ISSN 1063-7850, Technical Physics Letters, 2015, Vol. 41, No. 8, pp. 762–764. © Pleiades Publishing, Ltd., 2015.
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Control for Statistical Characteristics of Chaotic Oscillation Regimes by Noise Exposure

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Received March 6, 2015

Abstract—The possibility to control for statistical characteristics of chaotic oscillation regimes by use of noise is studied. For synchronous oscillation regimes in the dynamics of coupled oscillatory systems, it is shown that noise leads to significant changes in the mean Hölder exponent. The presence of a monotonic dependence of this quantity on the intensity of the external noise makes it possible to generate a chaotic signal with given correlation characteristics.

DOI: 10.1134/S1063785015080155

Generation of chaotic oscillation regimes with given statistical characteristics is a topical problem in communication engineering [1–3]. In particular, corresponding oscillation processes are applied as carrier or masking signals and provide higher protection of information transmitted by a communication channel, as well as channeling of a communication system [4–6]. In the transmission process, statistical characteristics of chaotic oscillations can be retuned, which favors higher protection of transmitted information. Oscillation regimes in the region of chaotic dynamics can be changed by varying the controlling parameter of the oscillatory system. However, this approach is complicated by the presence of periodicity "windows," which does not permit one to obtain smooth dependences of statistical characteristics when varying the parameter. As an alternative, one can use an approach that applies an additional source of noise affecting the chaotic oscillation generator. By varying the intensity of noise, one can implement switchings between different dynamic regimes in the multistability region, which has an effect on the statistical properties of generated oscillation processes.

In this work, investigations aimed to studying the possibility to control for statistical characteristics of chaotic oscillations by use of noise in the cases of synchronous and asynchronous regimes in the dynamics of coupled oscillatory systems are carried out. The investigations were performed based on analysis of sequences of recurrence times to the Poincaré secant. For a statistical characteristic of the analyzed dynamic regimes, mean Hölder exponent $h(0)$ reflecting the correlation properties of the process was chosen. It is related to such characteristics of spectral-correlation analysis as scaling exponents describing the frequency dependence of the power spectral density function of regularities in the decrease in the autocorrelation function [7]. Quantity $h(0)$ was determined based on the method of wavelet transform modulus maxima [8]. In the context of this method, a wavelet transform of the sequence of recurrence times to the Poincaré secant $x(i)$ is performed:

$$W(a, k) = \frac{1}{\sqrt{a}}\sum_{i=0}^{N} x(i)\psi^*\left(\frac{i-k}{a}\right), \tag{1}$$

where wavelet function ψ is subjected to scale transforms and translations that are specified by parameters a and k. In the presence of singularities in signal $x(i)$ at time instant k^*, coefficients of wavelet transform $W(a, k^*)$ are characterized by the presence of exponential dependence $W(a, k^*) \sim a^h$, where quantity h (the Hölder exponent) describes the local irregularity of the signal and characterizes its correlation properties. Usually, spectrum of Hölder exponents $h(q)$ is considered. Here, the index q characterizes the observation scale—whether the structure is small-scale ($q < 0$) or large-scale ($q > 0$). Spectrum $h(q)$ is calculated by use of an approach based on calculating partition functions [7]. After the calculation of coefficients of the continuous wavelet transform $W(a, k)$, a skeleton is distinguished—the set of lines of local extreme values of the surface of wavelet coefficients recorded at each fixed scale a. Then, generalized partition functions $Z(q, a)$ are calculated by the formula

$$Z(q, a) = \sum_{l \in L(a)} |W(a, k_l(a))|^q \sim a^{\tau(q)}, \tag{2}$$

where $L(a)$ is the set of lines l of modulus maxima of wavelet coefficients existing on the chosen scale a and

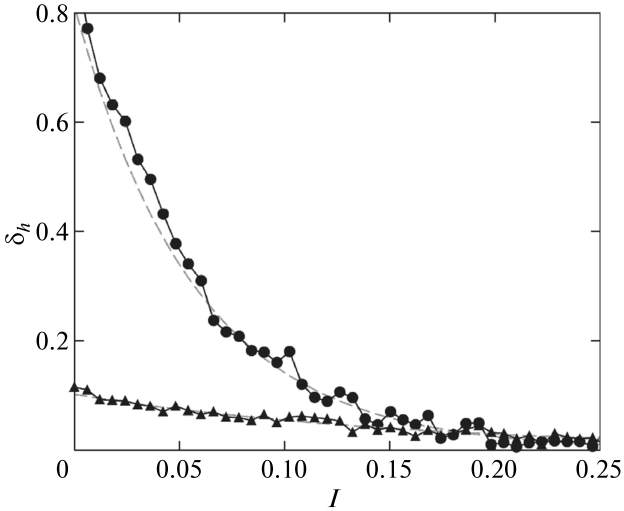
$k_l(a)$ determines the position of the maximum that corresponds to a line with a number l. Differentiating the function $\tau(q)$ numerically, one can calculate the sought spectrum $h(q) = d\tau(q)/dq$. This approach is more universal as compared to the classical correlation analysis and can be applied when studying nonstationary processes and signals of relatively short duration.

In contrast to investigations performed earlier [9], the method of wavelet transform modulus maxima was first modified for more exact calculation of Hölder exponents. This modification provided optimizing the choice of the range of scales a when approximating the power dependence of the partition function for determining the range of scales with the linear dependence $\log Z(\log a)$. This made it possible to reduce the effect of short lines of local extreme values of wavelet coefficients caused by oscillating "tails" of wavelet functions, as well as the effect of longest lines which lead to significant errors due to the insufficient statistics in the analysis of relatively short processes. Additionally, the reliability of calculation results was estimated by using different wavelet functions (WAVE, MHAT, and wavelets corresponding to higher derivatives of the Gauss function).

Special attention was devoted to studying noise-induced switchings between different dynamic regimes in the dynamics of coupled self-oscillating systems. For this purpose, values of controlling parameters were chosen in the range of pronounced multistability and the noise level was varied to change the frequency of switchings between different oscillation regimes. In particular, by the example of the dynamics of coupled Rössler systems

$$\begin{gathered}\frac{dx_1}{dt} = -\omega_1 y_1 - z_1 + \gamma(x_2 - x_1) + I\xi(t),\\ \frac{dy_1}{dt} = \omega_1 x_1 + a y_1, \quad \frac{dz_1}{dt} = b + z_1(x_1 - c),\\ \frac{dx_2}{dt} = -\omega_2 y_2 - z_2 + \gamma(x_1 - x_2),\\ \frac{dy_2}{dt} = \omega_2 x_2 + a y_2, \quad \frac{dz_2}{dt} = b + z_2(x_2 - c),\end{gathered} \tag{3}$$

considered for the following values of controlling parameters: $\omega_1 = 1.0093$, $\omega_2 = 0.9907$, $a = 0.15$, $b = 0.2$, and $\gamma = 0.02$ [10, 11], the effect of noise $I\xi(t)$ on statistical characteristics of synchronous and asynchronous self-sustained oscillatory regimes was compared (here, $\xi(t)$ is the standard normal white noise and I is the noise intensity). At larger values of I (e.g., $I > 0.5$), the noise leads to transitions between all possible dynamic regimes; as a consequence, statistical characteristics do not depend on the attractor that existed before introducing the fluctuations. However, for lower intensities of the noise, its effects on synchronous and asynchronous chaotic regimes of self-sustained oscillations are significantly different. In general, singularity spectra of synchronous regimes (both chaotic and regular ones) are more sensitive to fluctuations, and fine tuning of the noise intensity for them allows one to control for statistical characteristics over a wider range.



Variation in the mean value of the Hölder exponent as a function of noise intensity for the regime of synchronous (circle) and asynchronous (triangles) chaos. Values presented along the ordinate are absolute.

The figure presents calculation results for the variation in quantity $h(0)$ for synchronous and asynchronous chaotic regimes of self-sustained oscillations appearing due to the period-doubling bifurcation cascade. For the asynchronous regime, introducing an additive noise allows one to control for statistical characteristics in a small range (variation in the mean Hölder exponent does not exceed 0.1), while the corresponding variation in $h(0)$ for the synchronous chaos is larger almost by an order of magnitude. This is a significant change in correlation characteristics of the signal. Note, e.g., that a change in the Hölder exponent by unity for the white noise corresponds to the transition to the Wiener random process which is significantly "smoother." Thus, varying the parameter I, one can vary in a wide range statistical properties of the oscillation process generated by system (3). The calculated dependences (see the figure) are well approximated by a power function, and deviations from this approximation are insignificant. The obtained approximation allows one to determine the required value of intensity I that provides generation of a chaotic signal with given statistical properties.

Acknowledgments. This work was supported by the Russian Foundation for Basic Research (projects nos. 14-52-12002 and SFB-910), as well as by the Ministry of Education and Science of the Russian Federation in the scope of the project component of a state contract for scientific work no. 3.23.2014/K (topic

SSTU-157) and basic component of a state contract (topic SSTU-141).

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Translated by A. Nikol'skii