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Nonlinear dynamics and coherent resonance in a network of coupled neural-like oscillators

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ABSTRACT

In this paper we study the spiking behaviour of a neuronal network consisting of 100 Rulkov elements coupled to each other with randomly chosen coupling strength. We find periodical grouping forming in the signal from all neurons in the network. We discovered the phenomenon of coherent resonance when signal-to-noise ration takes the maximum value at certain values of such parameters as number of neurons in the system, number of stimulated neurons, amplitude of external stimulus and amplitude of internal noise.

Keywords: Nonlinear dynamics, Complex network, Rulkov map, Neural network, Coherent resonance, Neural-like oscillator

1. INTRODUCTION

As all real systems, the neural systems are noisy. Noise can lead to increase or decrease of order in the dynamical systems under noise.[1–3] To be mentioned here are the effects of noise induced order in chaotic dynamics,[4–6] synchronization by external noise,[7,8] and stochastic resonance.[9–13] Also, noise has been shown to play a stabilizing role in ensembles of coupled oscillators and maps.[14,15] Especially interesting is the phenomenon of stochastic resonance, which appears when a nonlinear system is simultaneously driven by noise and a periodic signal.[16–19] At a certain noise amplitude the periodic response is maximal.

The interest in mathematical modeling of neuronal synchronization has significantly increased after neurobiological experiments with two electrically coupled neurons,[20] where various synchronous states have been identified. Nowadays the interest of brain investigation is really high.[21–26]

In order to simulate cooperative neuron dynamics, numerous models based on either iterative maps of differential equations in various coupling configurations have been developed.[20] Depending on the coupling strength and synaptic delay time, coupled neurons generate spike sequences that are matching in their timings, or bursts either with lag or anticipation.[27] When three or more oscillators are accounted for a large number of coupling configurations can be realized. In the theory of graphs or complex networks, these basic configurations are called network motifs.

We explore a simple neural model, the Rulkov map.[28,29] Although this model is not explicitly inspired by physiological processes in the membrane, it is capable of generating extraordinary complexity and quite specific neural dynamics (silence, periodic spiking, and chaotic bursting), thus replicating to a great extent most of the experimentally observed regimes,[20] including spike adaptation, routes from silence to bursting mediated by subthreshold oscillations, emergent bursting, phase and antiphase synchronization with chaos regularization,[28] and complete and burst synchronization.[30]

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2. THE MODEL

Each neuron-like Rulkov element is described by the following system of equations with synaptic coupling:[29]

$$x_{n+1} = f(x_n, x_{n-1}, y_n + \beta_n), \tag{1}$$

$$y_{n+1} = y_n - \mu(x_n + 1) + \mu\sigma + \mu\sigma_n + \mu A^{\xi}\xi_n, \tag{2}$$

where x is a fast variable associated with membrane potential, y is a slow variable which has some analogy with gating variables, the parameters α, σ and $0 < \mu \leq 1$ control individual dynamics of the system, ξ is a Gaussian noise with a zero mean and standard deviation that equals 1, A^{ξ} is noise amplitude. β_n and σ_n are related to external stimuli, f is a piecewise function defined as

$$f(x_n, x_{n-1}, y_n) = \begin{cases} \alpha/(1-x_n) + y_n, & \text{if } x_n \leq 0 \\ \alpha + y_n, & \text{if } 0 < x_n < \alpha + y_n \text{ and} \\ & x_{n-1} \leq 0 \\ -1, & \text{if } x_n \geq \alpha + y_n \text{ or} \\ & x_{n-1} > 0 \end{cases} \tag{3}$$

It is constructed in a way to reproduce different regimes of neuron-like activity, such as spiking, bursting and silent regimes.

The parameters β_n and σ_n are defined as

$$\beta_n = \beta^e I_n^{ext} + \beta^{syn} I_n^{syn}, \tag{4}$$

$$\sigma_n = \sigma^e I_n^{ext} + \sigma^{syn} I_n^{syn}. \tag{5}$$

Coefficients β^e and σ^e are used to balance the effect of external current I_n^{ext}. β^{syn} and σ^{syn} are coefficients of synaptic coupling. I_n^{syn} is a synaptic current:

$$I_{n+1}^{syn} = \gamma I_n^{syn} - g_{syn} * \begin{cases} (x_n^{post} - x_{rp}), & \text{spike}^{pre}, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

where g_{syn} is the strength of synaptic coupling, $g_{syn} \geq 0$. Indexes *pre* and *post* correspond presynaptic and postsynaptic variables respectively. The first condition in (6) corresponds to the presynaptic impulse (spike) generation time moments and defined as $x_n^{pre} \geq \alpha + y_n^{pre} + \beta_n^{pre}$. Parameter γ is a relaxation time of the synapse, $0 \leq \gamma \leq 1$. It defines the part of synaptic current which preserve as in the next iteration. x_{rp} is a reversal potential that determines the type of the synapse: inhibitory or excitatory.

In our modeling we take values of the parameters $\alpha = 3.65$, $\sigma = 0.06$ and $\mu = 0.0005$ so that each neuron being autonomous demonstrates silent regime dynamics. Also we assume $\beta^e = 0.133$, $\sigma^e = 1.0$, $\beta^{syn} = 0.1$, $\sigma^{syn} = 0.5$ and $x_{rp} = 0.0$. Investigation system is a motif of N neurons coupled to each other with a random coupling strength g_{syn} and relaxation time γ. The values of them are randomly chosen from 0.0 to 0.1 and from 0.0 to 0.5 respectively. In the investigating system we apply an external stimulus to Na neurons. Stimulus is a current impulse of the following form: from the start it equals to 0, at the moment t_s when we apply it current starts equal to A. The values of variables are chosen so that without the external stimulus each neuron is in a silent regime but with starting the application of stimulus excited neurons start periodically generate spikes.

3. THE RESULTS

From the system we take signals as time series of the fast variable x from all neurons. In figure 1 one can see signals from all 100 neurons for different number of stimulated neurons. On them one can see phenomenon of grouping. It consists in periodically spiking unexcited neurons so that one can see areas of time on time series where all unstimulated neurons spike and areas where they all are silent and these areas periodically follows one by one. As one can see for small and relatively big number of stimulated neurons the phenomenon of grouping is not exist. For $N = 1$ there is only one group forming. For $N = 20$ one can see the time intervals of silent regime of all unstimulated neurons are decreasing and for bigger Na for all time period of stimulation all neurons starts chaotically generate spikes without forming groups.

We analyse influence of external stimulus amplitude. In figure 2 one can see the dependence of time series of x from this parameter. Increasing the stimulus amplitude leads to increasing frequency of grouping and grouping durations and decreasing time range between them. Also we can see decreasing of signal amplitude x with increasing A.

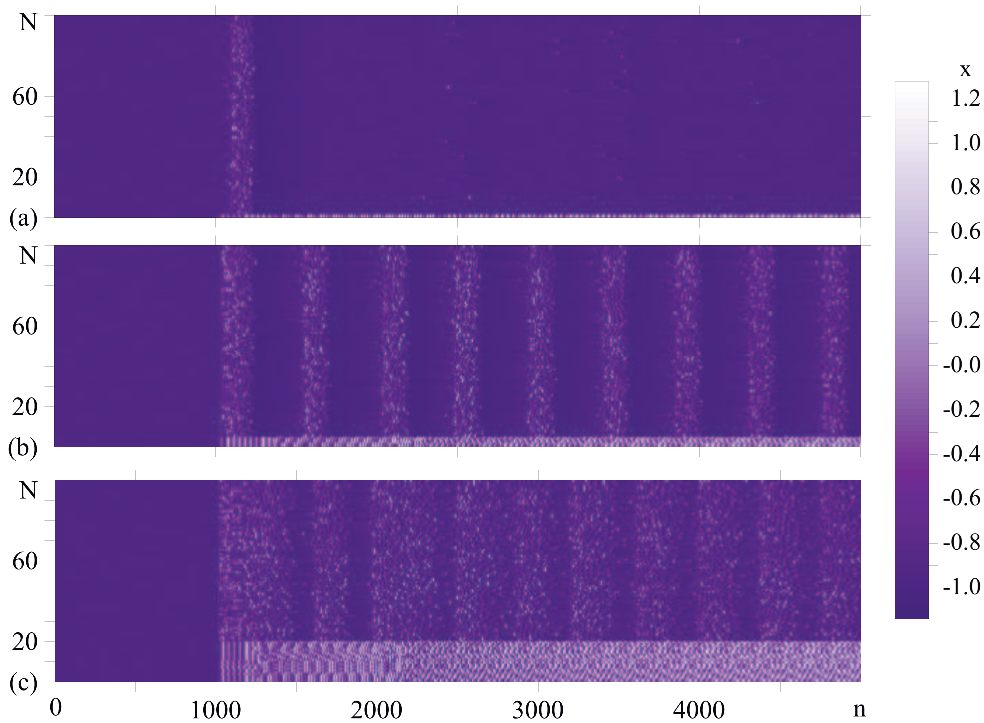

Figure 1. Time series of membrane potential x for all neurons in the network of $N = 100$ neurons when we apply external stimulus only on $Na = 1$(a), $Na = 5$(b) and $Na = 20$(c) neurons, $A = 1.0$, $A^{\xi} = 0.1$. Amplitude of x is defined by color.

For analyse phenomenon of periodical grouping we calculate dependencies of signal-to-noise ratio (SNR) from number of neurons in the system N, number of stimulated neurons Na, amplitude of external stimulus A and amplitude of internal noise A A^{ξ}. SNR measured from power spectra of average signal in dB as an excess of main frequency amplitude over background noise.[31,32] Average signal we calculate as follows:

$$x_{avr} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{7}$$

where i is an index of neuron, $N = 100$ is the number of neurons in the network.

In figure 3 (a) one can see the dependence of SNR from the number of stimulated neurons for the system of 100 neurons. There are two strong peaks, when $Na = 7$ and $Na = 12$. For this values of Na SNR takes the highest values. Moving away from it to $Na = 0$ and $Na = 30$ signal-to-noise ratio value decreases to 0.

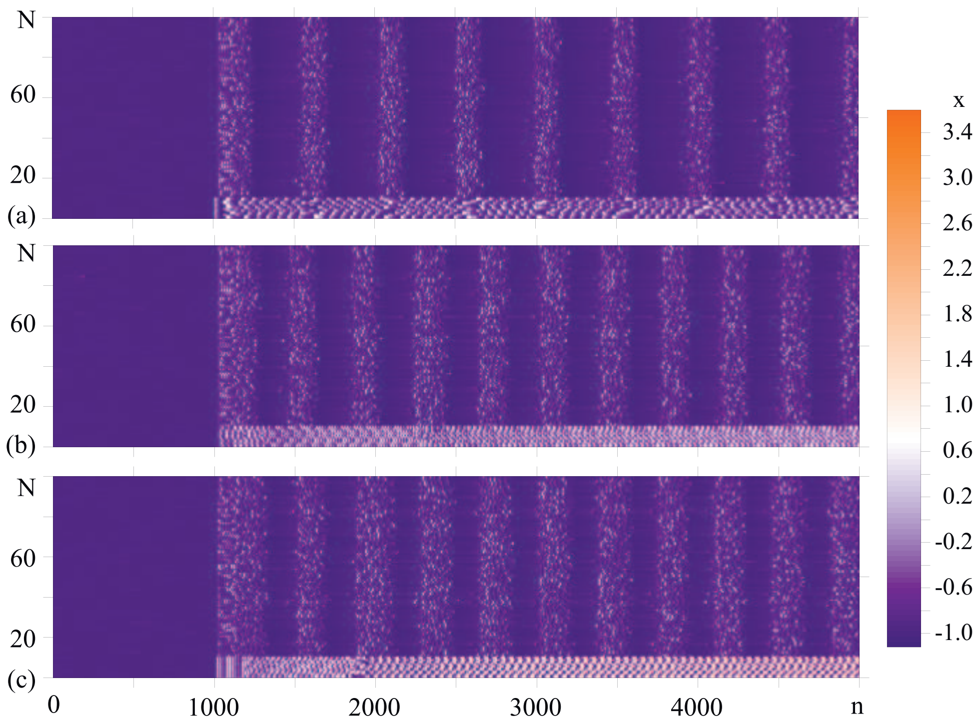

Figure 2. Time series of membrane potential x for all neurons in the network of $N = 100$ neurons when we apply external stimulus only on $Na = 10$ with amplitude $A = 0.5$ (a), $A = 1.0$ (b) and $A = 1.5$ (c), $A^{\xi} = 0.1$. Amplitude of x is defined by color.

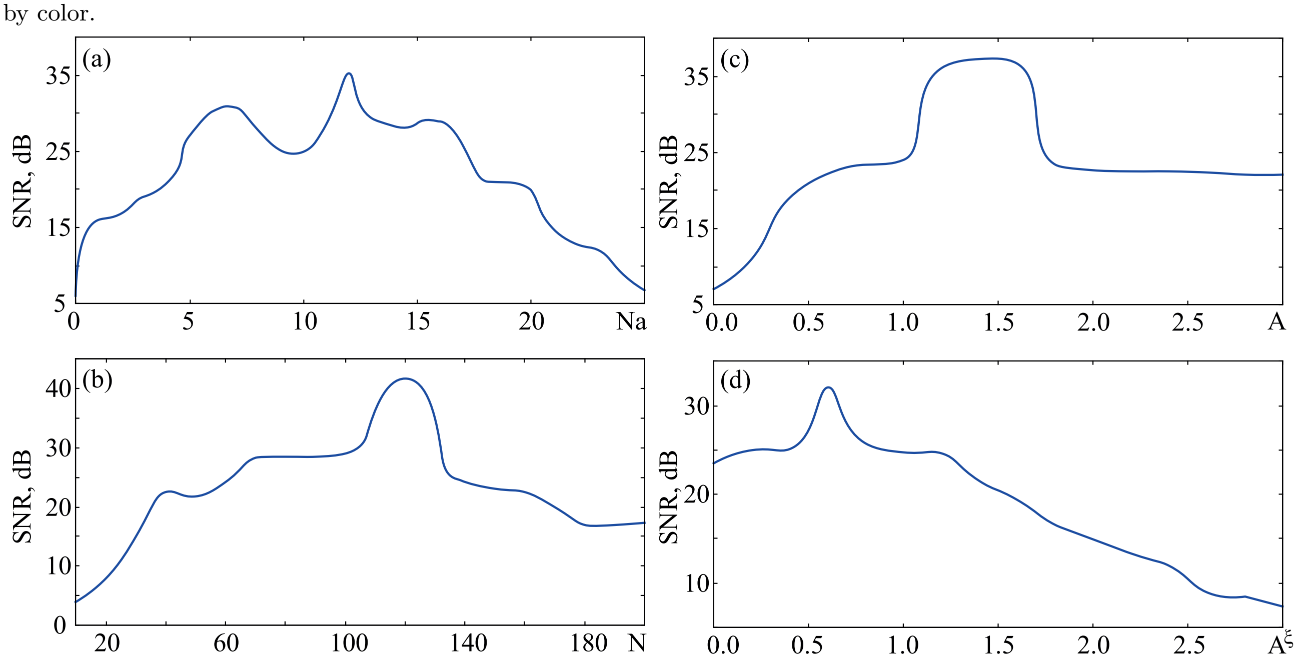

Figure 3. (a) Signal-noise ratio (SNR) versus number of stimulated neurons Na for $A^{\xi} = 0.1$, $A = 1.0$, and $N = 100$; (b) SNR versus network size N for $A^{\xi} = 0.1$, $A = 0.1$, and $Na = 10$, (c) SNR versus stimulus amplitude A for $A^{\xi} = 0.1$, $Na = 10$, and $N = 100$, (d) SNR versus internal noise amplitude A^{ξ} for $A = 1.0$, $Na = 10$, and $N = 100$

In figure 3(b) one can see dependence of SNR from number of neurons in the system when we excite 10 of them. At small values of $N(< 38)$ signal-to-noise ratio is small too but for increasing N from 38 leads to rapid increasing SNR from 5 to 30 and then it stays near of this level until $N = 110$ when SNR starts rapidly increase and reaches the maximum value for $N = 120$. With further increasing the network size SNR rapidly decreases and starting from $N = 138$ it slowly decreases. So for $Na = 10$ we have optimal values of $N = 130$ at which SNR takes the highest value.

Figure 3(c) shows signal-to-noise ratio dependence from external stimulus amplitude, on which one can see the phenomenon of coherent resonance when for a certain values of external stimulus amplitude ($A = 1.3 - 1.6$) SNR takes the maximum value. For $A > 1.6$ signal-to-noise ratio doesn't change. Decreasing external stimulus amplitude from 1.3 to 0 leads to decreasing SNR.

In figure 3(d) we can see influence of internal noise amplitude to signal-to-noise ratio. For $A^{\xi} = 0.6$ SNR takes the maximum value and decreases to 4 with decreasing A^{ξ}.

4. CONCLUSION

From the signals from all neurons in the network of 100 coupled to each other Rulkov neurons with presence of internal noise and external stimulus we have observed the phenomenon of periodical grouping when all unexcited neurons start spiking periodically during the time interval. Changing such parameters as number of neurons in the system, number of stimulated neurons, amplitude of external stimulus and amplitude of internal noise we've discovered phenomenon of coherent resonance when at the certain values of these parameters signal-to-noise ratio takes the maximal values.

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