

Serial identification of EEG patterns using adaptive wavelet-based analysis

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Abstract. A problem of recognition specific oscillatory patterns in the electroencephalograms with the continuous wavelet-transform is discussed. Aiming to improve abilities of the wavelet-based tools we propose a serial adaptive method for sequential identification of EEG patterns such as sleep spindles and spike-wave discharges. This method provides an optimal selection of parameters based on objective functions and enables to extract the most informative features of the recognized structures. Different ways of increasing the quality of patterns recognition within the proposed serial adaptive technique are considered.

1 Introduction

Development of new techniques for automatic recognition of specific oscillatory patterns in the scalp electroencephalogram (EEG) represents an important problem in neurophysiology and related fields of basic science. EEG signals contain a variety of rhythmic components whose frequencies provide important information about functional activity of neural structures [1]. EEG signal represents a linear mixture of co-existing oscillatory components, i.e., nonlinear effects do not complicate the process of recognition. The structure of EEG includes different types of specific oscillatory patterns such as sleep spindles (SS) and spike-wave discharges (SWD). The presence of SS-patterns is associated with sleep, when perceptual awareness is reduced [2]. They typically occur in EEG recorded in rats and other mammal species [3]. EEG structure of SS is similar to SWD in rats with absence epilepsy [4,5]. Although these

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two patterns can be visually identified, they have complex structure and often coexist with other rhythmic components. All these factors, including subjectivity (human factors), influence the quality of expert estimation. At present, different approaches are used in order to overcome the problem with routine EEG identification procedure including spectral analysis [6–8], wavelets [9–12], complexity measures [13], matching pursuit [14,15], etc.

Among various recognition techniques, wavelet-based approaches [16–18] are related to be the most powerful tools. However, they require an appropriate selection of parameters in order to minimize errors of patterns identification and, therefore, efficiency of these approaches can be significantly reduced if randomly chosen parameters are not optimal. This problem has already been tackled, for instance, the wavelet-based approach was used in an algorithm for spike sorting in extracellularly recorded electrical signals [19–21]. EEG patterns have more complicated structure in comparison to neuronal electrical activity at the extracellular level. Approaches based on the continuous wavelet-transform (CWT) assume a selection of at least two parameters characterizing translations and dilations of the “mother” wavelet, and a choice of the wavelet-basis that is constructed from a single soliton-like function.

Previously we developed several technical approaches to wavelet-based recognition of specific oscillatory patterns [22–25]. In this work we propose a modified method that we refer to as a serial adaptive method. This method provides a sequential recognition of patterns consequently as EEG spectral power is reduced. We evaluate the effectiveness of the proposed technique in the task of recognizing two types of coexisting oscillatory patterns, namely, sleep spindles and spike-wave discharges. We disclosed inconstancy of empirical selection of the wavelet-parameters, and our new approach is based on the optimization theory. Since optimal parameters are computed by means of the proposed technique, the quality of EEG pattern recognition does not depend on the experience of a researcher. We discuss further ways to improve the efficacy of the proposed serial adaptive method by additional optimization procedures.

2 Experiments

The experiments were performed at the Institute of the Higher Nervous Activity and Neurophysiology of Russian Academy of Sciences (Moscow). EEGs were recorded in six adult WAG/Rij male rats (7–9 months old). A recording electrode was implanted epidurally at the surface of the frontal cortex. Ground and reference electrodes were placed over the two symmetrical sides of the cerebellum. Continuous recording of EEG was performed during 24 hours. The corresponding signals were fed into a multi-channel differential amplifier, band-pass filtered between 0.5 and 200 Hz, digitized with 400 samples/second per channel.

SS-patterns were identified visually in EEG as sequences of 8–14 Hz waves with a minimal duration of 0.4 s. Further, first parts of EEG recordings (about 10% of the data) were analyzed by two experts that verified fragments of EEG associated with sleep spindles. Similar to that SWD-patterns appeared in the EEG as series of high-amplitude spikes and waves of the duration more than 1 s were identified by the same experts. Examples of the corresponding specific oscillatory patterns and their power spectra are given in Fig. 1.

3 Methods

3.1 Wavelet-based analysis of EEG: A general approach

The continuous wavelet-transform is a commonly used technique for time-frequency analysis of complex processes including nonstationary data. Most applications of the

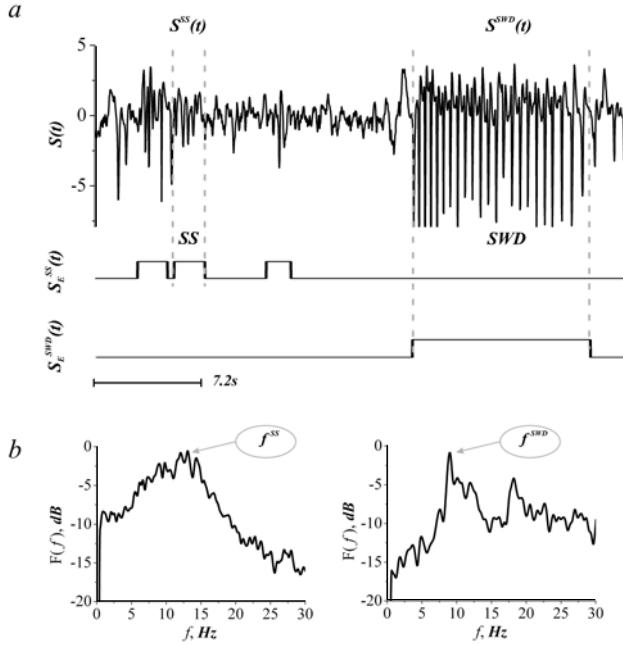


Fig. 1. EEG signal that contains SS and SWD patterns (a) and the corresponding power spectra (b). The identification of the SS and SWD is represented as expert logic signals $S_E^{SS}(t)$ and $S_E^{SWD}(t)$, respectively. f^{SWD} and f^{SS} indicate the main frequencies of the recognized EEG patterns.

CWT for spectral analysis of experimental signals are based on the Morlet-wavelet that represents harmonic oscillations modulated by Gaussian function

$$\psi_\omega(t) = \pi^{-1/4} \exp(j\omega t) \exp\left(-\frac{t^2}{2}\right). \quad (1)$$

The CWT-basis is constructed by scaling and translations of the Morlet-wavelet

$$\psi_{a,\omega,b}(t) = \frac{1}{\sqrt{a}} \psi_\omega\left(\frac{t-b}{a}\right), \quad (2)$$

where a is the scale and b is the translation parameters. Unlike other wavelets, the function (1) possesses an additional parameter ω called as the central frequency that adjusts time-frequency resolution of the Morlet-function. A relation between a and the frequency f of the Fourier-spectrum is given by Eq. (3) [26]

$$f = \frac{\omega + \sqrt{\omega^2 + 2}}{4\pi a}. \quad (3)$$

The CWT of an arbitrary signal $S(t)$ is estimated as follows

$$W^S(a, \omega, b) = \int_0^T S(t) \psi_{a,\omega,b}^*(t) dt \quad (4)$$

with asterisk denoting complex conjugation. An important characteristic of $S(t)$ is based on the instantaneous amplitudes or their averaged values assessed within a

selected range of scales (or frequencies) [22, 23, 27, 28]

$$A^S(t) = \frac{1}{a_{max} - a_{min}} \int_{a_{min}}^{a_{max}} |W^S(a, \omega, t)| da. \quad (5)$$

As an alternative, wavelet energy can be introduced and averaged by analogy with Eq. (5) to quantify the instantaneous wavelet power within a given frequency band. An additional filtering of the function $A^S(t)$ is performed in order to smooth time dependence $\bar{A}^S(t)$ and reduce effects of artifacts. In the simplest case, the corresponding filter Z is a procedure of averaging within a sliding window with the parameters N_S and N_H (the number of averaging and the window size) [29]:

$$\bar{A}^S(t) = Z(A^S(t), N_S, N_H). \quad (6)$$

As a result of patterns identification, a binary (logical) signal $\bar{S}(t)$ is obtained that takes the value “0” at the absence of specific oscillatory structures and the value “1” during time intervals associated with the detected patterns (Fig. 1(a)).

$$\bar{S}(t) = C(\bar{A}^S(t), \Theta_L, \Theta_H) = \begin{cases} 1, & \bar{A}^S(t) \in [\Theta_L; \Theta_H] \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

By analogy with our previous work [30], we consider here two thresholds Θ_L and Θ_H . The value Θ_L provides a separation of SS- and SWD-patterns from the background EEG. The value Θ_H provides selective identification of either SS- or SWD-pattern, since these patterns cannot present in EEG at the same time, and they are characterized by different wavelet energies. The described standard CWT-approach requires manual selection and adjustment of several parameters a_{min} , a_{max} , ω , N_S , N_H , Θ_L , Θ_H according to spectral features of the recognized type of patterns.

The accuracy of pattern identification can be estimated by comparison between the signal $\bar{S}(t)$ obtained within a wavelet-based approach and a binary signal $S_E(t)$ according to expert's visual estimations. An integral measure of accuracy is estimated as follows

$$E(\bar{S}, S_E) = \frac{1}{T} \int_0^T |\bar{S}(t) - S_E(t)| dt. \quad (8)$$

This measure takes the values in the range from 0 ($\bar{S}(t)$ and $S_E(t)$ are identical) to 1 ($\bar{S}(t)$ and $S_E(t)$ are never coincide).

Additional characteristics that quantify the quality of recognition are based on the number of patterns identified visually by an expert (N_E) and considering some numerical technique, e.g., the standard CWT-approach (N_D). The value N_E depends on the signal $S_E(t)$ in the following way

$$\begin{cases} N_E(S_E) = \int_0^T L(S_E(t)) dt \\ L(S_E(t)) = \begin{cases} \frac{dS_E(t)}{dt}, & \frac{dS_E(t)}{dt} > 0 \\ 0, & \text{otherwise} \end{cases} \end{cases} \quad (9)$$

The corresponding dependence for N_D is relatively more complicated and contains two parameters $\alpha_t, \beta_t \in [0; 1]$:

$$\begin{cases} N_D(\alpha_t, \beta_t, \bar{S}, S_E) = \int_0^T \tilde{\Delta}(\alpha_t, \beta_t, \bar{S}(t), S_E(t)) dt \\ \tilde{\Delta}(\alpha_t, \beta_t, \bar{S}(t), S_E(t)) = \begin{cases} \infty, & \Delta(\bar{S}(t), S_E(t)) \in [\alpha_t; \beta_t] \\ 0, & \text{otherwise} \end{cases} \\ y(t) = S_E(t)\bar{S}(t) \\ \Delta(\bar{S}(t), S_E(t)) = \begin{cases} \frac{G(S_E(t), S_E(t)) - G(y(t), S_E(t))}{G(S_E(t), S_E(t))}, & \frac{d^2G(S_E(t), S_E(t))}{dt^2} \rightarrow -\infty \\ 0, & \text{otherwise} \end{cases} \end{cases} \quad (10)$$

$$G(x(t), f(t)) = \begin{cases} \int_{t_0}^t x(t) dt, & f(t) = 1 \wedge f(t_0) = 1 \wedge t - t_0 < const \\ 1, & \text{otherwise} \end{cases}$$

Based on Eqs. (9) and (10), we introduced several measures quantifying accuracy of patterns recognition P , sensitivity RSE and specificity RSP . They were computed using formulae written below for SS-patterns, and the same formulae can be applied to other types of specific oscillatory activity [22]

$$P^{SS} = \frac{TP}{N_E} = \frac{N_D(0.4, 1.0, \bar{S}^{SS}, S_E^{SS})}{N_E(S_E^{SS})}, \quad (11)$$

$$\begin{aligned} RSE^{SS} &= \frac{TP}{TP + FN} = \frac{N_D(0.4, 1.0, \bar{S}^{SS}, S_E^{SS})}{N_D(0.4, 1.0, \bar{S}^{SS}, S_E^{SS}) + N_D(0, 0.2, \bar{S}^{SS}, S_E^{SS})}, \quad (12) \\ RSP^{SS} &= \frac{TN}{N_E} = \frac{N_D(0, 0.4, \bar{S}^{SS}, S_E^{SWD})}{N_E(S_E^{SWD})}, \end{aligned}$$

where TP , TN and FN are the number of true positive, true negative and false negative detections, respectively. In Eqs. (11) and (12), optimal values of the parameters α_t and β_t are indicated.

3.2 An adaptive wavelet-based technique and a serial method

In this work we propose an adaptive wavelet-based method that includes some aspects of the optimization theory [31, 32]. This method assumes that a considered EEG signal $S(t), t \in [0, T]$ can be divided into two parts T_1 and $T_2 (T_1 + T_2 = T, \alpha(T_1) = T_1/T)$. The first part is used to adjust the parameters a and ω of the wavelet-function and the parameters $\Theta_L, \Theta_H, N_S, N_H$ that affect the quality of patterns recognition. The first part T_1 should contain all types of recognized patterns in order to reveal their most essential distinctions from the background EEG. A visual inspection of this part by an expert provides a way to quantify the quality of patterns recognition.

In the adaptive wavelet-based method we consider instantaneous amplitudes associated with parameters a_j, ω_j that are denoted as $A^S(a_j, \omega_j, t)$. Then, using Eqs. (6) and (7), filtration and transition to a binary signal $\bar{S}(t)$ are performed. Further, we use optimization theory to define parameter values for the most effective and accurate recognition of specific EEG patterns. For this purpose, we consider a multi-parametric approach and introduce a series of $a_j, \omega_j, j \in [1; N_B]$ in order to quantify informative parameters of complex organization of EEG patterns. The values $a_j, \omega_j, j \in [1; N_B]$

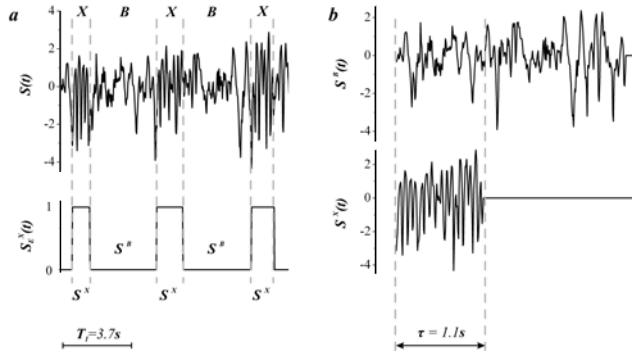


Fig. 2. EEG recording containing only one type of pattern, SS- (a) and the extracted time series formed by background activity $S^B(t)$ and by the detected patterns $S^X(t)$ (b).

are estimated according to objective functions that maximize distinctions between two investigated patterns (SS or SWD) and the background EEG activity. Further, optimization of the remaining parameters ($\Theta_L, \Theta_H, N_S, N_H$) is performed to reduce errors of patterns recognition.

Let us consider an arbitrary pattern on EEG that we further refer to as X-pattern (this may be SS, SWD or another type of patterns detected by an expert in the signal $S(t), t \in [0, T_1]$) – Fig. 2. At this stage, let us also suppose that we deal with a single type of specific oscillatory activity (X-patterns) that should be separated from the background EEG activity (indicated as “B” in Fig. 2).

Aiming to distinguish between $S^X(t)$ and $S^B(t)$ that are associated with X-patterns and the background activity, these fragments of EEG can be extracted and considered as new time series (Fig. 2b). This new time series are further divided into p parts of the duration m ($pm = \tau$, where τ is the length of shorter signal in Fig. 2b).

Instantaneous amplitudes of the signal $S^X(t)$ are averaged across each part to get a series of values $v_k^X(a_j, \omega_j)$ with $k \in [1; p]$. Using the same algorithm a series of averaged instantaneous amplitudes $v_k^B(a_j, \omega_j)$ corresponding to the signal $S^B(t)$ are obtained.

Functions $v_k(a_j, \omega_j)$ of both series are used to compute an objective function $R_d(a_j, \omega_j)$:

$$R_d(a_j, \omega_j) = \frac{\langle v_k^X(a_j, \omega_j) \rangle_k - \langle v_k^B(a_j, \omega_j) \rangle_k}{\sigma(v_k^X(a_j, \omega_j))_k + \sigma(v_k^B(a_j, \omega_j))_k}, \quad (13)$$

where angle brackets denote mean values and σ is the standard deviation.

Another way to compute the objective function does not require decomposition and fragmentation of EEG. An alternative computation of instantaneous amplitudes for signals $S^B(t)$ and $S^X(t)$, as well as R_I objective function is introduced as follows

$$R_I(a_j, \omega_j) = \frac{1}{\tau} \int_0^\tau r_I(a_j, \omega_j, t) db \quad (14)$$

$$r_I(a_j, \omega_j, t) = \begin{cases} \frac{A^X(a_j, \omega_j, t) - A^B(a_j, \omega_j, t)}{A^X(a_j, \omega_j, t)}, & A^X(a_j, \omega_j, t) \geq A^B(a_j, \omega_j, t), \\ \frac{A^X(a_j, \omega_j, t) - A^B(a_j, \omega_j, t)}{A^B(a_j, \omega_j, t)}, & A^X(a_j, \omega_j, t) < A^B(a_j, \omega_j, t), \end{cases}$$

where X and B correspond to S^X and S^B , respectively.

Optimal parameters a_j, ω_j of the Morlet-wavelet are selected according to the values of objective functions given by Eqs. (13) and (14). Optimal values of R_d vary between 1.0 and $+\infty$, but for the function (14) we should consider appropriate values between 0.3 and 1.0 that can be achieved by stochastic optimization based on Monte-Carlo methods or by a simpler method [31, 32] assuming a maximization procedure for R_d or R_I . Selection of the remaining parameters ($N_S, N_H, \Theta_L, \Theta_H$) can also be performed to improve the quality of patterns recognition.

At the first stage, fixed values n_S^j and n_H^j of the parameters N_S, N_H are considered, and the parameters Θ_L, Θ_H are optimized according to the minimum of $E(\bar{S}^X, S_E^X)$ or using a general objective function

$$R_\Theta(\Theta_L, \Theta_H, n_S^j, n_H^j) = \frac{1}{2}(1 - E(\bar{S}^X, S_E^X) + P(\bar{S}^X, S_E^X)). \quad (15)$$

At the fixed values n_S^j and n_H^j , optimal thresholds θ_L^j, θ_H^j are defined according to the maximum of (15). Further, the next pair of the parameters N_S, N_H is selected (n_S^{j+1}, n_H^{j+1}), etc. Parameters can be changed step-by-step or stochastically. Optimal settings are determined as

$$\exists k : P(\bar{S}^X(t, \theta_L^k, \theta_H^k, n_S^k, n_H^k), S_E^X) = \max, \quad t \in [0; T_1]. \quad (16)$$

The proposed serial adaptive method assumes a sequential detection of all specific oscillatory patterns that should be recognized in the structure of EEG. Automatic recognition procedure can be performed starting from patterns with the highest energies. In this method, visual inspection of the first part of EEG signal (T_1) has been done by an expert, and the signals $S_E^{X_j}$ are obtained for each type of specific oscillatory activity. Further, all types of patterns are reordered according to the decreasing spectral power.

For the validation of the proposed serial method (“adaptation” stage), recognition starts from a pattern X_0 characterized by the highest energy. The considered technique is applied to optimize the parameters (a_k, ω_k) , $k \in [1; N_B] |_{X_0}$ and, further, the parameters $(\Theta_L, \Theta_H, N_S, N_H)$. This procedure is repeated for all remaining types of patterns.

For the full processing mode (“working state”) the serial method assumes that the adapted CWT-algorithm is applied to the whole EEG data and aims to reveal the patterns X_j at the same order j as during the previous stage. In this work we basically concentrate on two types of oscillatory patterns (SS- and SWD-patterns).

4 Results

In order to compare accuracy of the standard CWT-based method and the proposed serial technique, we considered EEG signals contained both types of the discussed patterns (SS and SWD). At the first stage, the standard CWT-based method was applied with empirically selected parameters. The range of scales in Eq. (5) was chosen as $\pm 10\%$ of the scale associated with the spectral maximum, $\omega = 2\pi$. Table 1 summarizes the obtained results.

Application of the proposed serial adaptive method with the objective function R_d improves recognition abilities of wavelets (Table 2). The accuracy P is increased from 71,4% to 87,6% in SS-patterns and from 94,1% to 98,3% in SWD-patterns. Other measures such as the error E , the sensitivity RSE and the specificity RSP , confirm a higher efficiency of the adaptive technique. Similar results are achieved for the objective function R_I (Table 3).

Table 1. Results of the standard CWT-based approach applied to EEG recorded in 6 rats.

rat	$\alpha(T_1)$	E, %		P, %		RSE, %		RSP, %	
		SS	SWD	SS	SWD	SS	SWD	SS	SWD
1	0,0	18,8	1,6	71,6	100,0	78,0	100,0	100,0	95,0
2	0,0	9,3	0,8	92,9	100,0	95,5	100,0	66,7	97,3
3	0,0	9,8	1,6	76,0	97,3	83,3	97,3	100,0	97,1
4	0,0	4,8	2,3	57,4	67,9	64,2	95,0	96,4	72,5
5	0,0	19,9	9,2	58,3	100,0	61,1	100,0	87,1	47,9
6	0,0	12,4	1,3	72,0	100,0	78,3	100,0	87,5	95,6

Table 2. Results of the adaptive CWT-based approach with R_d -function for $N_B = 100$.

rat	$\alpha(T_1)$	E, %		P, %		RSE, %		RSP, %	
		SS	SWD	SS	SWD	SS	SWD	SS	SWD
1	13,4	8,0	0,3	80,4	100,0	83,4	100,0	100,0	100,0
2	9,0	7,0	0,4	87,5	100,0	90,2	100,0	100,0	98,3
3	6,0	7,5	1,3	88,5	100,0	92,7	100,0	94,6	99,8
4	10,8	3,2	0,6	91,7	92,3	96,5	92,9	92,9	98,4
5	18,2	8,3	0,5	91,4	96,8	93,7	96,8	93,6	98,0
6	10,0	10,3	1,1	85,9	100,0	87,9	100,0	93,8	98,4

Table 3. Results of the adaptive CWT-based approach with R_I -function for $N_B = 100$.

rat	$\alpha(T_1)$	E, %		P, %		RSE, %		RSP, %	
		SS	SWD	SS	SWD	SS	SWD	SS	SWD
1	13,4	9,0	0,3	85,1	100,0	90,1	100,0	100,0	100,0
2	9,0	8,4	0,3	86,8	100,0	91,5	100,0	100,0	99,0
3	6,0	9,5	2,1	92,3	78,4	95,5	90,6	81,1	99,8
4	10,8	4,9	1,2	80,0	85,7	85,1	100,0	96,4	95,6
5	18,2	8,0	0,5	87,3	96,8	91,0	96,8	93,5	98,0
6	10,0	10,4	0,8	83,3	87,5	85,7	100,0	87,5	99,6

Figure 3 illustrates averaged characteristics obtained within the proposed serial adaptive technique for SS-patterns that confirm an improvement of recognition abilities using this approach. The presented results demonstrate that a more effective recognition of EEG patterns can be achieved with the adaptive method as compared to the previously used method with the standard CWT-based approach. Objective function R_d provided more effective identifications of the examined patterns in EEG. In this case, the error E is equal to 7.38% in SS-pattern and 0.69% in SWD-pattern, respectively. This confirms that the difference between the signals $\bar{S}(t)$ and $S_E(t)$ is reduced. Numerical values of $E(\bar{S}, S_E)$ are controlled by appropriate selections of the thresholds Θ_L, Θ_H .

The quality of patterns recognition obviously depends on the duration of the initial time interval T_1 used by an expert for providing a visual identification of specific oscillatory structures. This interval should be long enough to adjust algorithm's parameters, which will be used for detection of patterns in the whole EEG recordings and for further identification of similar oscillatory phenomena in the other time series. Time interval T_1 should contain enough statistics of patterns that are necessary to assess the most informative features. The minimal length of T_1 used in this study is about 10% of the whole EEG recordings that corresponds to about 400 SS-patterns. A further increase in T_1 improves the recognition. Thus, a consideration of about 20% of the entire EEG durations increases the accuracy P by about 1–2%. However, this requires twice longer durations of data preprocessing by an expert.

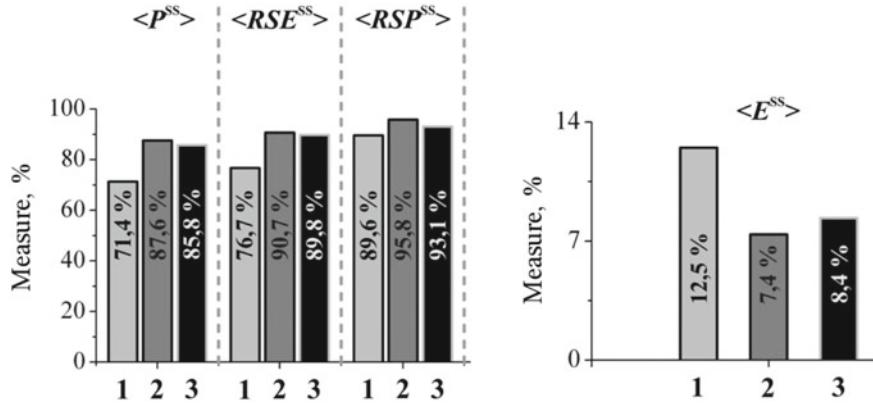


Fig. 3. Mean values of all estimated characteristics used for the automatic identification of SS-patterns. The values 1, 2, 3 correspond to the standard CWT-approach and to the proposed adaptive technique with the objective functions R_d and R_I , respectively.

5 Conclusions

In this work we introduce the adaptive wavelet-based method for recognition of specific oscillatory patterns in EEG, such as sleep spindles and spike-wave discharges. An important feature of this method is that all parameters affecting the quality of pattern recognition are automatically selected based on the optimization theory. These parameters are selected by means of the maximization of objective functions that reflects the most essential distinctions between the given patterns and the background EEG activity. Appropriate adjustment of all parameters enables us to increase the quality of EEG data preprocessing. Full-range tests in native EEG demonstrate that adaptive approach provided a significantly higher accuracy of identification for both, SS and SWD patterns. Sensitivity and specificity of the adaptive method are higher than in the standard wavelet-based technique.

Wavelet-analysis represents a “mathematical microscope” and it is able to retrieve characteristic structures in the signal only if its “optical properties” are appropriately adjusted. In the opposite case, efficiency of wavelet-based techniques may be lower than in simpler methods [33]. Inappropriate selection of parameters is the critical aspect that was recently discussed when solving the related problem of spike sorting [21]. The proposed serial adaptive method enables us to realize optimal tuning of all parameters in an automatic regime. The latter means that the recognition process is no longer dependent on subjective factor, such as the experience of a researcher that performs data processing. This method can be effectively used in a broad range of pattern recognition problems in physics, medicine, biology, etc.

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