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Structural properties of brain functional network during Schulte table solving

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ABSTRACT

In this paper, we analyze the inter-layer connectivity of multiplex functional network of the brain, where each layer represent the separate timescale. For this task we conduct the EEG experiments, which involve the solving of Schulte tables, the widespread psycological test. Using the wavelet bicoherence we reconstruct the functional network on various frequency bands of brain activity, that allows us to build multiplex functional network. Using the concept of betweenness centrality we analyze the inter-layer interaction in the brain functional network and reveal the regions, which demonstrate maximal inter-layer activity.

Keywords: Complex network, electroencephalogram, continuous wavelet analysis, oscillatory patterns, phase synchronization

1. INTRODUCTION

Currently, one of the most rapidly developing areas of radiophysics and nonlinear physics is the study of networks with a complex topology of inter-element connections (the so-called complex networks),[1] in particular, networks of interacting oscillators.[2] The theory of complex networks finds its application in such areas as neurodynamics,[3] bioinformatics,[4] neurophysiology,[5] social sciences[6] and others. The great interest in network theory was attracted by studies of the properties of real networks, which showed that many technogenic and social networks have a number of common properties, such as free scaling[7] and the presence of structural clusters.[8]

Thus, the property of free scaling networks (scale-free network)[7] is reflected in almost all real networks, in particular, the functional network of brain neurons.[5] In fact, this property indicates the presence of a heterogeneous structure and hub elements, concentrating connections with a large number of nodes. Also noteworthy is the important property of many real networks as a property of the small world,[9] reflecting the presence of a short path between two arbitrary nodes, namely, the shortest path in such networks increases in proportion to the logarithm of the number of nodes in the network.[10] An important actual task of the modern theory of networks is the study of classical and chaotic synchronization in networks of dynamic elements with a complex topology.[11] Topology has been shown to be the main factor determining the occurrence, stability, and propagation of synchronous states in complex networks.[12]

It should be noted that while the study of synchronization processes within complex networks is an actively studied problem, the issue of the interaction of networks of dynamic elements is still very poorly studied.[13] However, interest in this problem has increased dramatically in recent years, the attention of researchers has shifted towards studying the processes of competition and synchronization between interacting networks, which is motivated by the ability to describe many processes occurring in real systems.[14]It should be noted the work of scientific groups under the devoted to modeling attacks on interconnected networks,[15] as well as the rivalry of two connected networks. These works are motivated by the possibility of developing methods for constructing topologies that are most resistant to external influences and loss of individual nodes.[16,17]

One of the most complex systems, representing an ensemble of interacting networks, is the neural network of the brain.[18] Its various subnets, for example, the thalamo-cortical network, also represent the "network of networks".[19] As it has been shown in many papers, the neural networks of the brain have both the free scaling property and the small world,[20] while the appearance of pathologies is often reflected in the disappearance of these properties in local areas of the brain.[21,22] The study of the phenomena occurring in the neural ensemble

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of the brain is impossible without understanding the principles of synchronization of connected networks, and therefore, this problem is of great importance from the point of view of neuroscience and neurophysiology, it is of interest not only in the context of identifying the fundamental principles of synchronization and hierarchy in functional neural network,[23] but also in the context of the development of new diagnostic methods for synchronization between brain areas using neurophysiological EEG and MEG signals. This task is extremely relevant for clinical neurophysiology for the purpose of early detection of pathological hypersynchronous activity, for example, associated with absance epilepsy, and for the development of new approaches to the creation of brain-computer interfaces in the context of creating high-tech prostheses for people with disabilities.[24–26]

In this paper, we analyze the inter-layer connectivity of multiplex functional network of the brain, where each layer represent the separate timescale. For this task we conduct the EEG experiments, which involve the solving of Schulte tables, the widespread psycological test. Using the wavelet bicoherence we reconstruct the functional network on various frequency bands of brain activity, that allows us to build multiplex functional network. Using the concept of betweenness centrality we analyze the inter-layer interaction in the brain functional network and reveal the regions, which demonstrate maximal inter-layer activity.

2. METHODS

2.1 Experimental design

The experiments were carried out during the first half of the day at a specially equipped laboratory where the volunteer was sitting comfortably and effects of external stimuli, e.g. external noise and bright light, were minimized. The $S = 8$ subjects performed a series of attentional tests using the Schulte table. The Schulte table represents a square matrix with 5 columns and 5 rows, in which numbers from 1 to 25 are placed in random order. The volunteer is asked to search for numbers in ascending order (from 25 to 1) and point them in the table using a pen. The experiment for each subject contained $M = 5$ active stages (Schulte table evaluation), which were alternating with the passive stages (rest).

During all experiment, the multi-channel EEG data has been acquired using the BE Plus LTM amplifier, manufactured by EB Neuro S.P.A., Florence, Italy (www.ebneuro.com). It was recorded at 8 kHz sampling rate using the standard bipolar method of registration with two reference and $N = 19$ electrodes. The adhesive Ag/AgCl electrodes based in special prewired head caps were used to obtain the EEG signals. Two reference electrodes A1 and A2 were located on mastoids, while the ground electrode N was located above the forehead. The EEG signals were filtered by a band pass filter with cut-off points at 1 Hz (HP) and 300 Hz (LP) and a 50-Hz Notch filter. To accurately split the recording into the active (Shulte table evaluation) and passive (rest) phases we used the video recording during all stages of experiment.

Subjects participated in the experiment on a voluntary and gratuitous basis. All participants signed an informed medical consent to participate in the experimental work and received all necessary explanations about the process, including their agreement for further publication of the results. Acquired experimental data were processed with respect the confidentiality and anonymity of research respondents. The experimental studies were performed in accordance with the Declaration of Helsinki.

2.2 Reconstruction of multiscale connectivity

We use the wavelet bicoherence to estimate the strength of interaction between the brain regions. The wavelet bicoherence has proved itself as very powerful instrument to quantify the interactions on various scales of biological systems,[27–29] including brain activity.[30,31] Below the detailed description of the calculation of wavelet bicoherence for the pairs of EEG signals is presented.

We calculate the complex-valued wavelet coefficients $W_i(f, t_0)$ for each EEG channel $x_i(t)$ as

$$W_i(f, t_0) = \sqrt{f} \int_{t_0-4/f}^{t_0+4/f} x_i(t)\psi^*(f, t - t_0)dt, \tag{1}$$

where $i = 1, ..., N$ is the number of considered EEG channel, $N = 19$ is the total number of EEG channels, t_0 specifies the wavelet location on the time axis and "*" denotes the complex conjugation, and $\psi(f,t)$ is the mother wavelet function. We use the Morlet wavelet, which is often utilized for processing of biological signals[28]

$$\psi(f, t-t_0) = \sqrt{f}\pi^{-1/4}\mathrm{e}^{\mathrm{i}\omega_0 f(t-t_0)}\mathrm{e}^{-f^2(t-t_0)^2/2}, \tag{2}$$

where ω_0 is the wavelet scaling parameter and i is an imaginary unit. Previously we found that parameter $\omega_0 = 2\pi$ in the continious wavelet transform provides an optimal time-frequency resolution of EEG signal.[32,33]

To measure the degree of coherence between two EEG signals $x_i(t)$ and $x_j(t)$, we use the corresponding complex-valued wavelet coefficients $W_i(f,t) = a_i + \mathrm{i}b_i$ and $W_j(f,t) = a_j + \mathrm{i}b_j$.

Wavelet bicoherence, $\sigma_{ij}(f,t)$, is estimated based on the mutual wavelet spectrum $W_{i,j}(f,t)$ of the signals $x_i(t)$ and $x_j(t)$. Similarly to[34] the coefficients $\mathrm{Re}\,[\sigma_{ij}(f,t)]$ and $\mathrm{Im}\,[\sigma_{ij}(f,t)]$ represented as real and imaginary parts of mutual wavelet spectrum can be calculated via Eqs. (3) and (4), respectively:

$$\begin{aligned}&\mathrm{Re}\,[\sigma_{ij}(f,t)] = \\ &= \frac{a_i(f,t)a_j(f,t) + b_i(f,t)b_j(f,t)}{\sqrt{a_i^2(f,t)+b_i^2(f,t)}\sqrt{a_j^2(f,t)+b_j^2(f,t)}}\end{aligned} \tag{3}$$

and

$$\begin{aligned}&\mathrm{Im}\,[\sigma_{ij}(f,t)] = \\ &= \frac{b_i(f,t)a_j(f,t) - a_i(f,t)b_j(f,t)}{\sqrt{a_i^2(f,t)+b_i^2(f,t)}\sqrt{a_j^2(f,t)+b_j^2(f,t)}}.\end{aligned} \tag{4}$$

Next, we evaluate the degree of coherence between the different EEG signals, recorded during each EEG trial of rest or evaluation of cognitive task for each subject p. The values were averaged over time intervals, involved in each trial of experiment. As the result, time-averaged coefficients $\mathrm{Re}\,[\sigma_{ij}(f)]_{T_{mp},m,p}$ and $\mathrm{Im}\,[\sigma_{ij}(f)]_{T_{mp},m,p}$ were obtained as

$$\mathrm{Re}\,[\sigma_{ij}(f)]_{T_{mp},m,p} = \frac{1}{MS}\sum_{p=1}^{S}\sum_{m=1}^{M}\frac{1}{T_{mp}}\int\limits_{T_{mp}} \mathrm{Re}\,[\sigma_{ij}(f,t)]\,dt \tag{5}$$

and

$$\mathrm{Im}\,[\sigma_{ij}(f)]_{T_{mp},m,p} = \frac{1}{MS}\sum_{p=1}^{S}\sum_{m=1}^{M}\frac{1}{T_{mp}}\int\limits_{T_{mp}} \mathrm{Im}\,[\sigma_{ij}(f,t)]\,dt, \tag{6}$$

where $M = 5$ is the number of stages of cognitive task evaluation or rest, S is the number of subjects, T_{mp} is the duration of m–th stage of task evaluation by p–th subject determined by recorded video analysis or the duration of rest interval which was fixed $T_{mp} = 10\,\mathrm{s}$. Based on coefficients (5) and (6) the degree of coherence, $\sigma(f)$, between the EEG signals was estimated as the amplitude of mutual wavelet spectrum

$$\sigma_{ij}(f) = \sqrt{(\mathrm{Re}\,[\sigma_{ij}(f)]_{T_{mp},m,p})^2 + (\mathrm{Im}\,[\sigma_{ij}(f)]_{T_{mp},m,p})^2}. \tag{7}$$

The $\sigma_{ij}(f)$ function takes the values from 0 to 1, containing the information about the degree of phase coherence of the two signals $x_i(t)$ and $x_j(t)$ for the particular frequency. There $\sigma_{ij}(f) = 0$ implies that there is no phase coherence at the current frequency, for $\sigma_{ij}(f) > 0$ partial coherence takes place and $\sigma_{ij}(f) = 1$ indicates complete coherence.

Obtained values (7) were then averaged over EEG frequency bands. As the result, coefficients $\sigma_{ij}(s)$, defined the coherence between EEG signals in six typical EEG frequency bands ($\Delta f = \delta, \theta, \alpha, \beta_1, \beta_2, \gamma$):

$$\sigma_{ij}(\Delta f) = \frac{1}{\Delta f}\int\limits_{\Delta f} \sigma(f)\,df \tag{8}$$

2.3 Network analysis

Using the wavelet bicoherence between the pairs of channels $i, j = 1..N$ we reconstruct the $\sigma_{ij}(\Delta f)$ coupling matrixes in six considered frequency bands ($\Delta f = \delta, \theta, \alpha, \beta_1, \beta_2, \gamma$) for each $p = 1, \ldots S$ subject's stage of the experiment for the task evaluation stages (ES) and for the stages of rest (RS). It should be noted, that by definition of wavelet coherence measure σ the signals are self-correlated, so the elements of main diagonal $\sigma_{ii}(\Delta f) = 1$.

Due to the aspects of the betweenness calculation in the weighted networks,[35] more specifically, while the weight in considered as a distance, we should make the inverse transformation of each adjacency matrix:

$$w_{ij}(\Delta f) = 1 - \sigma_{ij}(\Delta f), \tag{9}$$

thus we have the inverse values within the range of $[0, 1]$.

To reveal the features of interaction between different timescales of neuronal activity during cognitive load we arrange the connectivity structures $w_{ij}(\Delta f)$ obtained for each frequency band, Δf, in the multiplex network:

$$\Omega = \begin{bmatrix} w_{ij}(\delta) & E & E & E & E & E \\ E & w_{ij}(\theta) & E & E & E & E \\ E & E & w_{ij}(\alpha) & E & E & E \\ E & E & E & w_{ij}(\beta_1) & E & E \\ E & E & E & E & w_{ij}(\beta_2) & E \\ E & E & E & E & E & w_{ij}(\gamma) \end{bmatrix}, \tag{10}$$

where E is identity matrix. It should be noted, that in this process each node (corresponding to specified EEG channel) becomes connected to itself in all other layers (EEG frequency bands). The magnitude of the inter-layer links is not chosen randomly: while the value of $w_{ij}(\Delta f)$ is always less than 1, although it can be very close to it, the shortest paths between the nodes located in the same layer will preferably go through that layer.

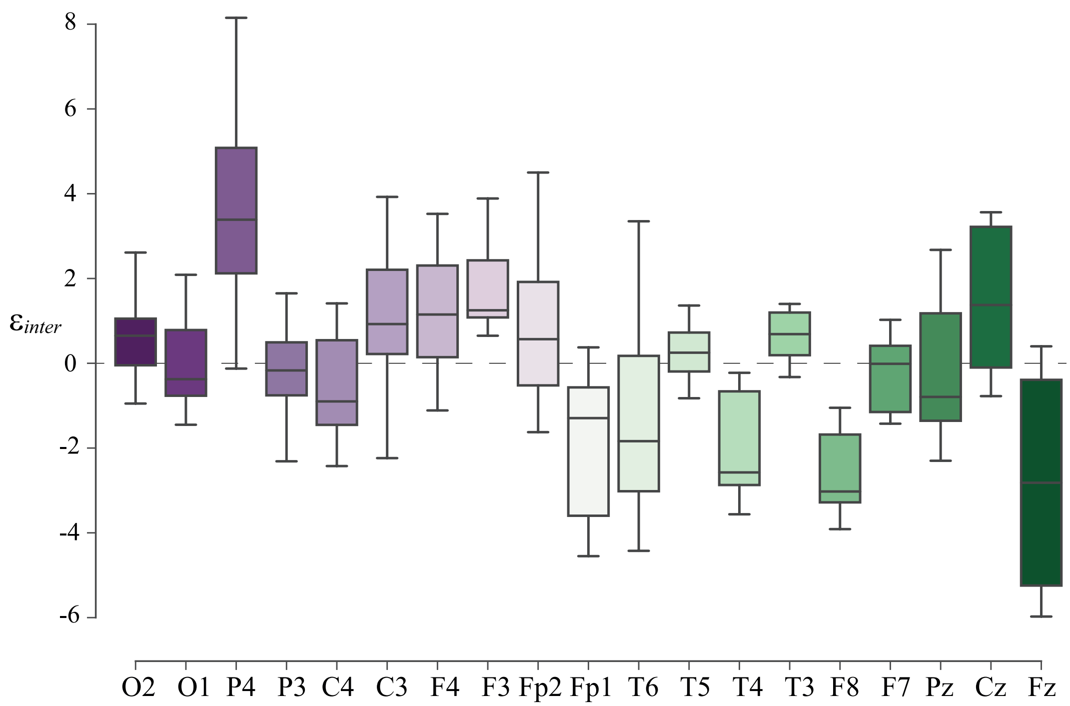

Figure 1. The relative value of betweenness centrality, which share each frequency band in multiplex network for passive (purple) and active (orange) phase of experiment are shown via box-plots.

Since we have obtained the final network structure, we are able to calculate the betweenness centrality for the each node, g_i, corresponding to i–th EEG channel:

$$g_i = \sum_{i \neq j \neq k} \frac{\Lambda^i_{jk}}{\Lambda_{jk}}, \tag{11}$$

where Λ^i_{jk} is the number of shortest paths between node j and k which pass through node i, and the Λ_{jk} is the total number of shortest paths between j and k. We use the algorithm for weighted networks proposed in[35] and calculate the values for the each node of multiplex network (10). After that we normalize the betweenness centrality of each node on the total value in the multiplex network:

$$g'_i = \frac{g_i}{\sum_{i=1}^{6N} g_i}. \tag{12}$$

3. RESULTS

The figure 1 shows the difference in the number of shortest paths which are going through each node's (specific region of the brain) inter-layer links between the active and passive phase of experiment:

$$\varepsilon_{inter} = g^{active}_{inter} - g^{passive}_{inter}, \tag{13}$$

where g^{active}_{inter} and $g^{passive}_{inter}$ are the number of shortest paths, which are going through the interlayer link in active and passive phase of experiment.

One can easyly see, that the most pronounced changes take place at frontal (F3, F3), prefrontal (Fp1, Fp2) and temporal lobe. At the same time, the hemispherical asymmetry can be observed in the temporal and frontal lobe: while channels T6, T4 and F8 demonstrate the decrease of shortest paths, the symmetrical regions does not show such behaviour. We should note, that the increase of the number of shortest paths can be interpreted as the emergence of strong interaction in the corresponding area. We also observe the strong decrease of the betweenness in channel Fz and its sharp increase in P4 channel.

Let us consider the detailed spatial plot of interaction between the different time scales. The figure 2 and represents the number of shortest paths, which are going through each inter-layer link in active (a) and passive (b) phase. Such representation allows us to direclty locate the regions of strong interaction in brain, both in passive wakefulness and cognitive load. We can see, that in passive phase (b), the interaction between frequency scales is mostly pronounced in central and frontoparietal lobes between the low-frequency ranges. But if we consider

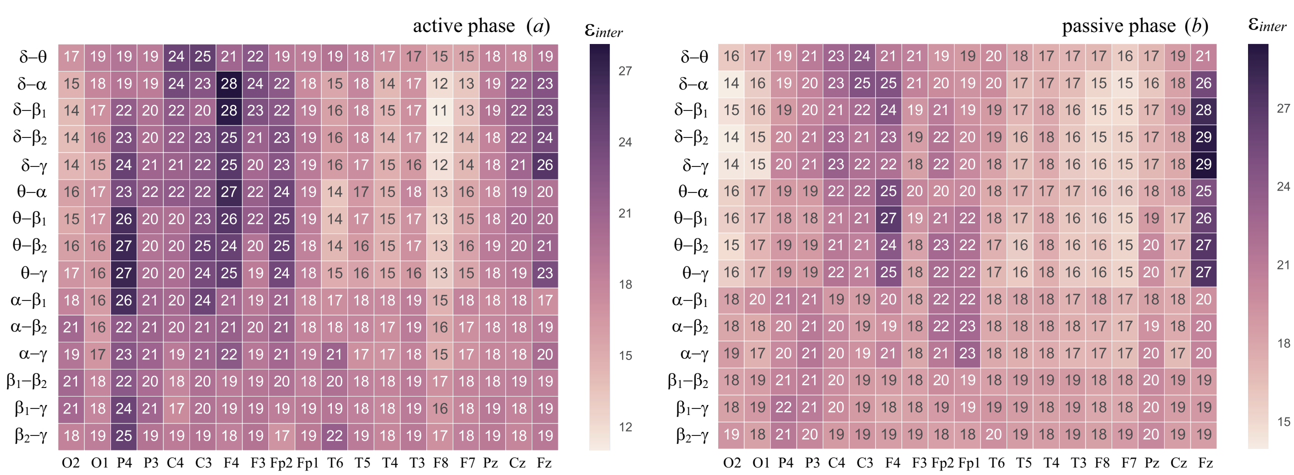

Figure 2. The number of shortest paths going through each inter-layer link in functional multiplex network for active (a) and (b) passive phase of experiment.

the active phase (a), we will see, that distribution of shortest paths becomes much more inhomogeneous. The main increase can be observed in parietal and frontal lobes, which includes both interaction between low and high frequencies.

For the clear representation we have depicted the arithmetic difference between this two plots in the figure 3. Here we can clearly see, that the emergence of cognitive load leads to the strong increase of interaction between θ range and other frequency bands in the area of parietal cortex. Also, the increase of shortest paths can be observed in frontal and frontoparietal lobes, especially in δ range. This area also demorstrate small increase of interaction in the high frequencies (β, γ ranges). We still can observe strong hemispherical assymmetry, considering temporal and occipital lobe. Note, than occipital lobe (channels O1, O2) demonstrate inverse behaviour on high and low frequencies, that marks the involvement of this region in the solving of such spatio-visual task as Schulte table solving.

4. CONCLUSION

In this paper, we analyze the inter-layer connectivity of multiplex functional network of the brain, where each layer represent the separate timescale. For this task we conduct the EEG experiments, which involve the solving of Schulte tables, the widespread psycological test. Using the wavelet bicoherence we reconstruct the functional network on various frequency bands of brain activity, that allows us to build multiplex functional network. Using the concept of betweenness centrality we analyze the inter-layer interaction in the brain functional network and reveal the regions, which demonstrate maximal inter-layer activity.

5. ACKNOWLEDGMENTS

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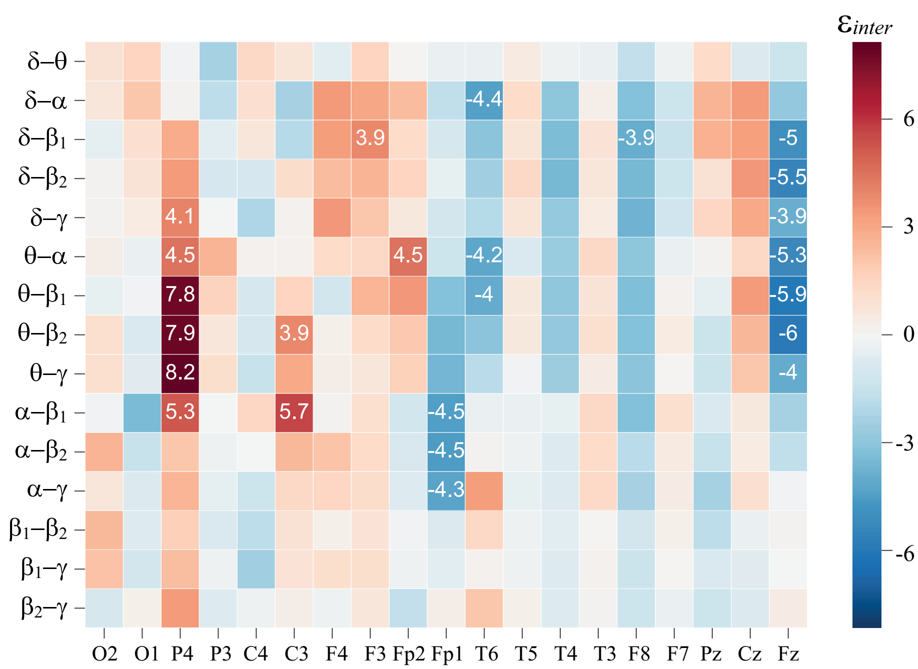

Figure 3. The difference in number of shortest paths going through each inter-layer link in functional multiplex network between active and passive phase of experiment.

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