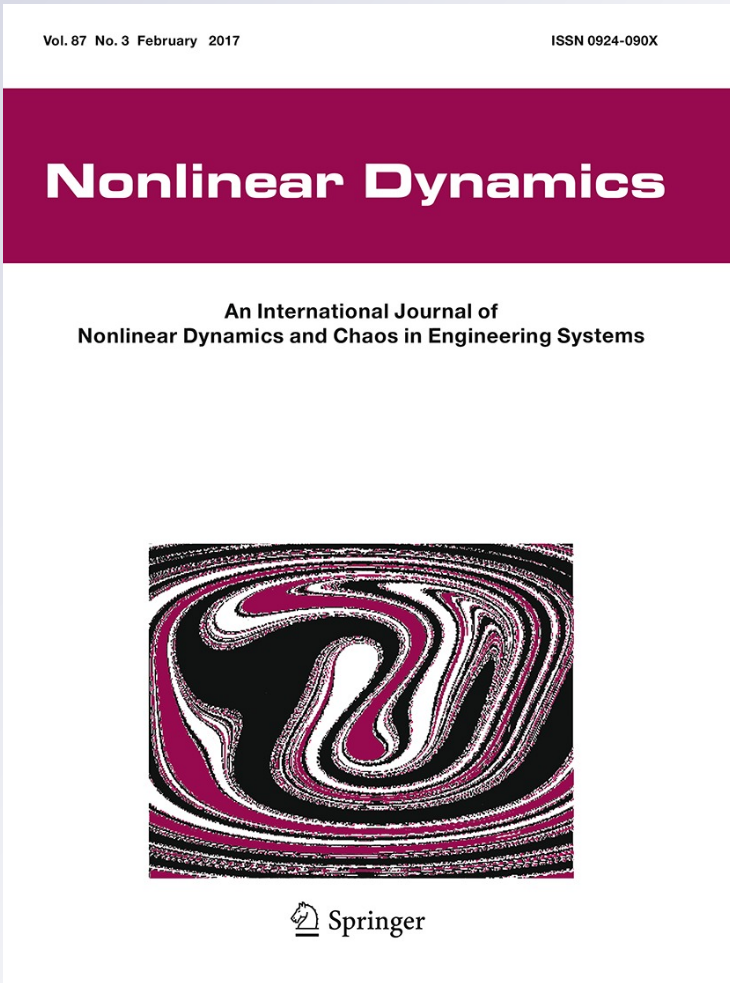
Resistant to noise chaotic communication scheme exploiting the regime of generalized synchronization

M. D. Prokhorov, V. I. Ponomarenko, D. D. Kulminskiy, A. A. Koronovskii, O. I. Moskalenko & A. E. Hramov

Nonlinear Dynamics
An International Journal of Nonlinear Dynamics and Chaos in Engineering Systems

ISSN 0924-090X
Volume 87
Number 3

Nonlinear Dyn (2017) 87:2039-2050
DOI 10.1007/s11071-016-3174-6



Springer

Nonlinear Dyn (2017) 87:2039–2050
DOI 10.1007/s11071-016-3174-6

ORIGINAL PAPER

Resistant to noise chaotic communication scheme exploiting the regime of generalized synchronization

M. D. Prokhorov · **V. I. Ponomarenko** · **D. D. Kulminskiy** · **A. A. Koronovskii** · **O. I. Moskalenko** · **A. E. Hramov**

Received: 14 June 2016 / Accepted: 27 October 2016 / Published online: 10 November 2016

Abstract We develop a chaotic communication system exploiting the regime of generalized synchronization between the transmitter and receiver. Two variants of communication scheme are considered. The first of them contains only one self-oscillating response system in the receiver, which is driven in turn by the signal from the drive system and delayed copy of this signal. In the second variant of the scheme, the receiver is composed of two response systems, but these systems could be nonidentical in contrast to the classical communication schemes based on the generalized synchronization. The efficiency of the proposed scheme is shown for the case, where the transmitter and receiver are constructed using time-delayed feedback oscillators. The communication scheme is studied numerically and realized in the physical experiment. It is shown that the proposed scheme possesses high tolerance to noise in the communication channel.

M. D. Prokhorov (✉) · V. I. Ponomarenko · D. D. Kulminskiy · A. A. Koronovskii · O. I. Moskalenko · A. E. Hramov
Saratov Branch of Kotel'nikov Institute of Radio Engineering and Electronics of Russian Academy of Sciences, Zelyonaya Street, 38, Saratov, Russia 410019
e-mail: mdprokhorov@yandex.ru

V. I. Ponomarenko · D. D. Kulminskiy · A. A. Koronovskii · O. I. Moskalenko · A. E. Hramov
Saratov State University, Astrakhanskaya Street, 83, Saratov, Russia 410012

A. E. Hramov
Yuri Gagarin State Technical University of Saratov, Politekhnicheskaya Street, 77, Saratov, Russia 410056

Keywords Chaotic communication · Generalized synchronization · Time-delay systems · Signal processing

1 Introduction

Development of communication schemes based on employment of various types of chaotic synchronization has been an active area of research since the early 1990s. A lot of communication schemes based on chaotic synchronization have been proposed [1–19]. The most of these schemes exploit the regime of identical synchronization between the oscillators in transmitter and receiver. It should be noted that identity of the receiver and transmitter parameters is of crucial importance in communication schemes based on the identical synchronization of chaotic systems. With the increase in mismatch of the receiver and transmitter parameters, the quality of chaotic synchronous response of the receiver deteriorates leading to a worse quality of the information signal extraction. However, the construction of identical oscillators in the experimental scheme is a difficult problem. Besides, chaotic communication schemes based on the identical synchronization have low resistance to noise and amplitude distortions of the signal in a communication channel [11].

The high tolerance to noise and signal distortions can be achieved in chaotic communication systems exploiting the regime of generalized synchronization [20,21]. Moreover, such communication schemes do not require the identity of the transmitter and receiver. The generalized synchronization can occur between two unidirectionally coupled chaotic systems described as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{F}[\mathbf{x}(t)], \quad (1)$$

$$\dot{\mathbf{y}}(t) = \mathbf{G}[\mathbf{y}(t), \mathbf{h}(\mathbf{x}(t))], \quad (2)$$

where $\mathbf{x}(t)$ is a drive system and $\mathbf{y}(t)$ is a response system, $\mathbf{x}(t) \in R^n$ and $\mathbf{y}(t) \in R^m$ [22]. $\mathbf{F}$ and $\mathbf{G}$ define the vector fields of the drive and response systems, respectively, and a function $\mathbf{h}\,(\mathbf{x}(t))$ characterizes the driving of the response system by the system $\mathbf{x}(t)$.

The generalized synchronization between the chaotic oscillations in the systems $\mathbf{x}(t)$ and $\mathbf{y}(t)$ means the presence of a transformation $\mathbf{H}$: $R^n \rightarrow R^m$ that takes the trajectories of the attractor in R^n space into the trajectories of the attractor in the R^m space, so that $\mathbf{y}(t) = \mathbf{H}[\mathbf{x}(t)]$. The properties of this transformation do not depend upon the initial conditions in the basin of attraction of the synchronized attractor [23]. The existence of a transformation $\mathbf{H}$ guarantees the ability to predict the state of the response system from measurements of $\mathbf{x}$(t) alone, once transients die out. The case when $\mathbf{H}$ is the identity transformation corresponds to the identical synchronization between $\mathbf{x}(t)$ and $\mathbf{y}(t)$.

To detect the regime of generalized synchronization, the following methods have been proposed: the nearest neighbor method [23], the conditional Lyapunov exponent calculation [24], the auxiliary system approach [25], the information-theoretic approach based on the coarse-grained information rate [26], the approaches based on the generalized angle calculation [27], the phase tube approach [28], the statistical modeling approach [29], the method based on dynamical Bayesian inference [30], and some other techniques [31]. Among these methods, the auxiliary system approach is the most popular one because it is rather simple and can be used in real time in distinction to other methods that require the recording of signals for further processing.

The auxiliary system approach is based on employment of the auxiliary system $\mathbf{v}(t)$ which is identical to the response system $\mathbf{y}(t)$. The initial conditions for $\mathbf{v}(t)$ are chosen to be different from those for $\mathbf{y}(t)$, but in the same basin of attraction as the initial conditions of $\mathbf{y}(t)$. If after termination of the transient process, the states of the response and auxiliary systems become identical, i.e., $\mathbf{y}(t) \equiv \mathbf{v}(t)$, then the generalized synchronization between $\mathbf{x}(t)$ and $\mathbf{y}(t)$ takes place.

Although the known communication schemes exploiting the generalized synchronization do not require the identity of oscillators in the transmitter and receiver, they need two identical oscillators (response and auxiliary ones) in the receiver, which creation is often a difficult technical problem especially in the case of high-frequency oscillators.

In the present paper, we propose a new resistant to noise chaotic communication scheme based on the generalized synchronization. The main novelty of this scheme, compared with the other communication schemes based on the generalized synchronization, is that our scheme does not require the identity of oscillators in the receiver. Two versions of the scheme are developed. In the first of them, the receiver contains only one oscillator that removes the problem of ensuring the identity of two oscillators in the receiver. In the second scheme, the receiver is composed of two response systems, but these systems could be nonidentical in contrast to the classical communication schemes based on the generalized synchronization. The proposed communication scheme is studied numerically and implemented in a real physical experiment using electronic ring oscillators with time-delayed feedback as the drive and response systems. It should be noted that in our paper we present the first experimental implementation of communication system based on the generalized synchronization. In the other papers, various communication schemes exploiting the generalized synchronization were studied only numerically.

The paper is organized as follows. In Sect. 2, a new chaotic communication system based on the generalized synchronization is proposed. Section 3 illustrates the results of numerical simulation of the proposed communication scheme. In Sect. 4, we consider the operation of the first experimental communication scheme based on the generalized synchronization. In Sect. 5, we summarize our results.

2 Communication scheme

A block diagram of the developed communication scheme exploiting the regime of generalized synchronization between the transmitter and receiver is shown in Fig. 1. A transmitter contains a drive system $\mathbf{x}(t)$ which parameters are modulated by the binary information signal $m(t)$. The chaotic signal from the trans-

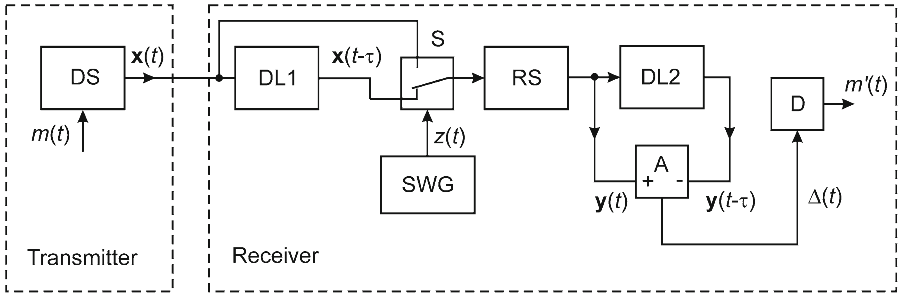


Fig. 1 Block diagram of a communication system based on the generalized synchronization: (DS) drive system, (RS) response system, (DL1 and DL2) delay lines, (SWG) square-wave generator, (S) commutator, (A) difference amplifier, and (D) detector

mitter output is transmitted into the communication channel. A receiver is composed of a self-oscillating response system, two identical delay lines with the delay time τ, square-wave generator, commutator, difference amplifier, and detector. The scheme parameters are chosen in such a way that the generalized synchronization between the transmitter and receiver is present at a transmission of binary 0 and absent at a transmission of binary 1. It is possible to choose the parameters in another way, where the transmission of binary 1 and 0 corresponds to the presence and absence of generalized synchronization, respectively.

For detecting the regime of generalized synchronization, we do not exploit an auxiliary system in the receiver as it is usually done in communication systems based on the generalized synchronization [20,21]. Instead of this, a single response system is driven in turn by the signal from the drive system and delayed copy of this signal. Similar approach has been used in [32] for studying consistency properties of a semiconductor laser in response to a coherent optical drive originating from delayed feedback.

In the absence of generalized synchronization, the response system shows different oscillations under the driving by the same signal. However, in the presence of generalized synchronization, the response system exhibits after the transient process identical oscillations in both cases of driving by the same signal.

To drive the response system two times by the same signal, we use the delay line DL1, switch, and square-wave generator (Fig. 1). The signal $z(t)$ of the square-wave generator controls a switch whose action changes the signal driving the response system. A half of period of $z(t)$, the response system is driven by the signal $\mathbf{x}(t)$ incoming from the communication channel. Another half of period of $z(t)$, the response system is driven by the signal $\mathbf{x}(t-\tau)$ incoming from the output of the delay line DL1. The period T of $z(t)$ is chosen so as to ensure that the transient process preceding the regime of generalized synchronization terminates at a time less than $T/2$. The delay time of DL1 is equal to $\tau = T/2$. Each bit of the information signal $m(t)$ is transmitted during the time interval $l = T$.

To compare the signals of the response system in the cases of its driving by $\mathbf{x}(t)$ and $\mathbf{x}(t-\tau)$, we use the second delay line DL2, having the same delay time τ as the delay line DL1, and difference amplifier. In the presence of generalized synchronization between the drive and response systems, the signal $\mathbf{\Delta}(t) = \mathbf{y}(t) - \mathbf{y}(t-\tau)$ at the difference amplifier output vanishes after the transient process in the second half of period of $z(t)$. In the absence of generalized synchronization, the signal $\mathbf{\Delta}(t)$ shows nonvanishing oscillations within the entire second half of period of $z(t)$.

During the first half of period of $z(t)$, the signal $\mathbf{\Delta}(t)$ always shows nonvanishing oscillations that are similar to those observed in the second half of period of $z(t)$ in the case of the absence of generalized synchronization between $\mathbf{x}(t)$ and $\mathbf{y}(t)$. Thus, during this time interval, $\mathbf{\Delta}(t)$ contains no useful information for the detection of generalized synchronization.

For extracting the information signal, we use the detector in the receiver. This detector evaluates the variance σ^2 of incoming difference signal $\mathbf{\Delta}(t)$ during the time interval $T/4$ and compares it with a threshold σ_c^2. At the detector output, we have a recovered information signal $m'(t)$ which is binary zero for $\sigma^2 < \sigma_c^2$ and binary unity for $\sigma^2 \geq \sigma_c^2$.

Since the response system is driven twice by the same signal in order to detect the regime of generalized synchronization and define, which bit is transmitted, the half of time the considered communication scheme is staying in a waiting mode. To eliminate this shortcoming and to increase the rate of data transmission in two times, we modified the scheme depicted in Fig. 1 by adding in the receiver the second response system

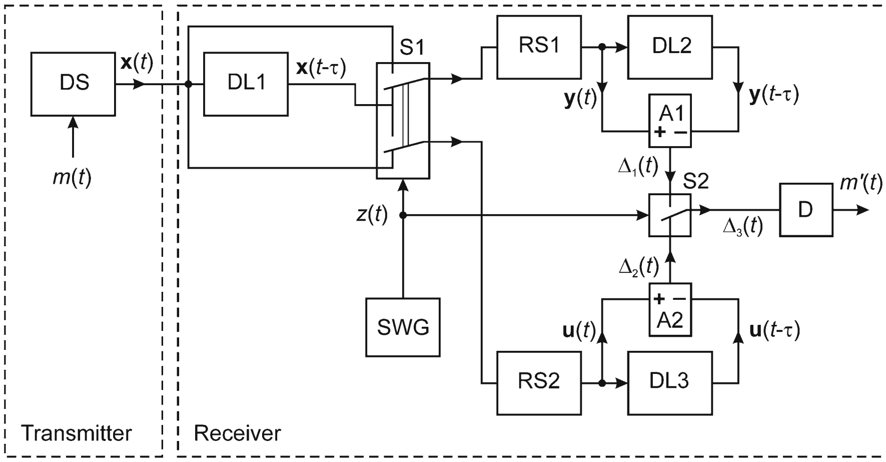


Fig. 2 Block diagram of a modified communication system based on the generalized synchronization: (DS) drive system, (RS1 and RS2) response systems, (DL1, DL2, and DL3) delay lines, (SWG) square-wave generator, (S1 and S2) commutators, (A1 and A2) difference amplifiers, and (D) detector

RS2, which operates in antiphase with the first response system, the delay line DL3 having the delay time τ, one more difference amplifier, and one more commutator.

A block diagram of the modified communication scheme is presented in Fig. 2. The signal $z(t)$ controls the commutator S1 which switches the driving signal in such a way that the first response system $\mathbf{y}(t)$ is driven by the signal $\mathbf{x}(t)$, while the second response system $\mathbf{u}(t)$ is driven by the signal $\mathbf{x}(t-\tau)$ and vice versa, $\mathbf{y}(t)$ is driven by the signal $\mathbf{x}(t-\tau)$, while $\mathbf{u}(t)$ is driven by the signal $\mathbf{x}(t)$. As the result, the first response system detects the transmitted binary signal as the second response system stays in the waiting mode. Then, the first response system switches to the waiting mode, and the transmitted signal is recovered by the second response system.

We denote the signals at the output of difference amplifiers in the first and second response systems as $\boldsymbol{\Delta}_1(t) = \mathbf{y}(t) - \mathbf{y}(t-\tau)$ and $\boldsymbol{\Delta}_2(t) = \mathbf{u}(t) - \mathbf{u}(t-\tau)$, respectively. The commutator S2 is controlled by the signal $z(t)$ in such a way that the signal $\boldsymbol{\Delta}_3(t)$ at the detector input represents the alternating fragments of the signals $\boldsymbol{\Delta}_1(t)$ and $\boldsymbol{\Delta}_2(t)$ with the duration of each fragment being equal to $T/2$. In the considered scheme, each bit of the information signal $m(t)$ is transmitted during the time interval $l = T/2$. Thus, the scheme operates twice as quicker in comparison with the scheme depicted in Fig. 1.

It should be noted that although the scheme in Fig. 2 contains two response systems in the receiver, these systems could be nonidentical in contrast to the classical communication schemes based on the generalized synchronization, which exploit the auxiliary system that is identical to the response system. In our scheme, the identity of systems $\mathbf{y}(t)$ and $\mathbf{u}(t)$ in the receiver is not necessary, because for detecting the regime of generalized synchronization we analyze not the difference between the signal from $\mathbf{y}(t)$ and the signal from $\mathbf{u}(t)$, but the differences of signals separately in the system $\mathbf{y}(t)$ and the system $\mathbf{u}(t)$ under the driving of each of these systems two times by the same signal.

3 Numerical simulation of communication system based on the generalized synchronization

We illustrate the operation of the proposed communication system based on the generalized synchronization for the case, where the drive and response systems represent time-delayed feedback oscillators. In the presence of time delays in coupled oscillators, the synchronization between these oscillators has some specific features including such phenomena as anticipating synchronization, lag synchronization, and a variety of transitions between different types of synchronization [33–38]. The delays play also an important role in the synchronization of various neuron systems [39–43] which can be considered as data transmission systems.

A block diagram of the drive system composed of two delay lines DL4 and DL5 with delay times τ_1 and τ_2, respectively, commutator, nonlinear element, and linear low-pass filter is shown in Fig. 3a. The binary information signal $m(t)$ switches the delay time in the system in such a way that the delay time is equal to τ_1 at a transmission of binary 0, and it is equal to $\tau_1 + \tau_2$ at a transmission of binary 1. The drive system is described by a first-order delay-differential equation

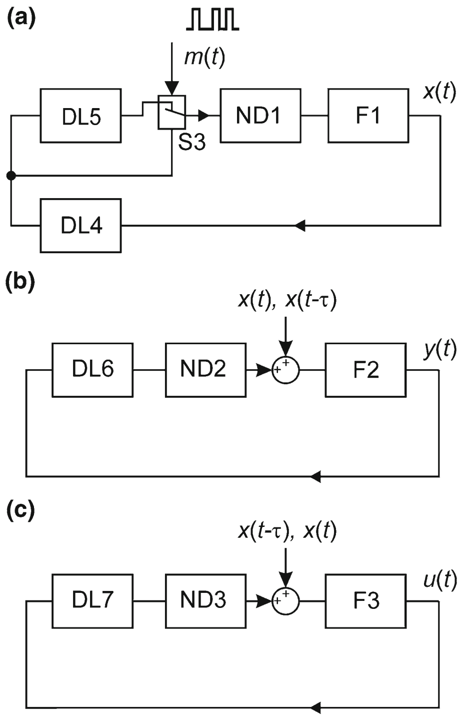


Fig. 3 Block diagrams of the drive system (**a**), first response system (**b**), and second response system (**c**): (DL4, DL5, DL6, and DL7) delay lines, (ND1, ND2, and ND3) nonlinear devices, (F1, F2, and F3) filters, and (S3) commutator

$$\varepsilon_1 \dot{x}(t) = -x(t) + f_1\left(x\left(t - (\tau_1 + m(t)\tau_2)\right)\right), \tag{3}$$

where $x(t)$ is the system state at time t, ε_1 is the parameter that characterizes the inertial properties of the system, and f_1 is a nonlinear function. The signal $x(t)$ from the filter output is transmitted into the communication channel. It should be noted that the transmitter signals must have similar spectral and statistical properties at τ_1 and $\tau_1 + \tau_2$ in order to ensure a security of message transmission.

Block diagrams of self-oscillating response systems composed of a delay line, nonlinear element, summator, and linear low-pass filter are shown in Fig. 3b, c. The delay lines DL6 and DL7 have the delay time τ_3 and τ_4, respectively. The nonlinear elements ND2 and ND3 and filters F2 and F3 in general case are different from the nonlinear element ND1 and filter F1, respectively, in the drive system. The response systems are driven in turn by the signals $x(t)$ and $x(t - \tau)$. They are described by the following equations:

$$\begin{aligned}\varepsilon_2 \dot{y}(t) &= -y(t) + f_2(y(t - \tau_3)) \\ &\quad + k(z(t)x(t) + \overline{z(t)}x(t - \tau)),\end{aligned} \tag{4}$$

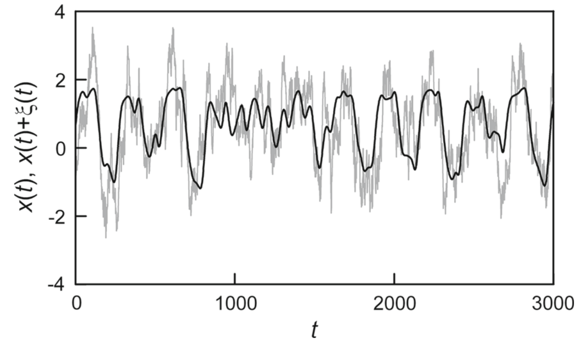


Fig. 4 Time series of the driving chaotic signal $x(t)$ in the absence of noise (*black color*) and in the presence of strong additive noise (*gray color*)

$$\begin{aligned}\varepsilon_3 \dot{u}(t) &= -u(t) + f_3(u(t - \tau_4)) \\ &\quad + k(z(t)x(t - \tau) + \overline{z(t)}x(t)),\end{aligned} \tag{5}$$

where ε_2 and ε_3 are the parameters of inertia, f_2 and f_3 are nonlinear functions, k is the parameter that characterizes the strength of the unidirectional coupling, and $\overline{z(t)}$ is the inversion of the signal $z(t)$ of the square-wave generator. At $z(t) = 1$, we have $\overline{z(t)} = 0$ and at $z(t) = 0$ we have $\overline{z(t)} = 1$.

At first, we consider the operation of the first scheme (Fig. 1) containing only one response system in the receiver. Let us consider the case, where the nonlinear elements ND1 and ND2 provide a quadratic transformation and the filters F1 and F2 are the low-pass first-order Butterworth filters with the cutoff frequencies $\nu_1 = 1/\varepsilon_1$ and $\nu_2 = 1/\varepsilon_2$, respectively. We choose the following values of the transmitter and receiver parameters: $\tau_1 = 110$, $\tau_2 = 10$, $\tau_3 = 100$, $\varepsilon_1 = 20$, $\varepsilon_2 = 25$, $f_1(x) = \lambda_1 - x^2$, $f_2(y) = \lambda_2 - y^2$, where $\lambda_1 = 1.8$ and $\lambda_2 = 1.3$ are the parameters of nonlinearity, $k = 0.07$, and $\tau = 20{,}000$ ($T = 40{,}000$). With these parameters, the transmitter generates a chaotic signal $x(t)$ (Fig. 4) and the receiver in the absence of coupling ($k = 0$) oscillates in a periodic regime. It was shown that in a communication system based on the generalized synchronization, the employment of periodic regime in the response system has an advantage over employment of chaotic regime [44]. Note that in our scheme, the transmitter and receiver have a mismatch of all parameters.

To investigate the tolerance of the proposed communication scheme to noise, we added a zero-mean Gaussian noise $\xi(t)$ filtered in the bandwidth of the chaotic carrier to time series of the signal $x(t)$ trans-

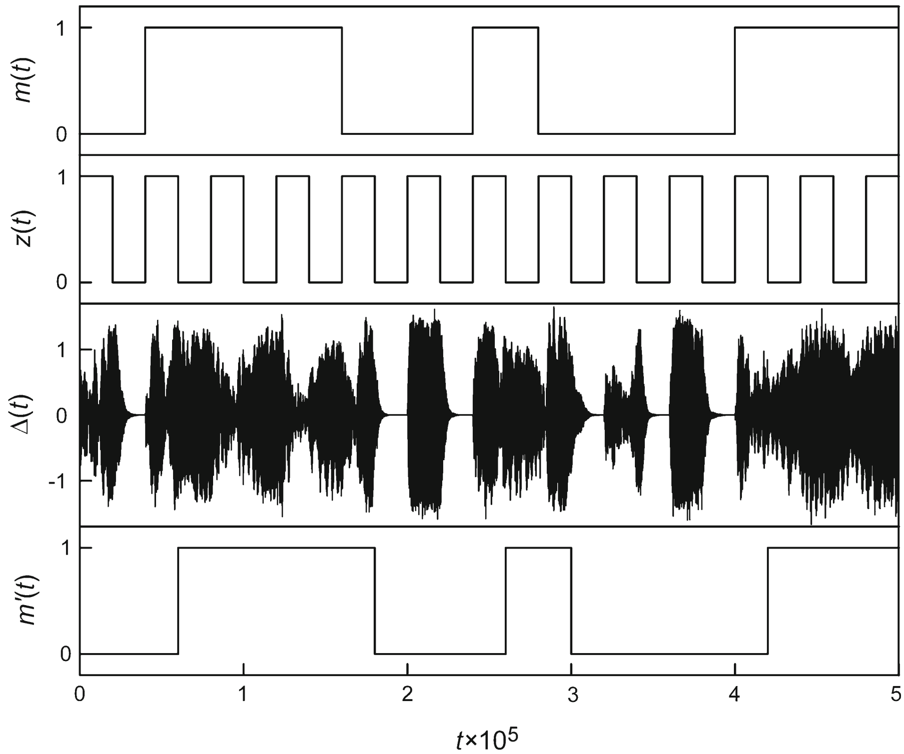


Fig. 5 Time series of the information signal $m(t)$, signal $z(t)$ of the square-wave generator, difference signal $\Delta(t)$, and signal $m'(t)$ extracted in the receiver

mitted into the communication channel. Figure 4 shows in gray color a part of the time series of $x(t)$ corrupted with noise for the case, where the variance of $\xi(t)$ is equal to the variance of $x(t)$ at the output of the drive system. The signal-to-noise ratio (SNR) in this case is equal to 0 dB.

Figure 5 depicts parts of the time series of the signals $m(t)$, $z(t)$, $\Delta(t)$, and $m'(t)$ for the case of SNR $=$ 0 dB. At a transmission of binary 0 ($m(t) = 0$), the difference $\Delta(t) = y(t) - y(t-\tau)$ oscillates at $z(t) = 1$ and vanishes after the transient process at $z(t) = 0$, indicating the presence of generalized synchronization between the drive and response systems. For $m(t) = 1$, the signal $\Delta(t)$ shows nonvanishing oscillations both at $z(t) = 1$ and $z(t) = 0$ indicating the absence of generalized synchronization between $x(t)$ and $y(t)$.

The threshold value of variance of the signal $\Delta(t)$, which is used for extracting the information signal $m'(t)$ in the detector, is chosen as $\sigma_c^2 = 0.001$. As shown in Fig. 5, the information signal is recovered accurately, but the signal $m'(t)$ at the receiver output is shifted by the time $T/2$ with respect to the signal $m(t)$ in the transmitter. This shift is explained by the algorithm of detector operation, which evaluates the variance of the signal $\Delta(t)$ during the time interval $T/4$ corresponding to the second half of time interval with constant value of $z(t)$. In the presence of generalized synchronization between $x(t)$ and $y(t)$, the signal $\Delta(t)$ is very small during this time interval and its variance $\sigma^2 < \sigma_c^2$, since the transient process already finished at the first half of time interval where $z(t) = 0$. In the absence of generalized synchronization, the variance of the signal $\Delta(t)$ is always great. Thus, the proposed scheme is efficient in spite of very high level of noise in the communication channel.

Let us consider now the operation of the second scheme (Fig. 2) containing two response systems in the receiver. We choose the same parameter values for the transmitter and receiver as in the considered above example. The parameters of the second response system are chosen as follows: $\tau_4 = 101$, $\varepsilon_3 = 25$, $f_3(u) = \lambda_3 - u^2$, where $\lambda_3 = 1.3$. It should be noted that the delay time τ_4 in the second response system differs from the delay time τ_3 in the first response system.

Figure 6 shows parts of the time series of the signals $m(t)$, $z(t)$, $\Delta_1(t)$, $\Delta_2(t)$, $\Delta_3(t)$, and $m'(t)$ for the case of SNR $=$ 0 dB. At $m(t) = 1$, the both signals, $\Delta_1(t)$

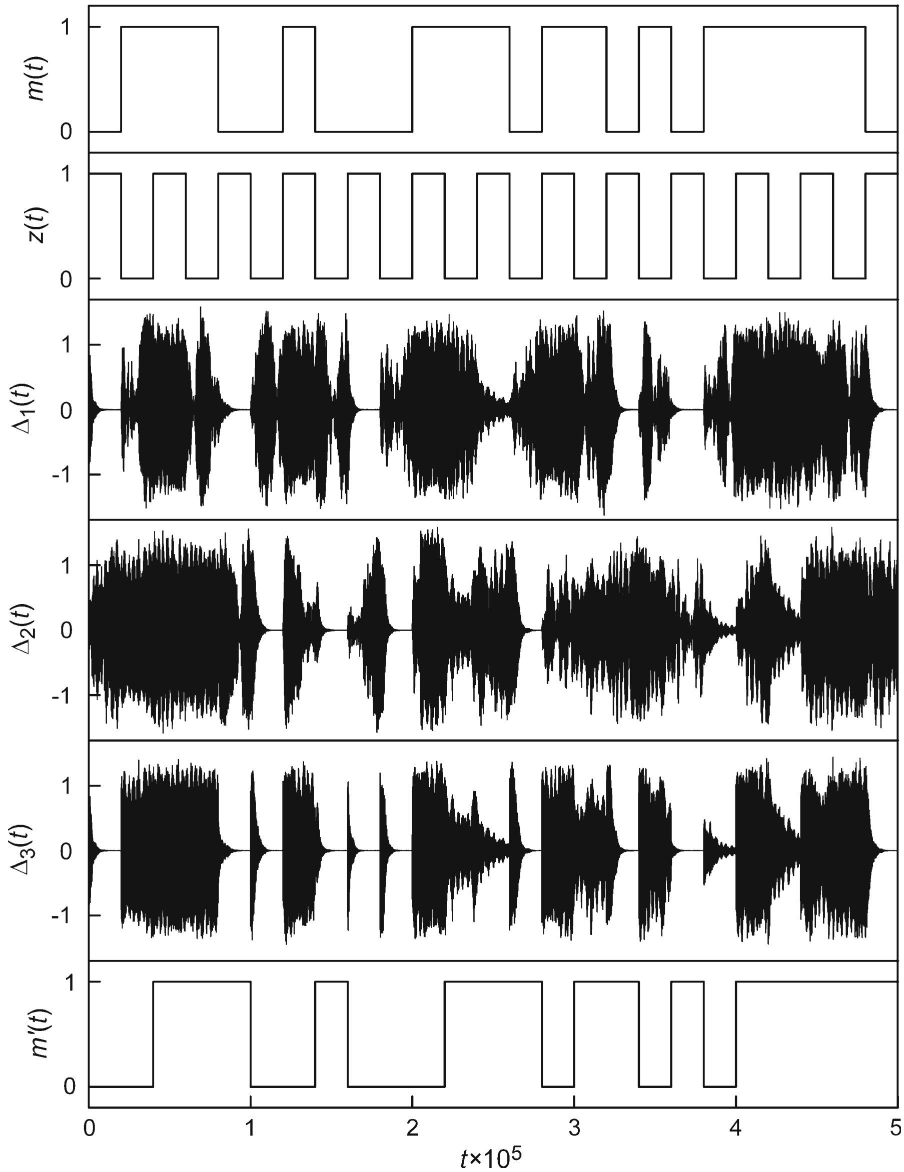


Fig. 6 Time series of the information signal $m(t)$, signal $z(t)$ of the square-wave generator, signals $\Delta_1(t)$, $\Delta_2(t)$, and $\Delta_3(t)$, and signal $m'(t)$ extracted in the receiver

and $\Delta_2(t)$, show similar nonvanishing oscillations both at $z(t) = 1$ and $z(t) = 0$ indicating the absence of generalized synchronization between $x(t)$ and $y(t)$ and between $x(t)$ and $u(t)$, respectively. Therefore, at a transmission of binary 1, the signal $\Delta_3(t)$ also does not vanish and has a great variance.

At a transmission of binary 0, the signal $\Delta_1(t)$ shows damping oscillations at $z(t) = 1$ that vanish in the second half of time interval where $z(t) = 1$. The second response system stays in a waiting mode at $z(t) = 1$ and the signal $\Delta_2(t)$ shows nonvanishing oscillations. At $m(t) = 0$ and $z(t) = 0$, on the contrary, the signal $\Delta_2(t)$ shows damping oscillations that vanish in the second half of time interval where $z(t) = 0$ while the first response system stays in a waiting mode and its signal $\Delta_1(t)$ shows nonvanishing oscillations. The signal $\Delta_3(t)$ is composed of the alternating fragments of the signals $\Delta_1(t)$ and $\Delta_2(t)$ with the duration of each fragment being equal to $T/2$. At the detector output, we have the recovered information signal $m'(t)$, which is binary zero, if the signal $\Delta_3(t)$ variance $\sigma^2 < \sigma_c^2$, and binary unity, if $\sigma^2 \geq \sigma_c^2$, where $\sigma_c^2 = 0.001$.

It is known that many chaotic communication schemes are not as secure as expected and can be suc-

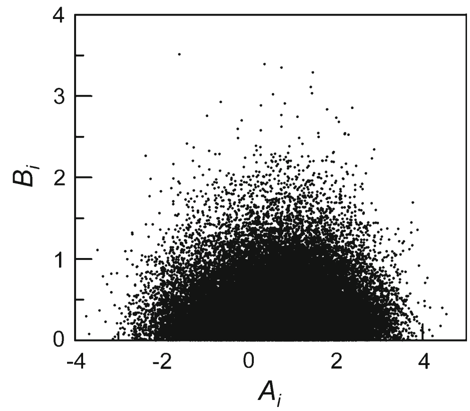


Fig. 7 Return map for the model communication system with one response system in the receiver

cessfully unmasked, for example by using the analysis of return maps [45,46]. To test the vulnerability of the proposed communication scheme against attacks, we applied the method of message extraction based on return maps.

Let $n = i_{\max}$ be the time when the signal $x(t)$ gets its ith local maximum M_i, and $n = i_{\min}$ be the time when $x(t)$ gets its ith local minimum N_i. We construct the return maps M_{i+1} versus M_i, N_{i+1} versus N_i, and B_i versus A_i, where $A_i = (M_i + N_i)/2$ and $B_i = M_i - N_i$. Figure 7 depicts the return map B_i versus A_i constructed from the maxima and minima of time series of $x(t)$ for the case of SNR = 0 dB shown in Fig. 5. In contrast to the return maps presented in [45], Fig. 7 does not show any 1D curves which correspond to different switched chaotic regimes and could be used for message extraction. The return maps M_{i+1} versus M_i and N_{i+1} versus N_i also do not show 1D curves. Moreover, the return maps are similar in the cases of fixed and switched delay time in the transmitter. Thus, the return map method which is efficient for unmasking low-dimensional chaotic communication systems fails being applied to the proposed communication scheme based on chaotic time-delayed feedback oscillator.

It should be mentioned that besides the employment of time-delay systems demonstrating chaotic dynamics of a very high dimension and possessing chaotic attractors with large number of positive Lyapunov exponents, one can use chaotic oscillators with multi-scroll attractors [47–50] in order to increase the security of chaotic communication systems. In numerical studies of such systems, sometimes it is preferred to use multi-step methods with automatic control of step size [51].

4 Results of the experimental scheme operation

We implemented the proposed chaotic communication scheme with one response system in the receiver in a radio physical experiment in which the drive and response systems were constructed using electronic ring oscillators with time-delayed feedback. These oscillators contain analog low-pass first-order *RC* filter and digital delay lines and nonlinear element implemented using programmable microcontrollers. The digital nonlinear elements in our scheme provide a quadratic transformation. We used 32-bit Atmel microcontrollers based on the ARM Cortex-M3 processor. They have a top clock speed in the range of 84 MHz. The delay lines DL1 and DL2 in the receiver (Fig. 1) are also implemented using microcontrollers having an integrated 12-bit analog-to-digital converter and digital-to-analog converter.

Note that for the hardware realization of the proposed scheme, one can use also field-programmable gate arrays (FPGA) similarly to [52,53]. Although FPGA-based schemes have a more elaborated design, they are more power-consuming than the schemes based on microcontrollers. Another technique for implementing chaotic oscillators is based on using integrated circuits [54].

In our scheme, we used a wire communication channel. This channel is supposed to be a nondispersive one since its bandwidth is much greater than the bandwidth of the transmitted signal, and it does not cause any frequency distortions in the transmitted signal.

The parameters of the drive system modeled by Eq. (3) are chosen as follows: $\tau_1 = 930\,\mu$s, $\tau_2 = 93\,\mu$s, $\varepsilon_1 = R_1C_1 = 47\ \mu$s, and $f_1(x) = \lambda_1 - x^2$, where $\lambda_1 = 1.7$. With these parameters, the transmitter generates a chaotic signal. The response system is described by Eq. (4) with $\tau_3 = 930\ \mu$s, $\varepsilon_2 = R_2C_2 = 95\ \mu$s, $f_2(y) = \lambda_2 - y^2$, where $\lambda_2 = 1.3$, $k = 0.1$, and $\tau = 90$ ms. In the absence of coupling, the response system generates periodic oscillations.

Figure 8 shows parts of the experimental time series of the driving chaotic signal $x(t)$, signal $z(t)$, and difference signal $\Delta(t) = y(t) - y(t - \tau)$ for the case of the transmission of binary 0. For better detection of generalized synchronization, the signal $\Delta(t)$ is passed through a low-pass filter with the cutoff frequency $\nu = 200$ Hz. The time scale over the horizontal axis is 200 ms/div. The scale over the vertical axis is 1 V/div,

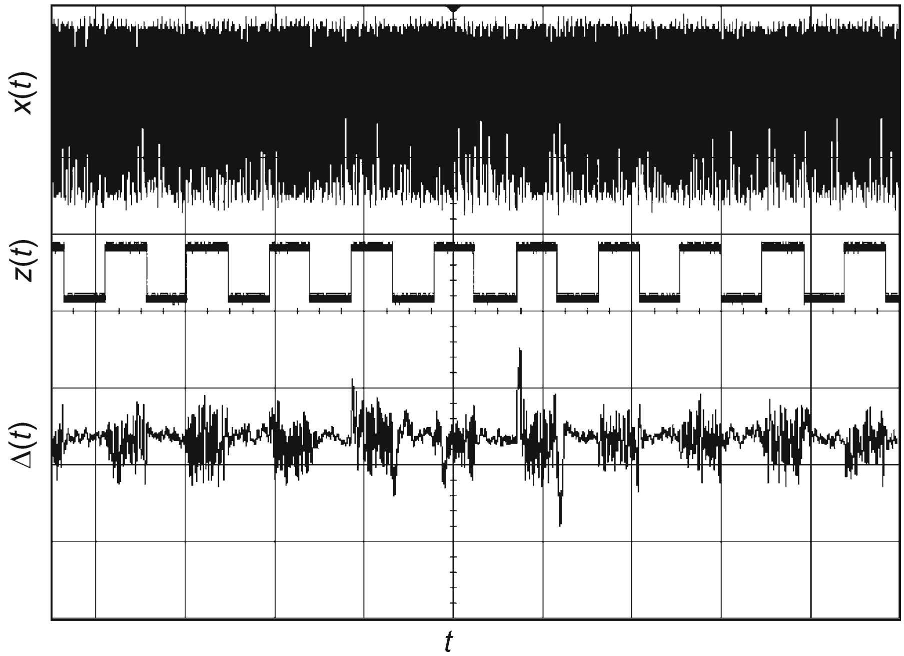


Fig. 8 Oscillograms of temporal realizations of the chaotic signal $x(t)$ in the communication channel (on *top*), signal $z(t)$ (in the *middle*), and filtered difference signal $\Delta(t)$ (*below*) in the case of $m(t) = 0$

5 V/div, and 200 mV/div for the signals $x(t)$, $z(t)$, and $\Delta(t)$, respectively.

As it can be seen from Fig. 8, the amplitude of oscillations of the signal $\Delta(t)$ is appreciably greater at high values of $z(t)$ than at small values of $z(t)$. This abrupt decrease in the amplitude of $\Delta(t)$ under variation of $z(t)$ indicates the presence of generalized synchronization between the drive and response systems.

Parts of the time series of the signals $x(t)$, $z(t)$, and $\Delta(t)$ for the case of the transmission of binary 1 are presented in Fig. 9. As well as in Fig. 8, the difference signal $\Delta(t)$ is passed through a low-pass filter with the cutoff frequency $\nu = 200$ Hz. The scales over the axes are the same as those in Fig. 8. In contrast to Fig. 8 corresponding to the transmission of binary 0, the amplitude of the difference signal $\Delta(t)$ in Fig. 9 is practically the same within the both halves of the period of $z(t)$. Such behavior of $\Delta(t)$ indicates the absence of generalized synchronization between $x(t)$ and $y(t)$.

Figure 10 shows parts of the experimental time series of the signals $\Delta(t)$, $m(t)$, and $z(t)$ for the case of the transmission of arbitrary sequence of binary 0 and 1 at the following values of the scheme parameters: $\tau_1 = 1.244$ ms, $\tau_2 = 113$ µs, $\tau_3 = 1.13$ ms, $\varepsilon_1 = 113$ µs, $\varepsilon_2 = 287$ µs, $\lambda_1 = 1.7$, $\lambda_2 = 1.3$, $k = 0.138$, and $\tau = 185$ ms. With these parameters, the transmitter generates a chaotic signal and the receiver oscillates in a periodic regime. In contrast to the cases depicted in Figs. 8 and 9, the delay time τ_3 in the response system differs from the delay time τ_1 in the drive system. Thus, the proposed communication scheme does not require the identity of the transmitter and receiver parameters. In the considered case, the parameter value of each element in the drive system differs from the parameter value of corresponding element in the response system.

The time scale over the horizontal axis in Fig. 10 is 500 ms/div. The scale over the vertical axis is 200 mV/div for the signal $\Delta(t)$ and 5 V/div for the signals $m(t)$ and $z(t)$. The signal $\Delta(t)$ is filtered by a low-pass filter with the cutoff frequency $\nu = 200$ Hz. At a transmission of binary 1, the signal $\Delta(t)$ shows nonvanishing oscillations both at $z(t) = 1$ and $z(t) = 0$ indicating the absence of generalized synchronization. At a transmission of binary 0, the amplitude of oscillations of $\Delta(t)$ is appreciably less at small values of $z(t)$ than at high values of $z(t)$ indicating the presence of generalized synchronization between the drive and response systems.

The considered example is the first experimental implementation of communication system based on the generalized synchronization. In further research, we plan to investigate this experimental system in more detail. It should be noted that the considered communication scheme is characterized by certain limitation of

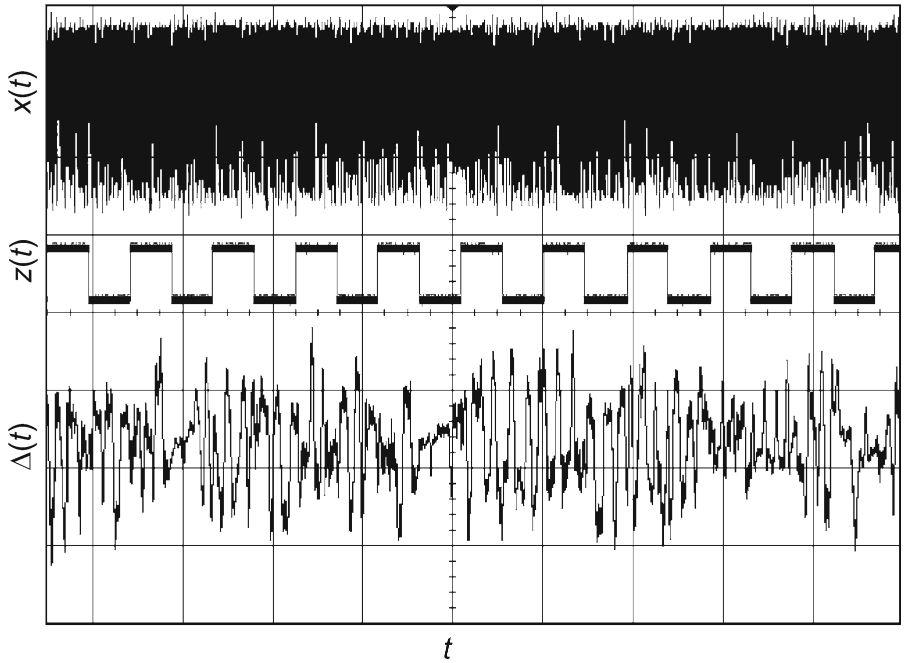


Fig. 9 Oscillograms of temporal realizations of the chaotic signal $x(t)$ in the communication channel (on *top*), signal $z(t)$ (in the *middle*), and filtered difference signal $\Delta(t)$ (*below*) in the case of $m(t) = 1$

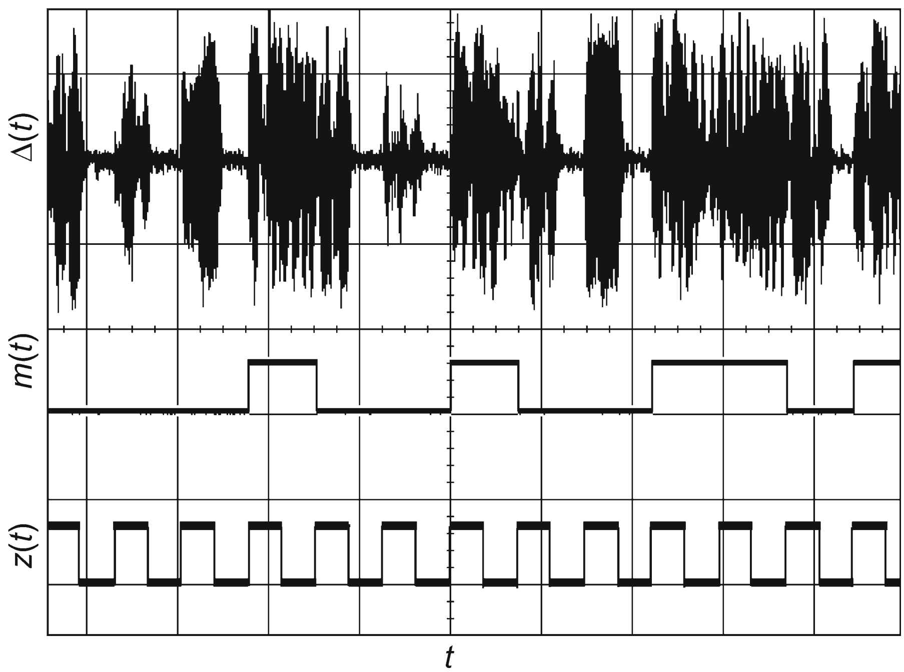


Fig. 10 Oscillograms of temporal realizations of the filtered difference signal $\Delta(t)$ (on *top*), information signal $m(t)$ (in the *middle*), and signal $z(t)$ (*below*) in the case of the transmission of arbitrary sequence of binary 0 and 1

the data transmission rate. This is because of long transient process that precedes the occurrence of the generalized synchronization regime in the time-delayed feedback oscillators used for the construction of the drive and response systems in our scheme. However, one can substantially increase the rate of information transmission by choosing other oscillators as the drive and response systems, which have a short time of transient process preceding the occurrence of the generalized synchronization.

5 Conclusion

We have developed the chaotic communication system exploiting the regime of generalized synchronization between the transmitter and receiver. Two variants of communication scheme are studied. In contrast to other communication schemes based on the generalized synchronization, the first variant of our scheme employs only one self-oscillating response system in the receiver. The absence of the auxiliary system in the receiver allows us to avoid the technical difficulties typical for communication systems based on the generalized synchronization caused by the necessity to create two identical oscillators in the receiver. In the second variant of the scheme, the receiver is composed of two response systems, but these systems could be nonidentical unlike the classical communication schemes exploiting the generalized synchronization.

For detecting the regime of generalized synchronization, the response system is driven in turn by the signal from the drive system and delayed copy of this signal. The efficiency of the proposed scheme is shown for the case, where the transmitter and receiver are constructed using time-delayed feedback oscillators.

The proposed communication scheme is studied numerically and realized in the physical experiment. We have illustrated the scheme efficiency for the transmission of binary information signal. It is shown that the proposed scheme possesses high resistance to noise in the communication channel. The security of the proposed scheme is studied by applying the return maps.

Acknowledgements This work is supported by the Russian Science Foundation, Grant No. 14–12–00324.

References

1. Kocarev, L., Halle, K.S., Eckert, K., Chua, L.O., Parlitz, U.: Experimental demonstration of secure communications via chaotic synchronization. Int. J. Bifurc. Chaos **2**, 709–713 (1992)
2. Parlitz, U., Chua, L.O., Kocarev, L., Halle, K.S., Shang, A.: Transmission of digital signals by chaotic synchronization. Int. J. Bifurc. Chaos **2**, 973–977 (1992)
3. Cuomo, K.M., Oppenheim, A.V.: Circuit implementation of synchronized chaos with applications to communications. Phys. Rev. Lett. **71**, 65–68 (1993)
4. Halle, K.S., Wu, C.W., Itoh, M., Chua, L.O.: Spread spectrum communication through modulation of chaos. Int. J. Bifurc. Chaos **3**, 469–477 (1993)
5. Dmitriev, A.S., Panas, A.I., Starkov, S.O.: Experiments on speech and music signals transmission using chaos. Int. J. Bifurc. Chaos **5**, 1249–1254 (1995)
6. Udaltsov, V.S., Goedgebuer, J.-P., Larger, L., Rhodes, W.T.: Communicating with optical hyperchaos: information encryption and decryption in delayed nonlinear feedback systems. Phys. Rev. Lett. **86**, 1892–1895 (2001)
7. Dmitriev, A.S., Panas, A.I.: Dynamical Chaos: New Information Carriers for Communication Systems. Fizmatlit, Moscow (2002)
8. Tao, Y.: A survey of chaotic secure communication systems. Int. J. Comput. Cogn. **2**, 81–130 (2004)
9. Argyris, A., Syvridis, D., Larger, L., Annovazzi-Lodi, V., Colet, P., Fischer, I., García-Ojalvo, J., Mirasso, C.R., Pesquera, L., Shore, K.A.: Chaos-based communications at high bit rates using commercial fibre-optic links. Nature **437**, 343–346 (2005)
10. Prokhorov, M.D., Ponomarenko, V.I.: Encryption and decryption of information in chaotic communication systems governed by delay-differential equations. Chaos Solitons Fractals **35**, 871–877 (2008)
11. Koronovskii, A.A., Moskalenko, O.I., Hramov, A.E.: On the use of chaotic synchronization for secure communication. Phys. Uspekhi **52**, 1213–1238 (2009)
12. Trejo-Guerra, R., Tlelo-Cuautle, E., Cruz-Hernández, C., Sánchez-López, C.: Chaotic communication system using Chua's oscillators realized with CCII+ s. Int. J. Bifurc. Chaos **19**, 4217–4226 (2009)
13. Gámez-Guzmán, L., Cruz-Hernández, C., López-Gutiérrez, R.M., García-Guerrero, E.E.: Synchronization of Chua's circuits with multi-scroll attractors: application to communication. Commun. Nonlinear Sci. Numer. Simul. **14**, 2765–2775 (2009)
14. Wang, M.-J., Wang, X.-Y., Pei, B.-N.: A new digital communication scheme based on chaotic modulation. Nonlinear Dyn. **67**, 1097–1104 (2012)
15. Mengue, A.D., Essimbi, B.Z.: Secure communication using chaotic synchronization in mutually coupled semiconductor lasers. Nonlinear Dyn. **70**, 1241–1253 (2012)
16. Ponomarenko, V.I., Prokhorov, M.D., Karavaev, A.S., Kulminskiy, D.D.: An experimental digital communication scheme based on chaotic time-delay system. Nonlinear Dyn. **74**, 1013–1020 (2013)
17. Deng, T., Xia, G.Q., Wu, Z.M.: Broadband chaos synchronization and communication based on mutually coupled VCSELs subject to a bandwidth-enhanced chaotic signal injection. Nonlinear Dyn. **76**, 399–407 (2014)
18. Karavaev, A.S., Kulminskiy, D.D., Ponomarenko, V.I., Prokhorov, M.D.: An experimental communication scheme based on chaotic time-delay system with switched delay. Int. J. Bifurc. Chaos **25**, 1550134 (2015)
19. Garza-González, E., Posadas-Castillo, C., Rodríguez-Liñan, A., Cruz-Hernández, C.: Chaotic synchronization of irregular complex network with hysteretic circuit-like oscillators in hamiltonian form and its application in private communications. Rev. Mex. Fis. E **62**, 51–59 (2016)
20. Moskalenko, O.I., Koronovskii, A.A., Hramov, A.E.: Generalized synchronization of chaos for secure communication: remarkable stability to noise. Phys. Lett. A **374**, 2925–2931 (2010)

21. Moskalenko, O.I., Hramov, A.E., Koronovskii, A.A., Ovchinnikov, A.A.: Effect of noise on generalized synchronization of chaos: theory and experiment. Eur. Phys. J. B **82**, 69–82 (2011)
22. Kocarev, L., Parlitz, U.: Generalized synchronization, predictability, and equivalence of unidirectionally coupled dynamical systems. Phys. Rev. Lett. **76**, 1816–1819 (1996)
23. Rulkov, N.F., Sushchik, M.M., Tsimring, L.S., Abarbanel, H.D.I.: Generalized synchronization of chaos in directionally coupled chaotic systems. Phys. Rev. E **51**, 980–994 (1995)
24. Pyragas, K.: Weak and strong synchronization of chaos. Phys. Rev. E **54**, R4508–R4511 (1996)
25. Abarbanel, H.D.I., Rulkov, N.F., Sushchik, M.M.: Generalized synchronization of chaos: the auxiliary system approach. Phys. Rev. E **53**, 4528–4535 (1996)
26. Paluš, M., Komárek, V., Hrnčíř, Z., Štěrbová, K.: Synchronization as adjustment of information rates: detection from bivariate time series. Phys. Rev. E **63**, 046211 (2001)
27. Liu, Z., Zhou, J., Munakata, T.: Detecting generalized synchronization by the generalized angle. Europhys. Lett. **87**, 50002 (2009)
28. Koronovskii, A.A., Moskalenko, O.I., Hramov, A.E.: Nearest neighbors, phase tubes, and generalized synchronization. Phys. Rev. E **84**, 037201 (2011)
29. Schumacher, J., Haslinger, R., Pipa, G.: Statistical modeling approach for detecting generalized synchronization. Phys. Rev. E **85**, 056215 (2012)
30. Stankovski, T., McClintock, P.V.E., Stefanovska, A.: Dynamical inference: where phase synchronization and generalized synchronization meet. Phys. Rev. E **89**, 062909 (2014)
31. Martínez-Guerra, R., Mata-Machuca, J.L.: Fractional generalized synchronization in a class of nonlinear fractional order systems. Nonlinear Dyn. **77**, 1237–1244 (2014)
32. Oliver, N., Jiingling, T., Fischer, I.: Consistency properties of a chaotic semiconductor laser driven by optical feedback. Phys. Rev. Lett. **114**, 123902 (2015)
33. Voss, H.U.: Anticipating chaotic synchronization. Phys. Rev. E **61**, 5115–5119 (2000)
34. Zhan, M., Wang, X., Gong, X., Wei, G.W., Lai, C.H.: Complete synchronization and generalized synchronization of one-way coupled time-delay systems. Phys. Rev. E **68**, 036208 (2003)
35. Sahaverdiev, E.M., Shore, K.A.: Generalized synchronization in time-delayed systems. Phys. Rev. E **71**, 016201 (2005)
36. Senthilkumar, D.V., Lakshmanan, M.: Transition from anticipatory to lag synchronization via complete synchronization in time-delay systems. Phys. Rev. E **71**, 016211 (2005)
37. Srinivasan, K., Senthilkumar, D.V., Murali, K., Lakshmanan, M., Kurths, J.: Synchronization transitions in coupled time-delay electronic circuits with a threshold nonlinearity. Chaos **21**, 023119 (2011)
38. Banerjee, T., Biswas, D., Sarkar, B.C.: Complete and generalized synchronization of chaos and hyperchaos in a coupled first-order time-delayed system. Nonlinear Dyn. **71**, 279–290 (2013)
39. Ma, J., Wang, C.-N., Jin, W.-Y., Wu, Y.: Transition from spiral wave to target wave and other coherent structures in the networks of Hodgkin–Huxley neurons. Appl. Math. Comput. **217**, 3844–3852 (2010)
40. Tang, J., Ma, J., Yi, M., Xia, H., Yang, X.: Delay and diversity-induced synchronization transitions in a small-world neuronal network. Phys. Rev. E **83**, 046207 (2011)
41. Shi, X., Wang, Z.: Adaptive synchronization of time delay Hindmarsh–Rose neuron system via self-feedback. Nonlinear Dyn. **69**, 2147–2153 (2012)
42. Ma, J., Qin, H., Song, X., Chu, R.: Pattern selection in neuronal network driven by electric autapses with diversity in time delays. Int. J. Mod. Phys. B **29**, 1450239 (2015)
43. Hettiarachchi, I.T., Lakshmanan, S., Bhatti, A., Lim, C.P., Prakash, M., Balasubramaniam, P., Nahavandi, S.: Chaotic synchronization of time-delay coupled Hindmarsh–Rose neurons via nonlinear control. Nonlinear Dyn. **86**, 1249–1262 (2016)
44. Koronovskii, A.A., Moskalenko, O.I., Pavlov, A.S., Frolov, N.S., Hramov, A.E.: Generalized synchronization in the action of a chaotic signal on a periodic system. Tech. Phys. **59**, 629–636 (2014)
45. Pérez, G., Cerdeira, H.A.: Extracting messages masked by chaos. Phys. Rev. Lett. **74**, 1970–1973 (1995)
46. Zhou, C.-S., Chen, T.-L.: Extracting information masked by chaos and contaminated with noise: some considerations on the security of communication approaches using chaos. Phys. Lett. A **234**, 429–435 (1997)
47. Muñoz-Pacheco, J.M., Zambrano-Serrano, E., Félix-Beltrán, O., Gómez-Pavón, L.C., Luis-Ramos, A.: Synchronization of PWL function-based 2D and 3D multi-scroll chaotic systems. Nonlinear Dyn. **70**, 1633–1643 (2012)
48. Jiménez-López, E., González Salas, J.S., Ontañón-García, L.J., Campos-Cantón, E., Pisarchik, A.N.: Generalized multistable structure via chaotic synchronization and preservation of scrolls. J. Frankl. Inst. **350**, 2853–2866 (2013)
49. Volos, C.K., Kyprianidis, I.M., Stouboulos, I.N.: An universal phenomenon in mutually coupled Chua's circuit family. J. Circuit Syst. Comput. **23**, 1450028 (2014)
50. Soriano-Sánchez, A.G., Posadas-Castillo, C., Platas-Garza, M.A., Cruz-Hernández, C., López-Gutiérrez, R.M.: Coupling strength computation for chaotic synchronization of complex networks with multi-scroll attractors. Appl. Math. Comput. **275**, 305–316 (2016)
51. Tlelo-Cuautle, E., Muñoz-Pacheco, J.M.: Simulation of Chua's circuit by automatic control of step-size. Appl. Math. Comput. **190**, 1526–1533 (2007)
52. Tlelo-Cuautle, E., Carbajal-Gomez, V.H., Obeso-Rodelo, P.J., Rangel-Magdaleno, J.J., Nunez-Perez, J.C.: FPGA realization of a chaotic communication system applied to image processing. Nonlinear Dyn. **82**, 1879–1892 (2015)
53. Tlelo-Cuautle, E., Rangel-Magdaleno, J.J., Pano-Azucena, A.D., Obeso-Rodelo, P.J., Nunez-Perez, J.C.: FPGA realization of multi-scroll chaotic oscillators. Commun. Nonlinear Sci. **27**, 66–80 (2015)
54. Trejo-Guerra, R., Tlelo-Cuautle, E., Jiménez-Fuentes, M., Muñoz-Pacheco, J.M., Sánchez-López, C.: Multiscroll floating gate—based integrated chaotic oscillator. Int. J. Circuit Theory Appl. **41**, 831–843 (2013)