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A Study of the Effect of Random Dopant-Concentration Fluctuations on Current in Semiconductor Superlattices

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Abstract—The influence exerted by random dopant-concentration fluctuations on the current–voltage characteristics of the current flowing through a semiconductor superlattice has been studied. It was shown that the characteristics of the current flowing through the superlattice noticeably vary with the amplitude of fluctuations of nanostructure parameters. It was possible to find for a small sample the probability-density distribution of the integrated absolute values of the difference of currents at various amplitudes of the dopant-concentration fluctuations.

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The object of the present study is a heterostructure constituted by alternating layers (with layer widths, as a rule, not exceeding several tens of nanometers) of various semiconductor materials with close lattice constants, i.e., a semiconductor superlattice [1, 2]. Superlattices serve as a convenient model for study of various quantum-mechanical effects [3, 4]. Of no less interest is the collective dynamics of electrons upon application of a dc voltage to a superlattice because spatial–temporal electronic structures, called "domains" (by analogy with the domains in a Gann diode [5, 6]), can be formed in the structure in this case. The transition of domains across the semiconductor superlattice gives rise to oscillations of the current flowing through the structure, with the frequency of these oscillations being, as a rule, several tens of gigahertz [7, 8]. It should be noted that researchers are paying steadily increasing attention to considering the electron transport across a semiconductor superlattice in terms of the nonlinear dynamics. In this way, various effects observed in the system under study can be revealed and explained [7–11].

It is an important that a large number of various spatial heterogeneities (e.g., random dopant-concentration fluctuations [12, 13]), which affect the characteristics of a superlattice, can appear in development of semiconductor heterostructures. In addition to the occurrence of random fluctuations, it is possible to deliberately change the dopant concentration so as to improve the characteristics of a device [14, 15]. In this study, we examine how the level of random dopant-concentration fluctuations affects the characteristics of a semiconductor superlattice.

A typical model in description of charge-transport processes in a semiconductor superlattice is a system of equations that includes the continuity equation, Poisson's equation, and expression for the current density with consideration for the electron-drift velocity [2, 9]:

$$e\frac{\partial n}{\partial t} = \frac{\partial J}{\partial x},$$
$$\frac{\partial F}{\partial x} = \frac{e}{\varepsilon_0\varepsilon_r}(n - n_{D0}),$$
$$J = env_d(\bar{F}). \quad (1)$$

Here, t designates time and coordinate x corresponds to the electron-motion direction in the semiconductor superlattice. Quantities $n(x, t)$, $F(x, t)$, and $J(x, t)$ identify, respectively, the concentration, electric-field strength, and current density. Parameters ε_0 and $\varepsilon_r = 12.5$ are the permittivity of free space and relative permittivity, $n_{D0} = 3 \times 10^{22}$ m^{-3} is the equilibrium electron concentration, v_d is the electron-drift velocity calculated for average electric-field strength $\bar{F}$, and $e > 0$ is elementary charge. The dependence of the drift velocity appearing in Eq. (1) on the electric-field strength is determined by the Esaki–Tsu formula [4].

We proceed from the assumption that the dopant concentration in each layer differs from the equilibrium concentration n_{D0} by a random value [12]. Then, to take into account random fluctuations, we modify Poisson's equation from (1) as follows:

$$\frac{\partial F}{\partial x} = \frac{e}{\varepsilon_0\varepsilon_r}[n - (n_{D0} + D_{nD_1}(x))], \quad (2)$$

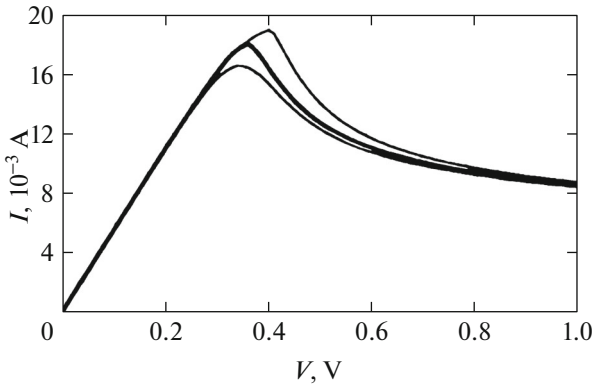

Fig. 1. Examples of I–V characteristics for several dopant-concentration profiles at fluctuation amplitude $D = 0.5$ (the heavy line represents the case without fluctuations).

where $n_{D1}(x)$ is the uniform distribution of a random quantity within the range [–0.5; 0.5] in accordance with [12] and D is the amplitude of the dopant-concentration fluctuations.

An important characteristic of the collective electron dynamics is the current–voltage (I–V) characteristic. The beginning of the falling portion in the I–V curve corresponds to the onset of generation of current oscillations. To plot this portion of the I–V characteristic, the current was time-averaged. The method for construction of I–V characteristics and their run for the case in which dopant-concentration fluctuations are disregarded ($n_{D1}(x) = 0$ along the entire superlattice) was described in [11] (Fig. 1). It can be seen in the figure that the I–V characteristics depart from the case without fluctuations for various dopant concentration profiles (at the same fluctuation amplitude D).

Let us introduce quantity Θ that determines the departure of the I–V characteristic from the case without dopant-concentration fluctuations. This quantity is calculated as follows:

$$\Theta = \frac{\int |I(V) - I_0(V)| dV}{\int I_0(V) dV}, \qquad (3)$$

where $I(V)$ is the current at a voltage V, and $I_0(V)$ is the current at the given voltage in the absence of fluctuations (fluctuation amplitude D tends to zero). This quantity will be different for each profile with random dopant concentrations.

In practice, it is important, when developing real devices, including those based on semiconductor superlattices, to evaluate the departure of characteristics at a fixed fluctuation amplitude of device parameters (on the assumption that this amplitude depends on the technological process). Methods for finding device characteristics from a small sample are important for obtaining this estimate, which seems reasonable for cost-cutting in development of device prototypes. The problem of finding from a small sample the manner in which device characteristics are distributed is also encountered in a computer simulation, because each calculation of device parameters requires a great expenditure of computer time. This problem can be solved by using a ranked distribution. The ranked distribution is more convenient in that no data reduction occurs in this case, in contrast to that in which the corresponding histogram is plotted [16, 17]. The ranked distributions for 200 random profiles and different amplitudes obtained by numerical simulation are shown in Fig. 2a. Rank 1 is assigned to the smallest value of Θ, rank 2 to the next larger value, and so on until the largest rank is assigned to the maximum value of Θ. For a great number of events, the rank can be considered a continuous quantity. Then, based on the ideology described in [18], we can find how the ranked distribution of Θ is related to the probability-density distribution of Θ; i.e.,

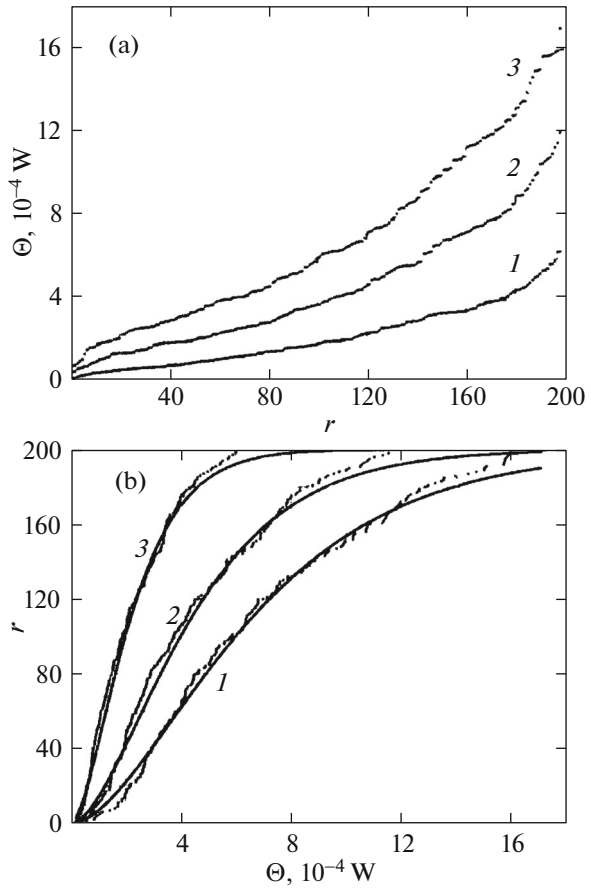

Fig. 2. (a) Integrated absolute value of the difference of currents, Θ, in increasing order for various fluctuation amplitudes D; (b) ranked distributions of the integrated absolute value of the difference of currents, Θ, for various fluctuation amplitudes D: (*1*) 0.25, (*2*) 0.5, (*3*) 0.75.

$$\rho(\Theta) = dr/d\Theta, \qquad (4)$$

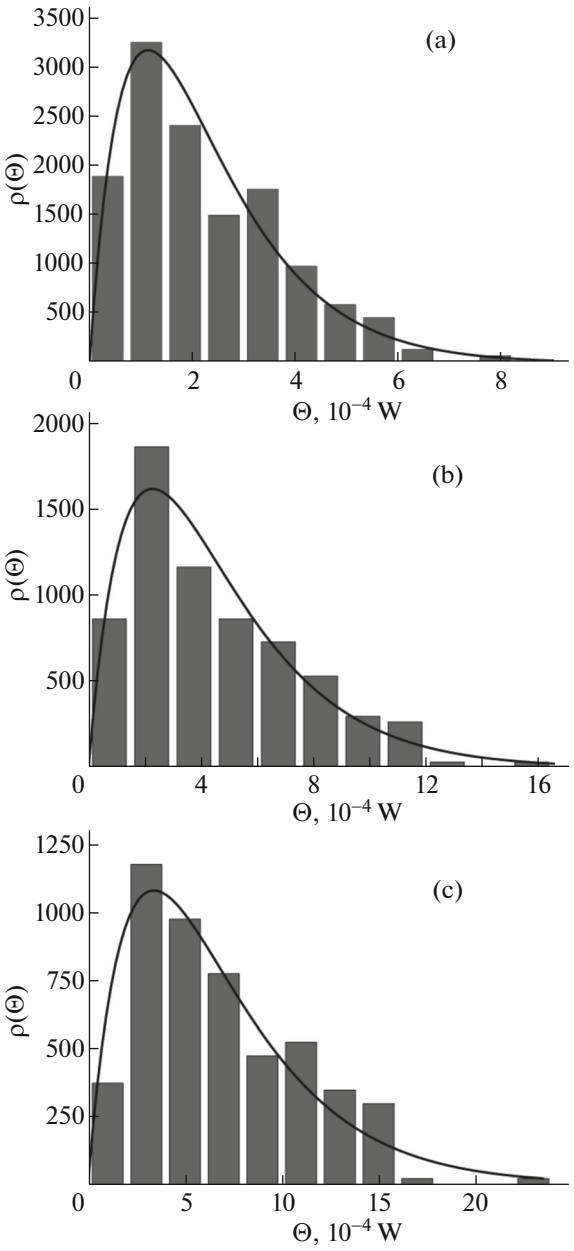

Fig. 3. Probability-density distributions of the integrated absolute value of the difference of currents, Θ, for various fluctuation amplitudes D: (a) 0.25, (b) 0.5, and (c) 0.75.

where r is rank. Figure 2b shows the ranked distributions as functions of Θ, which corresponds to expression (4) and simplifies further analysis. In this form, the dependence of the rank on Θ can be used to easily find the approximation, which appears as

$$r = N(1 - \exp(-b\Theta/D)(1 + b\Theta/D)). \quad (5)$$

Here, $N = 200$ is the total number of random profiles and $b = 2200$ is a constant parameter. These approximations for various fluctuations amplitudes D are represented in Fig. 2b by lines, with good agreement observed between the approximation and ranked dependences. Now, we can find, using (5), the distribution law of the probability density for Θ with the aid of relation (4). It can be easily seen that it has the form

$$\rho(\Theta) = \Theta \exp(-b\Theta/D)(b/D)^2. \quad (6)$$

Figure 3 shows the numerically obtained probability-density distributions for Θ in the form of histograms at various fluctuations amplitudes D. The lines in the figure are the curves that correspond to formula (6). It can be seen that, on the whole, the numerically found probability-density distributions for Θ are in a rather good agreement with the laws (6) previously obtained for various fluctuation amplitudes.

Thus, we demonstrated how random dopant-concentration fluctuations affect the I–V characteristics in a semiconductor superlattice. To take into account the related changes, we introduced integral quantity Θ and examined how it depends on the amplitude of dopant-concentration fluctuations. A comparatively small sample of dopant-concentration profiles (200 profiles at a fixed level of random fluctuations) was used to obtain via data ranking the probability-density distribution of Θ for an arbitrary amplitude of fluctuations. The proposed method can be applied to small samples of prototypes of real devices, including ones based on semiconductor superlattices, to find how departures of characteristics are distributed at random fluctuations of parameters of the devices, which appear in their fabrication.

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REFERENCES

1. A. Ya. Shik, Sov. Phys. Semicond. **8**, 1195 (1974).
2. A. Wacker, Phys. Rep. **357**, 1 (2002).
3. L. V. Keldysh, Sov. Phys. Solid State **4**, 1658 (1962).
4. L. Esaki and R. Tsu, IBM J. Res. Dev. **14**, 61 (1970).
5. R. Scheuerer, E. Schomburg, K. F. Renk, et al., Appl. Phys. Lett. **81**, 1515 (2002).
6. H. Eisele, S. P. Khanna, and E. H. Lineld, Appl. Phys. Lett. **96**, 072101 (2010).
7. L. L. Bonilla and H. T. Grahn, Rep. Prog. Phys. **68**, 577 (2005).
8. T. M. Fromhold, A. Patanè, S. Bujkiewicz, et al., Nature **428**, 726 (2004).
9. A. O. Selskii, A. A. Koronovskii, A. E. Hramov, et al., Phys. Rev. B **84**, 235311 (2011).
10. L. L. Bonilla and S. W. Teitsworth, *Nonlinear Wave Methods for Charge Transport* (Wiley-VCH, Weinheim, 2010).

11. A. E. Hramov, V. V. Makarov, A. A. Koronovskii, et al., Phys. Rev. Lett. **112**, 116603 (2014).
12. A. Wacker, G. Schwarz, F. Prengel, et al., Phys. Rev. B **52**, 13788 (1995).
13. M. Patra, G. Schwarz, and E. Scholl, Phys. Rev. B **57**, 1824 (1998).
14. A. G. Balanov, A. A. Koronovskii, O. I. Moskalenko, A. O. Selskii, and A. E. Hramov, Bull. Russ. Acad. Sci.: Phys. **81**, 43 (2017).
15. A. G. Balanov, A. A. Koronovskii, O. I. Moskalenko, et al., Uch. Zap. Fiz. Fakult. MGU **6**, 166907 (2016).
16. A. A. Koronovskii and D. I. Trubetskov, *Nonlinear Dynamics in Action* (Kolledzh, Saratov, 2002) [in Russian].
17. G. K. Zipf, *Human Behavior and the Principle of Least Effort* (Addison-Wesley, Reading, MA, 1949).
18. M. E. J. Newman, Proc. R. Soc. London B **263**, 1605 (1996).

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