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Synchronization in the inhibitory coupled Hodgkin-Huxley neural networks

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ABSTRACT

We consider two small-world networks of Hodgkin-Huxley neurons interacting via inhibitory coupling. We found that synchronization indices (SI) in both networks oscillate periodically in time, so that time intervals of high SI alternate with time intervals of low SI. Depending on the coupling strength, the two coupled networks can be in the regime of either in-phase or anti-phase synchronization. We suppose that the inherent mechanism behind such a behavior lies in the cognitive resource redistribution between neuronal ensembles of the brain.

Keywords: Complex network, Hodgkin-Huxley neuron, neural network, synchronization, small-world

1. INTRODUCTION

Perhaps one of the most exciting network theory applications is to describe collective neuronal activity underlying particular brain functions. One way of doing so would be to develop a mathematical model build on existing knowledge about the functioning of a single neuron and its interaction with the others. In this respect, advances in chemistry and biology allow the creation of neuron-like models reflecting the basic principles of the single neuron activity. Among them one can highlight the integrate-and-fire,[1] Morris-Lecar,[2] FitzHugh-Nagumo,[3,4] Hindmarsh-Rose,[5] Hodgkin-Huxley.[6] The Hodgkin-Huxley neuron (HH) is one of the most complex bio-inspired models describing the initiation and propagation of action potentials in neurons and taking into account the majority of biological processes. The spiking activity produced by this model coincides with the one generated by the real neuron.[7]

Investigation of dynamics of spiking neural networks has attracted much attention in recent years.[8–12] Although there is bulk information about certain aspects of neuronal dynamics, the features of collective neuronal activity remain poorly understood. At the same time, it plays an essential role in the functioning of brain neuronal networks. Study of brain activity is a very important task at the present time.[13–17] According to the functional magnetic resonance imaging (fMRI) studies, the whole-brain network activity is generated through the interaction of multiple functional subnetworks during either a resting state or task accomplishing. These functional subnetworks include a dorsal attention network, a fronto-parietal network, an executive control network, a default mode network, etc.[18]

The collective processes resulted from the functional interaction between the remote populations of the cortical neurons subserve the cognitive performance during the demanding tasks. For instance, when the task complexity is high brain engages the additional resource by involving multiple neuronal populations. In the visual processing tasks, the small amount of the sensory information can be processed by the occipito-parietal network, while increasing information complexity requires additional activation of the prefrontal regions.[19,20] Finally, collective neuronal activity underly cognitive performance during prolonged cognitive tasks. In this case, the brain dynamically redistributes the cognitive load among the multiple cortical regions.

The current view on neuronal communication highlights a vital role of the phase coherence in functional interaction between remote neuronal ensembles. Let us consider the interaction between a pair of neurons. A presynaptic neuron fires the neurotransmitter as a result of an action potential entering its axon terminal. A

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postsynaptic neuron receives the neurotransmitter and may experience an action potential if the neurotransmitter is strong enough. In the brain, a postsynaptic neuron receives input signals from several presynaptic neurons simultaneously. At the same time, it responds primarily to those neurons with which it is coherent. In the absence of coherence, input signals come to postsynaptic neuron at random phases of its excitability cycle, having a low connectivity efficiency. Thus, effective communication between the neurons requires the phase synchronization of their spiking activity.

We consider two small-world networks of Hodgkin-Huxley neurons interacting via inhibitory coupling. We found that synchronization indices (SI) in both networks oscillate periodically in time, so that time intervals of high SI alternate with time intervals of low SI. Depending on the coupling strength, the two coupled networks can be in the regime of either in-phase or anti-phase synchronization. We suppose that the inherent mechanism behind such a behavior lies in the cognitive resource redistribution between neuronal ensembles of the brain.

2. NUMERICAL MODEL

We consider the network of $N = 100$ Hodgkin-Huxley neurons. The time evolution of the transmembrane potential of the HH neurons is given by[6]

$$\begin{aligned} C_m \frac{dV_i}{dt} &= -g_{Na}^{max} m_i^3 h_i (V_i - V_{Na}) - g_K^{max} n_i^4 (V_i - V_K) - \\ &- g_L^{max}(V_i - V_L) + I_i^{ex} + I_i^{syn} \end{aligned} \tag{1}$$

where $C_m = 1\mu F/cm^3$ is the capacity of cell membrane, I_i^{ex} is an external bias current injected into a neuron in the network, V_i is the membrane potential of i-th neuron, $i = 1,...,N$, $g_{Na}^{max} = 120mS/cm^2$, $g_K^{max} = 36mS/cm^2$ and $g_L^{max} = 0.3mS/cm^2$ receptively denote the maximal sodium, potassium and leakage conductance when all ion channels are open. $V_{Na} = 50mV$, $V_K = -77mV$ and $V_L = -54.4mV$ are the reversal potentials for sodium, potassium and leak channels respectively. m, n and h represent the mean ratios of the open gates of the specific ion channels. n^4 and m^3h are the mean portions of the open potassium and sodium ion channels within a membrane patch. The dynamics of gating variables ($x = m, n, h$) are given:

$$\frac{dx_i}{dt} = \alpha_{x_i}(V_i)(1 - x_i) - \beta_{x_i}(V_i)x_i, \qquad x = m, n, h \tag{2}$$

$\alpha_x(V)$ and $\beta_x(V)$ are rate functions, described by[21]

$$\alpha_m(V) = \frac{0.1(25 - V)}{\exp[(25 - V)/10] - 1} \tag{3}$$

$$\beta_m(V) = 4\exp(-V/18) \tag{4}$$

$$\alpha_h(V) = 0.07\exp(-V/20) \tag{5}$$

$$\beta_h(V) = \frac{1}{1 + \exp[(30 - V)/10]} \tag{6}$$

$$\alpha_n(V) = \frac{0.01(10 - V)}{\exp[(10 - V)/10] - 1} \tag{7}$$

$$\beta_n(V) = 0.125\exp(-V/80) \tag{8}$$

I_i^{syn} is the total synaptic current received by neuron i. We consider coupling via chemical synapses. The synaptic current takes the form[22]

$$I_i^{syn} = \sum_{j \in neigh(i)} g_c \alpha(t - t_0^j)(E_{rev} - V_i) \tag{9}$$

where the alpha function $\alpha(t)$ describes the temporal evolution of the synaptic conductance, g_c is the maximal conductance of the synaptic channel and t_0^j is the time at which presynaptic neuron j fires. We suppose $\alpha(t) =$

$e^{-t/\tau_{syn}}\Theta(t)$, there $\Theta(t)$ is the Heaviside step function and $\tau_{syn} = 3ms$. The initial conditions of all neurons correspond to the oscillatory basin of attraction of individual neuron.

To investigate synchronization inside each network we calculate synchronization index as follows:[23, 24]

$$S = \sqrt{\frac{1}{T}\sum_{n=1}^{T}\xi_n}, \tag{10}$$

where ξ_n is the standard deviation given as

$$\xi_n = \frac{1}{N}\sum_{i=1}^{N}\left(x_n^{(i)}\right)^2 - \left(\frac{1}{N}\sum_{i=1}^{N}x_n^{(i)}\right)^2. \tag{11}$$

where T is a number of iterations, N is a number of neurons in the network. The smaller S, the better the synchronization; $S = 0$ means complete synchronization. We apply filtering in [0.004,0.015] Hz frequency band.

To investigate correlation between synchronization indexes $S^{(1)}$ and $S^{(2)}$ of N_1 and N_2 networks respectively we calculate Pearson's linear correlation coefficient as follows[25]

$$r = \frac{\sum_{n=1}^{T}(S_n^{(1)} - \overline{S}^{(1)})(S_n^{(2)} - \overline{S}^{(2)})}{\sqrt{\sum_{n=1}^{T}(S_n^{(1)} - \overline{S}^{(1)})^2}\sqrt{\sum_{n=1}^{T}(S_n^{(2)} - \overline{S}^{(2)})^2}} \tag{12}$$

where T is a number of iterations. The value $r = 1$ means a perfect positive correlation and the value $r = -1$ means a perfect negataive correlation.

3. RESULTS

We investigate the dynamics of the network presented on Fig. 1. The external stimulus of constant current with amplitude A is applied to the input network of $N^{ex} = 5$ neurons. All of them are connected to each other with the coupling strength chosen randomly from the range [0,0.15]. This network is connected to the two other networks of $N_1 = N_2 = 50$ neurons by one-directional excitatory couplings with coupling strength $g_c = 0.05$

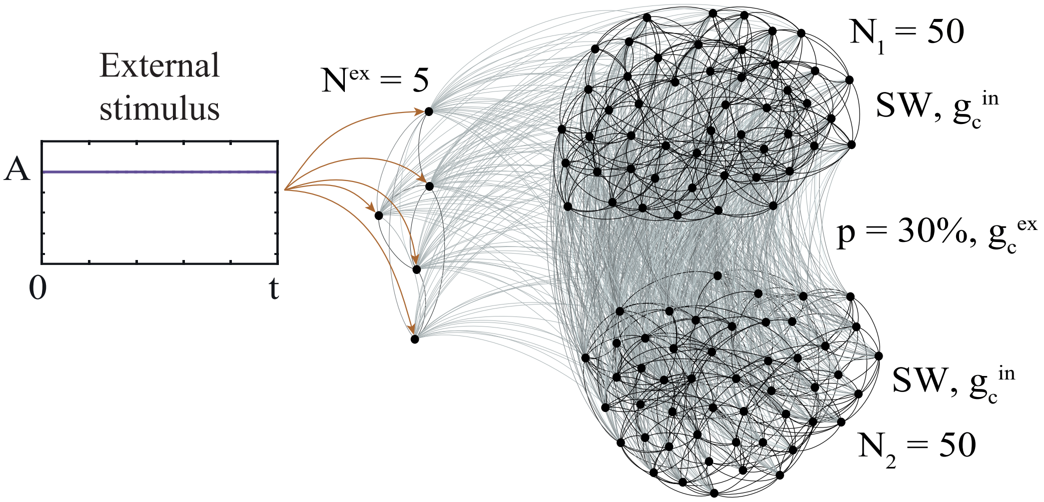

Figure 1. Network model. The external stimulus with amplitude A is applied to the input network of $N^{ex} = 5$ neurons. The network is connected to the two other networks of $N_1 = N_2 = 50$ neurons by one-directional excitatory couplings. The networks N_1 and N_2 are connected to each other by two-directional inhibitory couplings with coupling strength g_c^{ex} and probability $p = 30\%$. Inside N_1 and N_2 networks neurons are connected to each other according to "small-world" (SW) topology with coupling strength g_c^{in}.

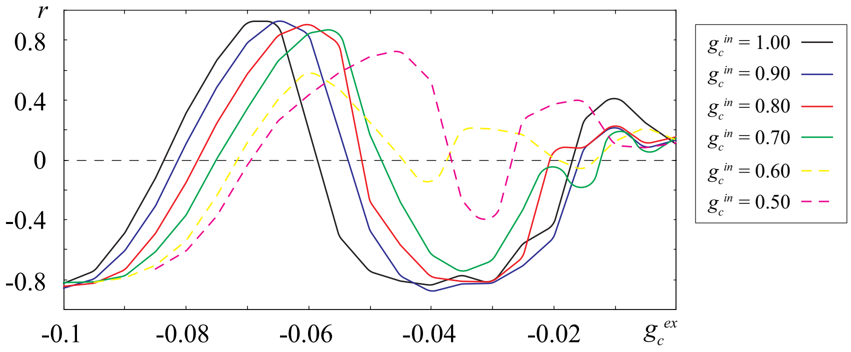

Figure 2. Correlation of synchronization indexes $S^{(1)}$ and $S^{(2)}$ versus coupling strength between the networks g_c^{ex} for different values of the strength of inside couplings $g_c^{in} = 1.0$ (black line), 0.9 (blue line) 0.8 (red line), 0.7 (green line), 0.6 (yellow line), 0.5 (purple line).

and probability $p = 30\%$. The networks N_1 and N_2 are connected to each other by two-directional inhibitory couplings with coupling strength g_c^{ex} and probability $p = 30\%$. Inside them neurons are connected to each other according to "small-world" (SW) topology with coupling strength g_c^{in}.

We analyse neural dynamics of N_1 and N_2 networks. Excitatory coupling inside each network leads to synchronization of these neurons. Since networks are interconnected via an inhibitory coupling, depending on the coupling strength an anti-phase dynamics in the activities of them can be achieved. To investigate it we calculate synchronization index (Eq. 10) for networks N_1 and N_2 and apply filtering in [0.004,0.015] Hz frequency band corresponding to the low-frequency modulation of macroscopic signal of each network.

Fig. 2 illustrates the dependencies of synchronization indexes correlation on the coupling strength between the networks for different values of the strength of inside couplings. Without connection between the networks, when $g_c^{ex} = 0$, correlation r is close to 0.15. Changing the coupling strength towards negative values at first leads to small increasing of correlation up to $0.2 - 0.4$. Then, with further decrease of g_c^{ex} synchronization indexes correlation goes to the negative values and reaches -0.8 value for $g_c^{ex} \approx -0.3$. After that, decrease is replaced by growth, and r reaches 0.9 for $g_c^{ex} \in [-0.055, -0.07]$ depending on g_c^{in}. At last, correlation decreases and reaches -0.8 value again for $g_c^{ex} \approx -0.095$. The less the coupling strength inside the networks the less the coupling strength between the networks is needed to reach the maximal and minimal values of the correlation.

Figure 3 illustrates time series of synchronization indexes $S^{(1)}$ and $S^{(2)}$ for the networks N_1 (a) and N_2 (f) and time-space diagrams of membrane potential for $g_c^{ex} = -0.1$ (a-e) and $g_c^{ex} = -0.07$ (f-j) sorted by the sums of coupling strength of internal couplings g^{in} (k,l) and external couplings g^{ex} (m,n). First 50 neurons correspond to N_1 network, second ones correspond to N_2 neurons. Black color illustrates the inhibition of neurons of one network by other neurons. Yellow color corresponds to spike generation, and one can see that thickness of yellow lines of each network changes through time which is connected to the synchronization of neurons. The thickness of yellow line and synchronization index are well correlated with each other: lower thickness means better synchronization, hence lower S.

4. CONCLUSION

Having summarized, we have investigated the dynamics of complex network of Hodgkin-Huxley neurons. It consists of 2 sub-networks. The input small network N^{ex} receives external signal which is transferred into a spike sequence. Then it is transmitted to two small-world N_1 an N_2 networks interacting via an inhibitory coupling and working together to process the signal.

We have observed that the synchronization index in both networks periodically changes in time: the time intervals with the high SI alternate with the time intervals where SI is low. We have calculated correlation between them and found that when adjusting the strength of the inhibitory coupling one can observe SI in these networks changes either in phase or out of phase.

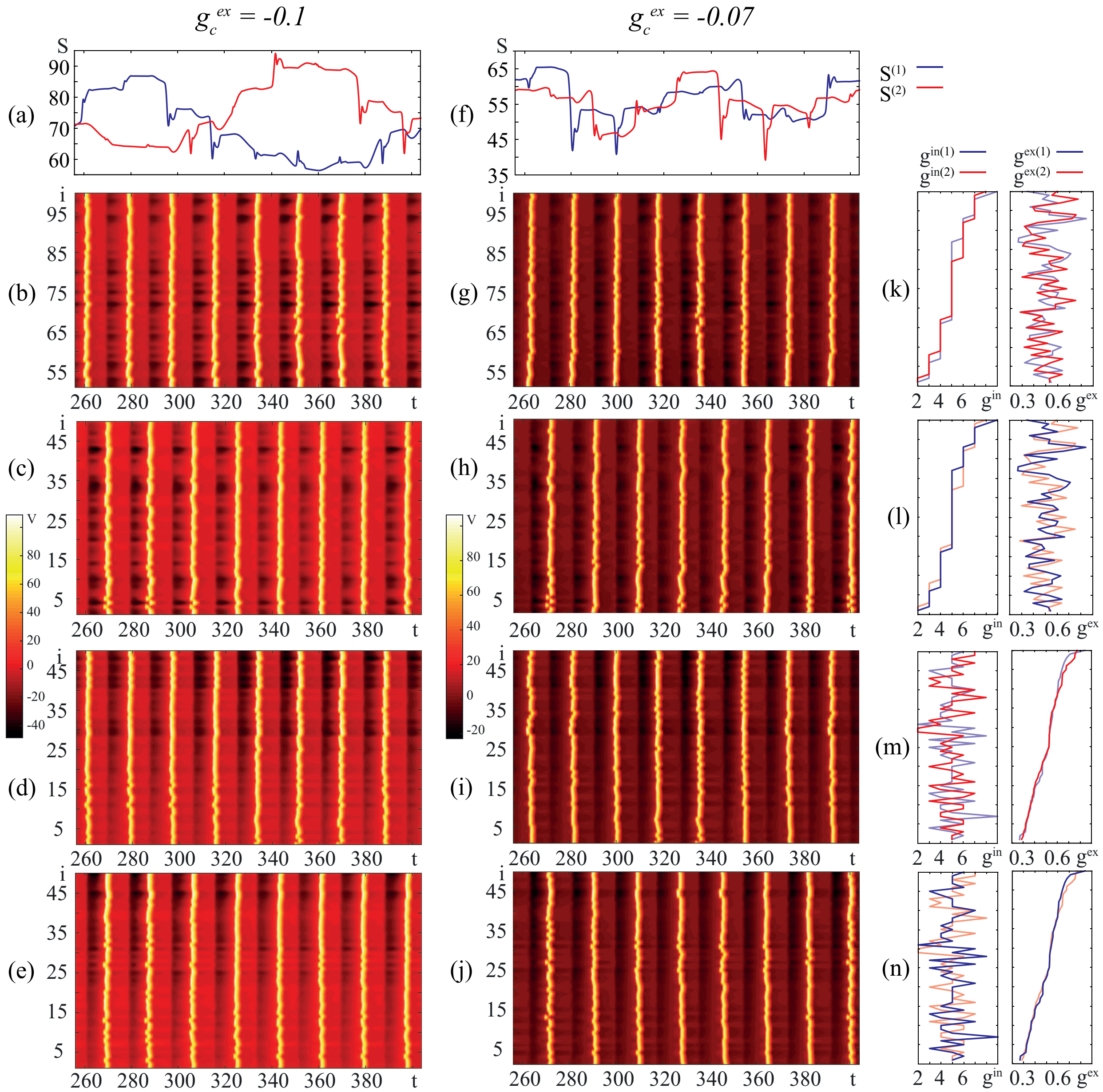

Figure 3. (a,f) Time evolution of synchronization indexes $S^{(1)}$ and $S^{(2)}$ for the networks N_1 and N_2 respectively. (b-e,g-j) Time-space diagrams of membrane potential V and (k-n) corresponding to them sums of coupling strength of internal couplings g^{in} (left) and external couplings g^{ex} (right) versus the number of neuron i. $1 \leq i \leq 50$ corresponds to N_1 network, $51 \leq i \leq 100$ corresponds to N_2 network. For the left column (a-e) $g_c^{ex} = -0.1$, for the right column (f-j) $g_c^{ex} = -0.07$.

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