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Studying Transitions between Different Regimes of Current Oscillations Generated in a Semiconductor Superlattice in the Presence of a Tilted Magnetic Field at Various Temperatures

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Abstract—The mechanisms of transitions between different regimes of current oscillations in a semiconductor superlattice in the presence of a tilted magnetic field at various temperatures have been studied. At relatively low temperatures, an increase in the applied voltage leads to a period-doubling bifurcation that causes a change in the dynamic regime. At increased temperatures, the transition takes place with the quenching of current oscillations.

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Semiconductor superlattices are complex nanostructures consisting of several thin (on the order of 10 nm) alternating layers of various semiconducting materials, usually with close crystalline lattice parameters (e.g., GaAs and AlGaAs) [1, 2]. These structures can exhibit a large variety of quantum-mechanical phenomena [3, 4]. The application of an electric field to semiconductor superlattices leads to the formation of electron domains [5, 6] and the corresponding current oscillations. In recent years, increasing research attention has been devoted to analysis of the transport of electrons in semiconductor superlattices from the standpoint of nonlinear dynamics, which allows the phenomena observed in the system to be elucidated and explained [7–13]. Topical issues related to these phenomena include the bifurcations that take place in semiconductor superlattices and lead to transitions between characteristic dynamic regimes.

The present work was devoted to elucidating the mechanism of transitions between various regimes of current oscillations observed with increasing voltage applied to a semiconductor superlattice (Fig. 1) in the presence of a tilted magnetic field at various temperatures.

Processes in a semiconductor superlattice in the presence of a tilted magnetic field have been modeled using a system of equations that includes the equation of continuity, the Poisson's equation, and an expression for the current density with allowance for the electron drift velocity [2, 7]:

$$\begin{aligned} e\frac{\partial n}{\partial t} &= \frac{\partial J}{\partial x}, \\ \frac{\partial F}{\partial x} &= \frac{e}{\varepsilon_0\varepsilon_r}(n - n_D), \\ J &= en v_d(\bar{F}), \end{aligned} \tag{1}$$

where t is the current time; x is the coordinate in the direction of electron motion (perpendicular to layers of the superlattice); variables $n(x, t)$, $F(x, t)$, and $J(x, t)$ describe the electron concentration, electric field strength, and current density, respectively; ε_0 and ε_r = 12.5 are the absolute and relative dielectric permittivi-

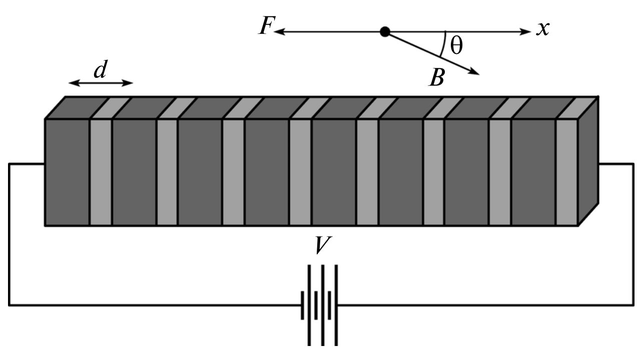

Fig. 1. Schematic diagram of a semiconductor superlattice in the presence of a tilted magnetic field. Alternating shades of gray indicate different semiconductor materials (e.g., GaAs and AlGaAs): (d) superlattice period, (F) electric field vector, (B) magnetic field vector, (θ) tilt angle, and (V) voltage applied to the superlattice.

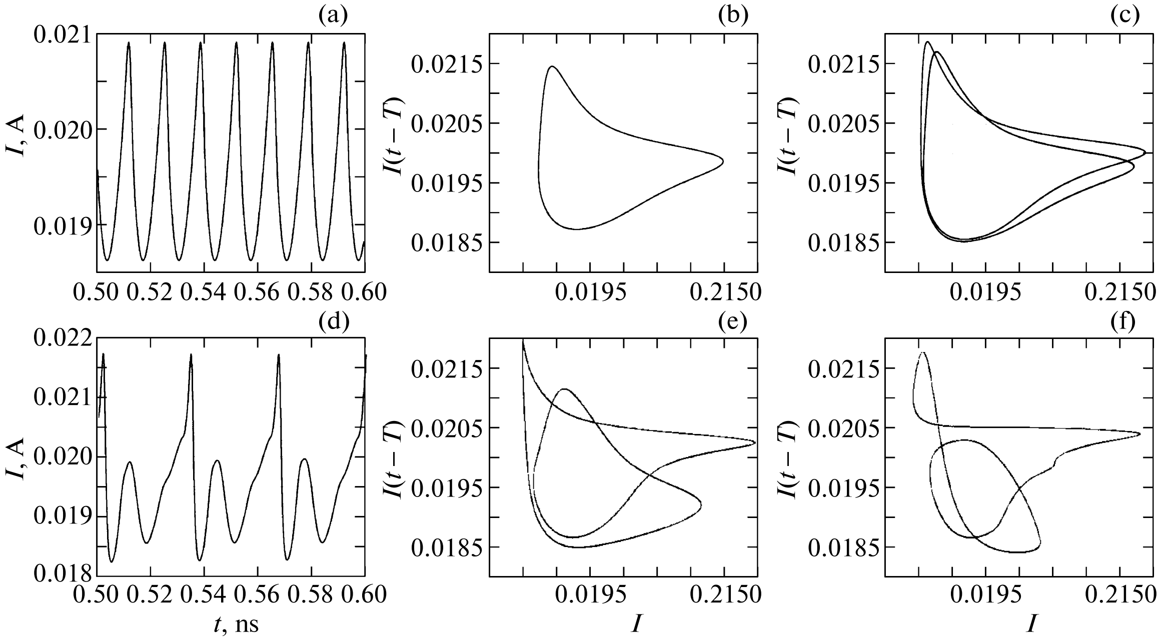

Fig. 2. (a, d) Time series of the current passing through the semiconductor superlattice at applied voltages $V = 0.56$ and 0.585 V, respectively; (b, c, e, f) phase portraits reconstructed using the Takens method (for a delay time of quarter period) at applied voltages $V = 0.56$, 0.565, 0.57, and 0.585 V, respectively; temperature, $T = 4.2$ K.

ties, respectively; n_D is the equilibrium electron concentration; v_d is the electron drift velocity calculated for the average electric field $\bar{F}$; and $e > 0$ is the electron charge.

The dependence of the electron drift velocity in Eq. (1) on the electric field is influenced by the temperature and the tilt angle of magnetic field. The drift velocity decreases with increasing temperature, while the presence of a tilted magnetic field leads to the appearance of resonant maxima [11] and substantially complicates dynamics of the system. Based on system of equations (1), dependences of the drift velocity on the electric field were numerically calculated for various temperatures in the framework of a model described in detail elsewhere [5, 10, 11]. Calculations were performed for the following values of parameters typical of real nanostructures: miniband width, $\Delta = 19.1$ meV; superlattice period, $d = 8.3$ nm; magnetic field tilt angle, $\theta = 40°$; magnetic induction, $B = 15$ T; and scattering time, $\tau = 0.25$ ps.

It was established that, immediately after the onset of generation, the oscillations of current in the semiconductor superlattice are close to harmonic. Then, as the applied voltage is increased, the character of oscillations rapidly changes so that their shape significantly complicates. Figures 2a and 2d present the time series of oscillating current for different applied voltages at a low temperature ($T = 4.2$ K). As can be seen, an increase in the voltage applied to the semiconductor superlattice leads to a significant change in the character of current oscillations. For a detailed analysis of changes in the system dynamics, it is necessary to use the Takens delay method for reconstructing phase portraits from time series [14]. According to this method, a temporal variation of some variable $I(t)$ (in this case, the magnitude of current passing through the superlattice) for a certain time step (delay) τ and integer m, it is possible to construct the m-dimensional vector with components representing I values at time moments $t, t - \tau, t - 2\tau, ..., t - (m - 1)\tau$. This vector determines a point in the m-dimensional space, which moves along a certain trajectory with the time. Assuming that we are dealing with a stationary regime of oscillations in a dissipative system, the obtained trajectory can be considered as representing a reconstructed portrait of attractor in the phase space [15]. For analysis of the dynamics of current, it is sufficient to construct a 2-dimensional phase portrait at a time delay of quarter period.

Based on the shapes of reconstructed phase portraits (Figs. 2b, 2c, 2e, and 2f), it can be concluded that the system with the indicated values of parameters exhibits a period-doubling bifurcation. As the applied voltage increases, the regime of current oscillations changes at the point of bifurcation. It should be emphasized that the system exhibits a single period-doubling bifurcation rather than a cascade of such bifurcations. The appearance of a period-doubling bifurcation leads to a significant decrease in the frequency of oscillations—in the given case, from 72 to 36 GHz.

Figure 3 shows the analogous time series and phase portraits of oscillations of the current passing through the semiconductor superlattice at $T = 200$ K. In this case, the transition between different regimes of oscil-

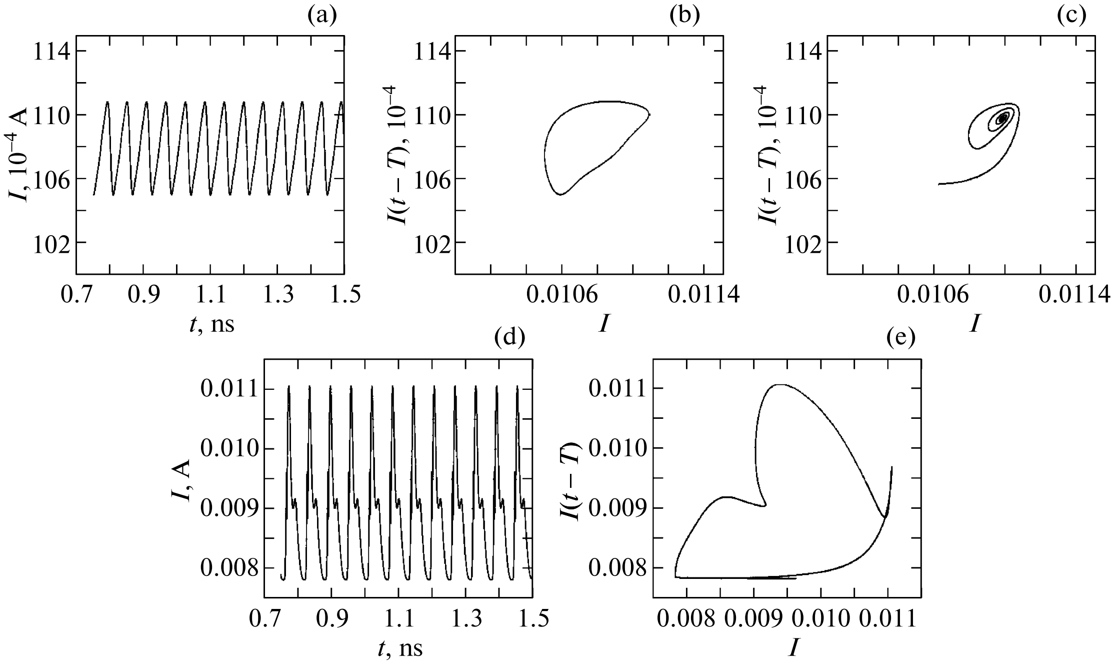

Fig. 3. (a, d) Time series of the current passing through the semiconductor superlattice at applied voltages $V = 0.542$ and 0.56 V, respectively; (b, c, e) phase portraits reconstructed using the Takens method (for a delay time of quarter period) at applied voltages $V = 0.542$, 0.55, and 0.56 V, respectively; temperature, $T = 200$ K.

lations (Figs. 3a and 3d) proceeds by the quenching of generation, rather than through a period-doubling bifurcation. A comparison of the reconstructed phase portraits (Figs. 3b, 3c, and 3e) shows that, as the applied voltage increases, the existing oscillations of current through the superlattice initially decay (Fig. 3c) and then fully vanish. However, the further increase in the voltage leads to the restoration of current oscillations (e.g., for $V = 0.56$ V). The character of oscillations before and after the quenching of oscillations is significantly different. At the same time, the amplitude and frequency (18 kHz) of oscillations remain unchanged.

Thus, it can be concluded that semiconductor superlattices in the presence of a tilted magnetic field at various temperatures can exhibit different variants of transitions between various regimes of current oscillations at increasing applied voltage. At low temperatures, the transition proceeds via a single period-doubling bifurcation, while the transition at higher temperatures takes place with the quenching of oscillations of the current passing through the semiconductor superlattice.

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