consideration, e.g., the ones with numbers i and j. It is necessary to choose control point $\mathbf{x}_1^i$ on the attractor in the phase space of one of the oscillators (e.g., the ith one); find N of its nearest neighbors $\{\mathbf{x}_{1k}^i\}_{k=1,\ldots,N}$, such that $\|\mathbf{x}_1^i - \mathbf{x}_{1k}^i\| < \delta$ for the chosen control point; and fix the corresponding to them images of the nearest neighbors, $\mathbf{x}_1^{ji}$ and $\{\mathbf{x}_{1k}^{ji}\}_{k=1,\ldots,N}$, in the phase space of the jth oscillator [2, 20]. Average distance S between the images of the control point and the nearest neighbors in the phase space of the jth oscillator should be considered as the quantitative characteristic of the degree of synchronism (in the sense of generalized chaotic synchronization) of the ith and jth oscillators under consideration. If we define its value as

$$S_1^{ji} = \frac{\sum_{k=1}^{B} \|\mathbf{x}_1^{ji} - \mathbf{x}_{1k}^{ji}\|}{N} \quad (1)$$

the character of dynamics for two nodes of the network (synchronous/asynchronous) at the moment of time corresponding to the ith system location at selected control point $\mathbf{x}_1^i$ can be determined based on the relation between the value of S_1^{ji} and some preliminarily specified threshold value S_c (larger/smaller). Covering the attractor of the ith oscillator by a large number of control points $\{\mathbf{x}_1^i\}_{l=1,\ldots,M}$ with a high coverage density and using interpolation, it becomes possible to get an idea as to what dynamics characterizes interaction of the selected network oscillators at any point of the attractor. Such an approach allows estimating the average distance between the images of nearest neighbors $S(t)$ at any moment of time and, correspondingly, determining the instant phase of the dynamics of interaction of the two oscillators and identifying the intervals of synchronous and asynchronous dynamics for the two discussed network nodes.

To illustrate the method, in the present work, we chose a model system in the form of a network of coupled flow systems consisting of $K = 5$ mutually coupled Ressler oscillators. The evolution of the nth network element ($n = 1, \ldots, K$) is described by the following set of equations:

$$\begin{aligned} \frac{dx_n}{dt} &= -\omega_n y_n - z_n + \varepsilon \sum_{n=1}^{K} C_{nm} x_m, \\ \frac{dy_n}{dt} &= \omega_n x_n + a y_n, \\ \frac{dz_n}{dt} &= p + z_n(x_n - c), \end{aligned} \quad (2)$$

where $a = 0.15$, $p = 0.2$, and $c = 10$ are the control parameters; ε is the coupling parameter; and $\mathbf{C} = \{C_{ij}\}$ is the matrix characterizing the coupling topology between the network oscillators ($C_{nm} = 0$ corresponds to the absence of action of the mth element on the nth oscillator, $C_{nm} = 1$ corresponds to the presence of action of the mth element on the nth oscillator, and $C_{nn} = -\sum_{m \neq n} C_{nm}$). To ensure that there is a mismatch between the interacting oscillators, the parameters responsible for the eigenfrequencies of oscillation of the partial systems were chosen as follows: $\omega_1 = 0.95$, $\omega_2 = 0.9525$, $\omega_3 = 0.955$, $\omega_4 = 0.9575$, and $\omega_5 = 0.96$. The type of coupling between the network elements was chosen to be bidirectional in the "each-to-each" form, i.e., $C_{nm} = 1$ ($n \neq m$). We chose the first ($n = 1$) and the second ($n = 2$) nodes of the described network for illustration of the method. The set of control points $\{\mathbf{x}_l^2\}_{l=1,\ldots,M}$ was specified for the second oscillator, for which points $\{\mathbf{x}_{lk}^{12}\}_{l=1,\ldots,M}$ were found in the first oscillator. After that, we determined the values of S_l^{12}, which were used, in turn, to interpolate the values of $S(t)$.

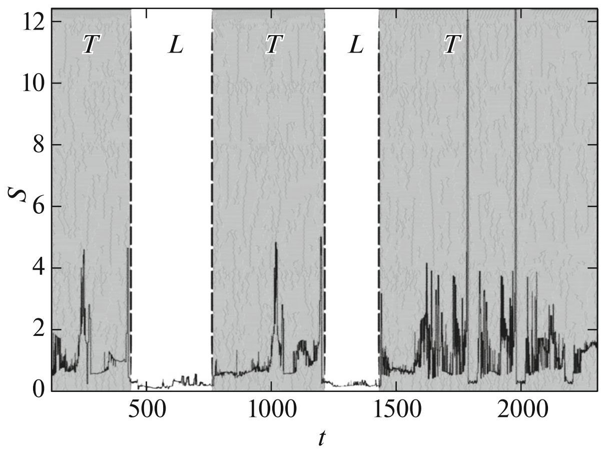

Fig. 2. The time dependence of the average distance $S(t)$ between the images of the nearest neighbors. Symbols "T" mark the regions of asynchronous dynamics of the first and second oscillators ("turbulent phase"), while symbols "L" mark the regions of synchronous behavior ("laminar phase").

The coverage of the chaotic attractor of the second oscillator ($n = 2$) by control points $\{\mathbf{x}_l^2\}_{l=1,\ldots,M}$ is illustrated in Fig. 1. Information about the regions of the chaotic attractor in which synchronous dynamics already takes place allows detecting the beginning and the end of the interval of synchronous behavior of the discussed oscillators. Part of the time dependence of the average distance $S(t)$ between the images of the nearest neighbors is presented in Fig. 2. Regions "T" correspond to the intervals of asynchronous dynamics, in which the condition $S(t) > S_c$ is fulfilled. In turn, the regions marked by the symbol "L" correspond to the synchronous behavior regime.

Thus, the proposed method allows distinguishing between the intervals of characteristic laminar and turbulent behavior in the system exhibiting a complex intermittent dynamics, which will allow analyzing the transition to the generalized synchronization regime in a network of oscillators through intermittence in the future.

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