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Synchronization in interacting networks of Hodgkin-Huxley neurons

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ABSTRACT

We consider a network of networks consisting of small input neural network and four small-world subnetworks Hodgkin-Huxley neurons. Input network receives an external signal and transfers it to subnetworks via excita-tory couplings while the subnets interact with each other via inhibitory couplings. We show that the subnets are divided into 2 clusters under the influence of an inhibitory couplings between them. The synchronization indexes of subnetworks periodically change in time. We found that SIs can oscillate either in-phase or anti-phase depending on the couplings between subnetworks.

Keywords: Hodgkin-Huxley neuron, network of networks, synchronization index, complex network

1. INTRODUCTION

Nowadays, the study of the dynamics of complex networks is relevant and promising,[1–3] in particular, due to the use of network analysis to study brain signals.[4–8] Various methods of network theory are used to analyze the interaction between brain regions in cognitive activity and pathologies, based both on the analysis of experimental data (for example, multichannel records of electrical,[5, 9–14] magnetic[15–17] and blood oxygenation [18–20] activity), and on numerical modeling of the interaction of individual neurons and their groups by constructing mathematical models of networks of nonlinear elements[21–25] .

In a numerical simulation as models of neurons, the models of Hodgkin-Huxley[26] , FitzHugh-Nagumo[27] , Hindmarsh-Rose[28] are commonly used. The Hodgkin-Huxley model is the most complete model that describes the initiation and propagation of an action potential, taking into account ionic currents in the neuron membrane. The spike activity generated by this model simulates the electrical activity of a real neuron.

Collective neuronal activity plays an important role in brain functioning. According to the functional magnetic resonance imaging (fMRI) studies, the whole-brain network activity is generated through the interaction of multiple functional subnetworks during either a resting state or task accomplishing. The functional subnetworks include a dorsal attention network, a frontoparietal network, an executive control network, a default mode network, and other neuronal networks.[29]

In this work, we consider a network of networks consisting of small input neural network and four small-world subnetworks Hodgkin-Huxley neurons. Input network receives an external signal and transfers it to subnetworks via excitatory couplings while the subnets interact with each other via inhibitory couplings. The subnets are divided into 2 clusters under the influence of inhibitory couplings between them. We calculate the synchronization indices (SI) between all neurons of each large subnetwork to analyze the dynamics. SIs of subnetworks periodically change in time. We find that SIs can oscillate either in-phase or anti-phase depending on the couplings between subnetworks.

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2. NUMERICAL MODEL

As a model of neuron we consider Hodgkin-Huxley neuron. The time evolution of the transmembrane potential of the HH neurons is given by[26]

$$C_m \frac{dV_i}{dt} = -g_{Na}^{max} m_i^3 h_i (V_i - V_{Na}) - g_K^{max} n_i^4 (V_i - V_K) - \\ - g_L^{max} (V_i - V_L) + I_i^{ex} + I_i^{syn} \tag{1}$$

where $C_m = 1\mu F/cm^3$ is the capacity of cell membrane, I_i^{ex} is an external bias current injected into a neuron in the network, V_i is the membrane potential of i-th neuron, $i = 1,...,N = 205$, $g_{Na}^{max} = 120mS/cm^2$, $g_K^{max} = 36mS/cm^2$ and $g_L^{max} = 0.3mS/cm^2$ receptively denote the maximal sodium, potassium and leakage conductance when all ion channels are open. $V_{Na} = 50mV$, $V_K = -77mV$ and $V_L = -54.4mV$ are the reversal potentials for sodium, potassium and leak channels respectively. m, n and h represent the mean ratios of the open gates of the specific ion channels. n^4 and m^3h are the mean portions of the open potassium and sodium ion channels within a membrane patch. The dynamics of gating variables $(x = m, n, h)$ are given:

$$\frac{dx_i}{dt} = \alpha_{x_i}(V_i)(1 - x_i) - \beta_{x_i}(V_i)x_i, \qquad x = m, n, h \tag{2}$$

$\alpha_x(V)$ and $\beta_x(V)$ are rate functions, described by[30] . I_i^{syn} is the total synaptic current received by neuron i. We consider coupling via chemical synapses. The synaptic current takes the form[31]

$$I_i^{syn} = \sum_{j \in neigh(i)} g_c \alpha(t - t_0^j)(E_{rev} - V_i) \tag{3}$$

where the alpha function $\alpha(t)$ describes the temporal evolution of the synaptic conductance, g_c is the maximal conductance of the synaptic channel and t_0^j is the time at which presynaptic neuron j fires. We suppose $\alpha(t) = \mathrm{e}^{-t/\tau_{syn}}\Theta(t)$, there $\Theta(t)$ is the Heaviside step function and $\tau_{syn} = 3ms$. The initial conditions of all neurons correspond to the oscillatory basin of attraction of individual neuron.

To investigate synchronization inside each network we calculate synchronization index as follows:[32,33]

$$S = \sqrt{\frac{1}{T}\sum_{n=1}^{T} \xi_n}, \tag{4}$$

where ξ_n is the standard deviation given as

$$\xi_n = \frac{1}{N}\sum_{i=1}^{N} \left(x_n^{(i)}\right)^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_n^{(i)}\right)^2. \tag{5}$$

where T is a number of iterations, N is a number of neurons in the network. The smaller S, the better the synchronization; $S = 0$ means complete synchronization. We apply filtering in [0.004,0.015] Hz frequency band.

To investigate correlation between synchronization indexes $S^{(i)}$ and $S^{(j)}$ of i-th and j-th subnetworks respectively we calculate Pearson's linear correlation coefficient[34] .

3. RESULTS

We consider a network of networks consisting of small input neural network and four small-world subnetworks Hodgkin-Huxley neurons (Fig. 1). We apply an input signal of the constant current $I^a = 9\ \mu\mathrm{A/cm}^2$ to the small input network of 5 neurons. The neurons inside the input network are connected to each other with a coupling strength chosen randomly from the range [0,0.15] $\mathrm{mS/cm}^2$. Each neuron of the input network is unidirectionally connected to each of the neurons of the four large subnets by excitatory synapses with a coupling strength $g_c = 0.05\ \mathrm{mS/cm}^2$ and probability $p = 30\%$. The subnetworks consisting of 50 neurons interact with each

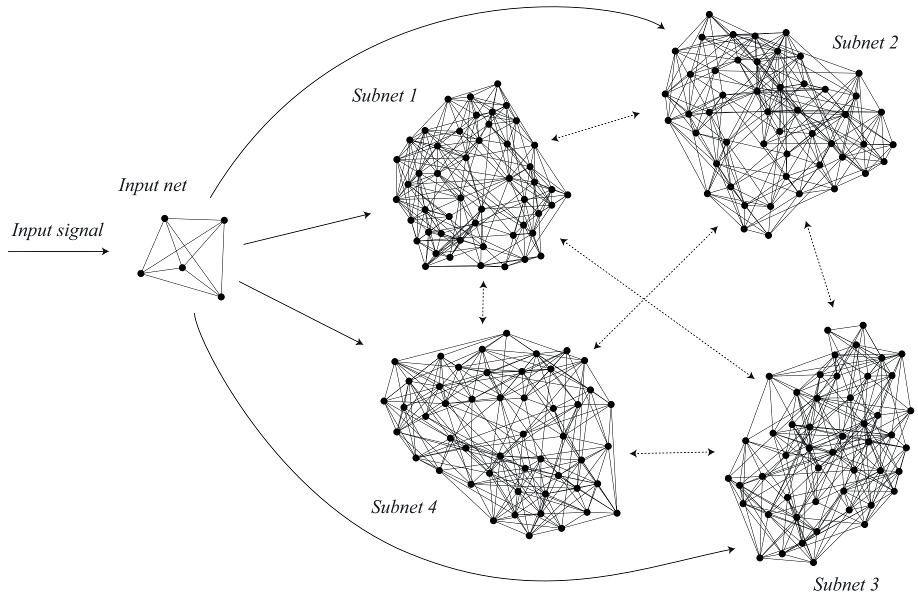

Figure 1. Network of networks model. An input signal is applied to a small input network consisting of 5 neurons which is connected to four subnetworks via unidirectional excitatory couplings. The subnets interact with each other via bidirectional inhibitory couplings with coupling strength g_c^{in}. Between the subnetworks, the neurons are connected via bidirectional excitatory couplings with coupling strength g_c^{ex}.

other via bidirectional inhibitory couplings with coupling strength g_c^{ex} and probability $p = 30\%$. Inside each subnetwork, the neurons are connected to each other according to a "small-world" (SW) topology with coupling strength g_c^{in}. The SW network is generated using the Watts-Strogatz model[35] with parameters $\beta = 0.3$ and $K = 5$. The parameter β is the probability for a particular link in the initially regular topology to be randomly rewired, and K is the mean degree.

We investigate the dynamics of the considered network of networks by analyzing the averaged over each subnet signals:

$$V_{avr,i} = \frac{1}{N_i} \sum_{j=1}^{N_i} V_i \tag{6}$$

where $N_i = 50$ is a number of neurons in the i-th subnet.

Fig. 2 illustrates the averaged signals of 4 subnetworks. One can see that four subnets are divided into two clusters under the influence of the inhibitory couplings between them: the first (solid black line) and fourth (dashed black line) networks generate spikes synchronously at the same time intervals. In contrast, the second (solid blue line) and third (dashed blue line) networks are synchronized with each other, their activity is observed in antiphase to the rest.

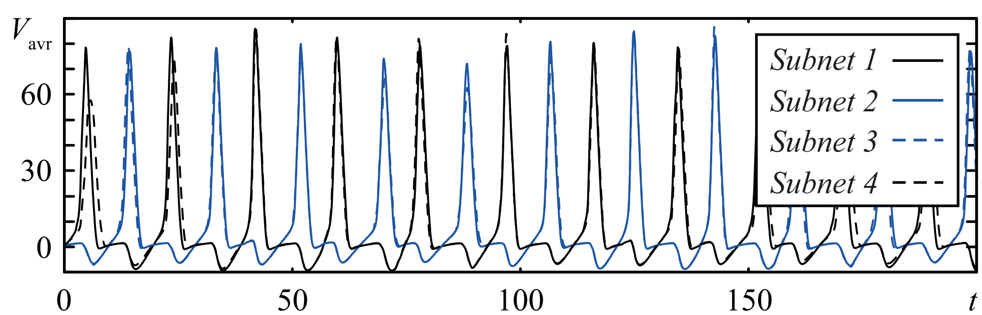

Figure 2. Time series of the averaged signals of 4 subnetworks.

We calculate the synchronization indices (SI) between all neurons of each large subnetwork (Eqs. 4-5) to analyse the dynamics. Then, we filter the resulting time series in the 0.004-0.015 Hz frequency band to visualize slow changes. The SIs can be in varying degrees of correlation with each other depending on the strength of the couplings between subnets and the one between the elements within the subnets. We calculate the linear Pearson correlation coefficient r for each pair of subnets to analyse the correlation.

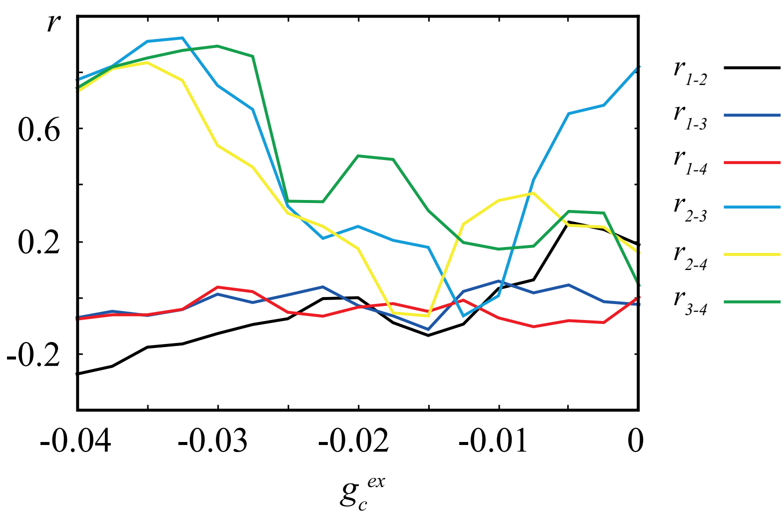

Figure 3. Correlations of synchronization indexes r between four subnetworks versus coupling strength between them g_c^{ex} for $g_c^{in} = 1.0$.

The dependencies of the correlations of synchronization indexes between four subnetworks on the coupling strength between them g_c^{ex} for $g_c^{in} = 1.0$ are illustrated on Fig. 3. One can see that the first subnet for the entire range of considered values of the coupling strength demonstrates almost zero correlation with all other subnets at low values of the inter-network coupling strength, which decreases to $r = -0.2$ with an increase in inhibitory connections (lines $r_{1-2}, r_{1-3}, r_{1-4}$). At the same time, the other three subnets behave completely differently: with weak inter-network coupling, the correlation between their synchronization indices is close to 0, but with an increase in this coupling, the correlation increases up to 0.9 (lines $r_{2-3}, r_{2-4}, r_{3-4}$). It should also be noted that an increase in the strength of intra-network couplings leads to an increase in the correlation between all networks.

4. CONCLUSION

Having summarized, we have investigated the dynamics of the complex network of Hodgkin-Huxley neurons. It consists of 4 sub-networks. The small input network receives an external signal and transfers it to subnetworks via excitatory couplings while the subnets interact with each other via inhibitory couplings.

We have shown that the subnets are divided into 2 clusters under the influence of inhibitory couplings between them. We have calculated the synchronization indices (SI) between all neurons of each large subnetwork to analyze the dynamics. We have found that SIs can oscillate either in-phase or anti-phase depending on the couplings between subnetworks. We showed that the first subnet for the entire range of considered values of the coupling strength demonstrates almost zero correlation with all other subnets while the other three subnets behave completely differently. Increasing the strength of intra-network couplings, the correlations between SIs of them increases up to 0.9.

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