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Chaos and Its Suppression in a System of Two Coupled Rydberg Atoms

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Abstract—The nonlinear behavior and chaos in a system of two coupled Rydberg atoms is investigated. A map of the regimes in which oscillations with different periods are plotted is produced, and the chaotic behavior of the system is revealed. A procedure for suppressing chaos in the system by means of an external parametric effect is presented.

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INTRODUCTION

The problems of controlling the quantum systems with Rydberg atoms are currently of great interest, since they are related to the problems of producing quantum computers [1, 2]. It is known that Rydberg atoms are hydrogen-like atoms in which the external electron is in a highly excited state up to a level of around 1000 [3, 4]. At present, such objects are of great interest [5, 6] since they can be used for the quantum control of one atom by another, due to the Rydberg (highly excited) state. The wave functions of atoms in the basic state are no higher than 0.1 nm, while in the Rydberg state they can be as high as several nanometers or more. This allows atoms that are far enough apart to prevent their interaction while they are in the ground state to strongly interact when exited [7].

The problem of chaotic behavior in a quantum system is of special interest. It is also interesting from a practical point of view when solving problems with quantum calculations for clusters of atoms introduced into a solid while in the Rydberg state [8].

Such systems with Rydberg atoms are promising for storing and transmitting information. The problem of analyzing how to suppress chaotic behavior in such systems is important since chaos can damage stored or transmitted information.

In this work, we investigate systems of two coupled Rydberg atoms and show that chaotic behavior can emerge in such a system. We also investigate whether it is possible to control chaotic behavior by means of external parametrical effect for a system with two coupled Rydberg atoms.

ANALYZING A SYSTEM WITH TWO COUPLED ATOMS

Our system with two coupled Rydberg atoms is described by the system of equations obtained in [9] using the mean field theory and assuming that each atom has the ground and Rydberg states only:

$$\begin{aligned} \dot{w}_1 &= -2\Omega\,\mathrm{Im}\,q_1 - w_1 - 1, \\ \dot{w}_2 &= -2\Omega\,\mathrm{Im}\,q_2 - w_2 - 1, \\ \dot{q}_1 &= i[\Delta - c(w_2+1)]q_1 - \frac{q_1}{2} + i\frac{\Omega}{2}w_1, \\ \dot{q}_2 &= i[\Delta - c(w_1+1)]q_2 - \frac{q_2}{2} + i\frac{\Omega}{2}w_2, \end{aligned} \quad (1)$$

where Ω is the Rabi frequency with which the population of the exited state oscillates due to resonance laser radiation; Δ is the laser radiation's detuning against the frequency of the resonance atom transition; c is the Rydberg interaction; $w_{1,2}$ are inversions, i.e., the difference between atom level populations; and $q_{1,2}$ are the nondiagonal elements of the atom density matrix, which is an analog of the wave function and is used to describe the state of the quantum-mechanical system.

In [9], it was found that there are three typical regimes: uniform, antiferromagnetic, and oscillating. In the uniform regime, inversions $w_1 = w_2$ and are time-independent; in the antiferromagnetic regime, inversions $w_1 \neq w_2$ and are also time-independent; in the oscillating regime, inversions $w_1 \neq w_2$ and are not stationary over time; i.e., they oscillate.

We investigated the area with an oscillating effect in which we found different periodic regimes: T_1, T_2, T_3, ..., chaotic. Using our results, we created a map of regimes for the oscillating area shown in Fig. 1 with areas with different periodic regimes. Areas with chaotic oscillations are colored in black.

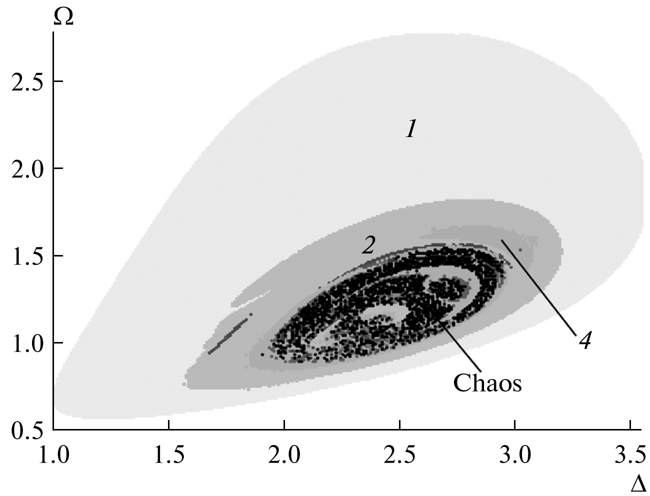

Fig. 1. Map of regimes for the oscillating area. The numbers denote the areas with the respective periods of oscillations; the area of chaotic behavior is marked in black.

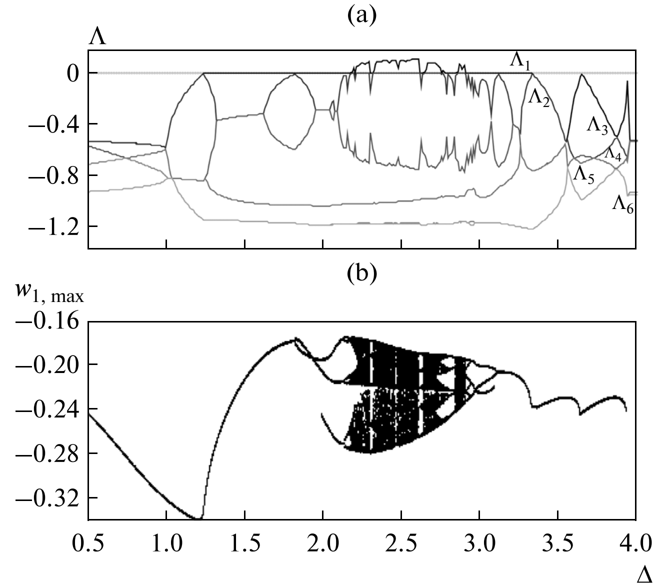

Fig. 2. (a) Spectrum of Lyapunov exponents and (b) bifurcation diagram for an independent system consisting of two coupled Rydberg atoms when $\Omega = 1.3$.

For qualitative analysis and diagnosing the oscillating regimes, we created bifurcation diagrams in the form of the relationship between the local maxima of w_1 and controlling parameter Δ at a constant value of second controlling parameter Ω, along with the spectra of Lyapunov exponents with the same parameters. To calculate the spectrum of Lyapunov exponents, we introduced six disturbing vectors (since there are 6 variables in our system), each of which has six components and we monitored their evolution along the considered phase trajectory. In equal time periods, the vectors are renormalized and are orthogonalized by the Gramm-Schmidt procedure. As a result, we removed the effects of all previous vectors from each vector in numerical order; this allows us to calculate not just the highest exponent, but their whole spectrum. After each orthogonalization and before renormalizing, we calculate the natural logarithm for each vector and calculate the sums of logarithms for each disturbing vector. Dividing these sums over time, we obtained the Lyapunov exponent [10]. Figure 2 shows the spectrum of all Lyapunov exponents, denoted as Λ_{1-6}, along with the bifurcation diagram for controlling parameter $\Omega = 1.3$. The diagram and spectrum for the Lyapunov factors are in good agreement with each other while qualitatively and quantitatively reflecting all changes in the system's behavior, e.g., the transition from the stationary to the oscillating state, bifurcations of period doubling, and moments in time when the system's behavior shifts from periodic to chaotic and vice versa; in addition, the windows of periodicity in the chaotic areas are clearly seen. The above shows that our instruments for investigating nonlinear behavior in the considered system are quite valid.

We also discovered there was bi-stability in the considered system. Under different initial conditions but at the same values of the controlling parameters, we landed in the gravity well of different attractors. In addition, we observed coexisting regimes of periodic and chaotic oscillation in the area of controlling parameters Ω and Δ.

SUPPRESSING CHAOTIC BEHAVIOR IN THE CONSIDERED SYSTEM

We used an external parametric effect for suppressing chaotic oscillations to control the complicated behavior in the considered system [11, 12]. We selected the modulation of Rabi frequency Ω as our external parametric effect since it was possible to employ it experimentally. In the system of equations that describes two coupled Rydberg atoms, the above external effect is written by modifying the Rabi frequency:

$$\Omega = \Omega_m[1 + M\sin(2\pi ft)], \quad (2)$$

where Ω_m is the Rabi frequency in the independent system, M is the depth, and f is the parameter's modulation frequency.

We analyzed Eqs. (1) and parametric effect (2) with controlling parameters corresponding to chaotic behavior in the independent system at different values of amplitude M and frequency f of the external effect. For analysis, we used a bifurcation diagram and the spectrum of Lyapunov conditional exponents [13] while varying one parameter of the external effect.

Figure 3 shows the spectrum of Lyapunov conditional exponents and the bifurcation diagram corre-

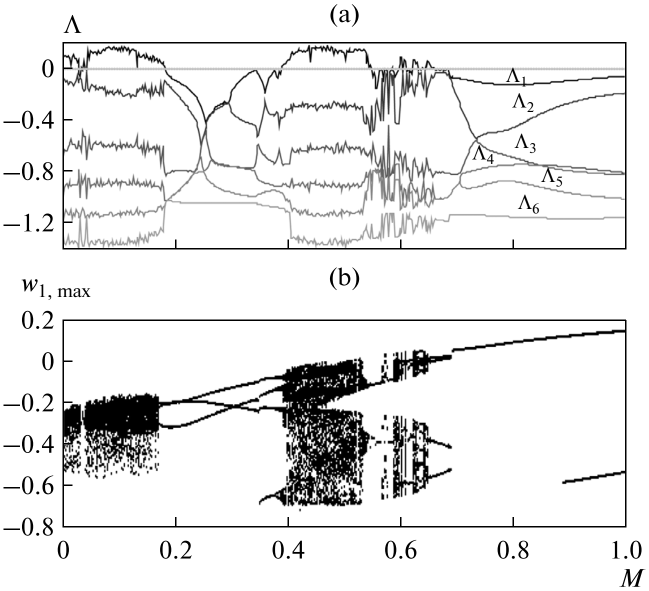

Fig. 3. (a) Spectrum of Lyapunov's exponents and (b) bifurcation diagram for an independent system consisting of two coupled Rydberg atoms upon external parametric effect when $\Omega = 1.2$, $\Delta = 2.6$, and $f = f_0$, where f_0 is the self-resonant frequency of the independent system.

sponding to the considered system under our external effect for the Ω and Δ at which chaotic behavior occured in the independent system. The relations were generated by varying amplitude M of the external effect at constant frequency f. Since these exponents were calculated for the system under our external effect, they were considered conditional and contained no zero exponent [14]. Upon periodic behavior, the highest exponent was negative in the spectrum of Lyapunov exponents.

In the figure, we can clearly see the windows of periodicity in the area of chaotic oscillations in both the spectrum of Lyapunov exponents and the bifurcation diagram. In the bifurcation diagram, we can also see classical transition from chaotic to periodic oscillations via the reverse cascade of the bifurcation of doubling periods. From the figure, we can also see that there were areas of parameter M in which chaos was suppressed in the system and periodic behavior was observed.

We also investigated whether it was possible to suppress chaotic oscillations in the same manner and in the same area of parameter M, but at another values of frequency f. The results from these investigations were similar to those presented in Fig. 3: there were areas of parameters M in which chaotic behavior was suppressed in the system. This means that with parameters Ω and Δ, at which we see chaos in the independent system, there are areas where we see periodic behavior in the area of parameters (M, f); i.e., chaos is suppressed.

CONCLUSIONS

We investigated the behavior of a system with two coupled atoms. In the area of the oscillating regime, we observed oscillating areas with different periods and chaotic behavior. Using an external parametric effect that depended on the frequency and amplitude parameters of external effect, we suppressed chaos in the system.

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