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Intermittent behavior near the synchronization threshold in system with fluctuating control parameter

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Abstract – In this paper the characteristics of the intermittent behavior observed near the synchronization threshold of a periodic oscillator driven by the external harmonic signal are studied in the case when the amplitude of the external signal fluctuates, *i.e.*, it changes its value randomly in some moments of time. The analytical law for the distribution of the laminar phases lengths has been deduced. The very good agreement between the discovered theoretical law and the results of numerical simulation has been shown.

Introduction. – Intermittency is well known to be an ubiquitous phenomenon in nonlinear science. Its arousal and main statistical properties have been studied and characterized already since long time ago, and different types of intermittency have been classified as types I–III intermittencies [1,2], on-off intermittency [3–6], eyelet intermittency [7] and ring intermittency [8]. There are no doubts that the different types of the intermittent behavior may take place in a wide spectrum of systems, including cases of practical interest for applications in radio engineering [1,9], medical [10], physiological [11,12], and other applied sciences.

In most of the cases the intermittent behavior taking place in a system is considered when the control parameters are constant. Since the random perturbations existing in Nature can modify the system behavior sufficiently [13–17], the intermittent behavior of different types has also been studied in the presence of noise [18–23]. It seems also worthy of note, that intermittency of epochs of synchronous and asynchronous behaviors is quite essential to the problem of synchronization by common noise [24,25]. However, the parameters of the system demonstrating an intermittent behavior can fluctuate. Since the characteristics of the intermittent behavior depend greatly on the control parameters, the fluctuations of the parameters will also result in the modification of these characteristics. In particular, such circumstance can be observed very often in living systems. For example, intermittency is known to arise near the threshold of chaotic synchronization regimes [26–28]. In turn, the chaotic synchronization regimes (such as phase synchronization) take place in living systems, *e.g.*, in the human cardiovascular system (see, *e.g.*, [29,30] where synchronization between cardio-rhythm and breathing has been observed). At the same time, the control parameters of the living systems (such as, *e.g.*, the breathing frequency) are very changeable over time. As a result, near the boundary of the synchronous regime an intermittent behavior occurs, with its characteristics being dependent sufficiently on the fluctuating system control parameters. In other words, this situation is generic for the living systems as well as for other types of complex objects.

In this paper the characteristics of the intermittent behavior near the boundary of the synchronous regime are considered in the case when one of the system control parameters changes its value randomly and slowly. To reveal the main features of the phenomenon under study we consider the behavior of an oscillator driven by an external harmonic signal whose amplitude A is changeable. Let A_c be its value corresponding to the onset of the synchronous regime when the amplitude of the external signal is constant. For $A > A_c$ synchronous dynamics is observed in the system under study, whereas for $A < A_c$ the oscillator demonstrates asynchronous behavior with intermittency manifestations.

We suggest that the control parameter value is close to the critical point A_c and changes its value at certain moments of time t_n randomly, whereas between t_n and

$t_{n+1} = t_n + \tau_\xi$ (τ_ξ is supposed to be constant) the amplitude of the external signal remains unchanged. In other words, the random amplitudes are chosen in the form of periodic switchings. In the different moments of time the value A of the amplitude can be both above and below the critical value A_c that results in the distortion of the typical statistical characteristics of the intermittent behavior taking place near the onset of synchronization.

Sample model. – As a sample model the Van der Pol oscillator under the external harmonic signal with the amplitude $A = A_0 + D\xi$ is considered

$$\ddot{x} - (\lambda - x^2)\dot{x} + x = (A_0 + D\xi)\sin(\omega_e t). \tag{1}$$

Here A_0 and ω_e are the amplitude and the frequency of the external harmonic force, correspondingly, λ is the nonlinearity parameter, D is the value characterizing the intensity of fluctuations [1], ξ is supposed to be a random quantity which changes its value over the fixed (and equal) intervals of time with duration τ_ξ (see fig. 1). We have also supposed that ξ is described by the normal distribution $N(0,1)$

$$p(\xi) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\xi^2}{2}\right). \tag{2}$$

The control parameter values have been selected as $\lambda = 0.1$, $\omega_e = 0.98$. $A_0 = 0.024$ exceeds the boundary value $A_c = 0.0238$ corresponding to the threshold of synchronization in the absence of the fluctuations of the amplitude (*i.e.*, in the case $D = 0$). To integrate eq. (1) we have used Ito integral [31] and the one-step Euler method with time step $h = 5\times 10^{-4}$.

The amplitude A of the external force changes its value randomly over the equal intervals of time (see fig. 1). The parameter being responsible for the intensity of fluctuations is set as $D = 5\times 10^{-4}$. The value of the amplitude of the external signal, $A = A_0 + D\xi$, takes the new values over the time intervals $\tau_\xi = 900$. We consider *the slow* fluctuations of the amplitude, *i.e.* $\tau_\xi \gg 2\pi/\omega_e$. The distribution of the probability density $p(A)$ may be written in the following form:

$$p(A) = \frac{1}{\sqrt{2\pi}D}\exp\left(-\frac{(A-A_0)^2}{2D^2}\right). \tag{3}$$

Figure 1 illustrates the typical fragment of the behavior of the system under study. The dynamics of the external signal amplitude $A(t)$ is shown in fig. 1(a). To detect the synchronous and asynchronous intervals of motion the instantaneous phase $\varphi(t)$ of the signal $x(t)$ can be introduced in the traditional way, as the rotation angle

$$\tan\varphi(t) = \frac{\dot{x}(t)}{x(t)} \tag{4}$$

[1]Note, for certain purposes D may be considered as a percentage instead of an absolute amplitude (in this case the symbol D should be replaced by δ).

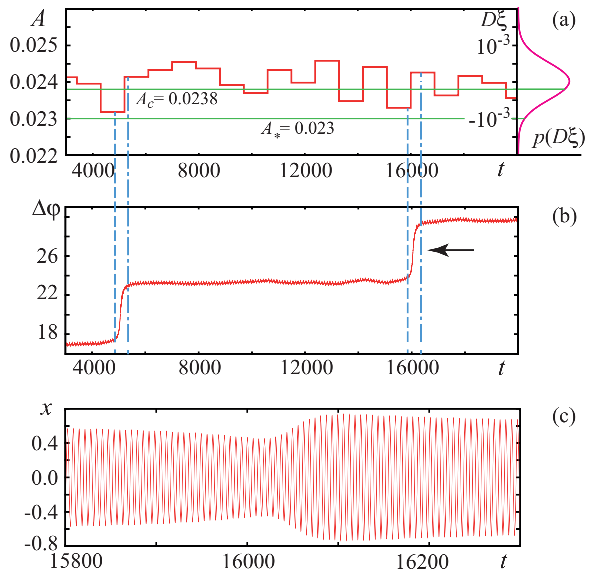

Fig. 1: (Colour on-line) (a) The dependence of the external signal amplitude $A(t)$ (and, correspondingly, random fluctuations $\xi(t)$) on time t (the left axis corresponds to the amplitude A, whereas the right axis represents the fluctuations $D\xi$). The values of A_* and A_c are shown by the solid horizontal lines. The probability density $p(D\xi)$ is also shown in the right graph. (b) The dependence of the phase difference $\Delta\varphi$ on time t. Both the laminar and turbulent phases can be seen easily. The starts and the ends of the laminar phases are marked by the dotted and dashed lines, respectively. (c) The small fragment of time series $x(t)$ illustrating the system behavior during the phase slip is marked by the arrow in panel (b).

on the projection plane $(x,\dot{x})$ of system (1). The synchronous interval of motion can be detected by means of monitoring the time evolution of the instantaneous phase difference, that has to obey the phase locking condition [32]

$$|\Delta\varphi(t)| = |\varphi(t) - \omega_e t| < \text{const}. \tag{5}$$

Due to the external signal amplitude variation the dynamics of the phase difference $\Delta\varphi(t)$ features time intervals of phase-synchronized motion (laminar phases) persistently and intermittently interrupted by sudden phase slips (turbulent phases) during which the value of $|\Delta\varphi(t)|$ jumps up by 2π (see fig. 1(b)). To separate the laminar and turbulent phases the approved method proposed in [33] has been used. The starts and the ends of the laminar phases detected by means of this method are shown in fig. 1(b) with the help of the dotted and dashed lines, respectively. The small fragment of time series $x(t)$ illustrating the behavior during the phase slip is shown in fig. 1(c).

Theory of intermittent behavior. – First of all, we consider the behavior of the system under study during the period between fluctuations of the parameter A. In particular, we discuss the probability of the phase slips corresponding to the appearance of the turbulent dynamics.

It is known that for the periodically forced weakly nonlinear isochronous oscillator (in the case of a small frequency mismatch) the complex amplitude method may be used to find the solution describing the oscillator behavior in the form

$$u(t) = \mathrm{Re}\, a(t)e^{i\omega t}. \quad (6)$$

For the complex amplitude $a(t)$ one obtains the averaged (truncated) equation

$$\dot{a} = -i\nu a + a - |a|^2 a - ik, \quad (7)$$

where ν is the frequency mismatch, and k is the (renormalized) amplitude of the external force [34,35]. For the small ν and large k the stable solution

$$a(t) = Ae^{i\phi} = \mathrm{const} \quad (8)$$

corresponds to the synchronous regime. So, in the case when $A > A_c$ the system is in the synchronous state and, as a consequence, there are no phase slips during this time interval τ_ξ. Therefore, the whole considered time interval is treated as the laminar dynamics.

The situation seems to be more complicated if due to fluctuations the control parameter A takes a value less than A_c. If the value of the amplitude of the external signal is constant and lies below the synchronization threshold, the driven oscillator demonstrates the behavior with the features of type-I intermittency [36]. Indeed, for the truncated equation (7) the synchronization destruction corresponds to the local saddle-node bifurcation associated with the global bifurcation of the limit cycle birth [34,35]. Therefore, below the boundary of the synchronization area the dynamics of the phase difference

$$\Delta\varphi(t) = \varphi(t) - \omega_e t \quad (9)$$

(where $\varphi(t)$ is the phase of the driven oscillator) demonstrates time intervals of phase-synchronized motion (laminar phases) interrupted by phase slips (turbulent phases). For the periodic oscillator these laminar phases are regular and have a fixed duration (in contrast to chaotic oscillations), but the dependence of their duration on the criticality parameter also obeys the law of type-I intermittency. So, for the considered case the dependence of the length of the laminar phases (pieces of synchronous dynamics) on the criticality parameter is governed by the power law

$$T(A) = C_1 \cdot (A_c - A)^{-1/2}, \quad (10)$$

where C_1 is a constant. For the considered system and chosen values of the control parameters we have found numerically that $C_1 \approx 26$.

If the average length $T(A)$ of the laminar phase turns out to be sufficiently shorter than the duration of time interval τ_ξ, then with the probability close to one a phase slip corresponding to the end of the laminar phase will be observed in the system. We use the notation A_* for the value of the A-parameter for which the average length of the laminar phase is equal to τ_ξ, *i.e.*,

$$T(A_*) = \tau_\xi. \quad (11)$$

From eq. (10) and eq. (11) one can obtain that

$$A_* = A_c - (C_1/\tau_\xi)^2\,. \quad (12)$$

For the given control parameter values $A_* \simeq 0.023$.

Let us consider the phase slip which is observed during the fixed time interval τ_ξ between fluctuations of the parameter A. For all $A < A_*$ the condition $T(A) < \tau_\xi$ is satisfied and, hence, the phase slip occurs with probability close to one. When the value of the externals signal amplitude falls in the range $A_* < A < A_c$ the probability of the phase slip can be estimated as $\tau_\xi/T(A)$. Therefore, the expression for the probability of the phase slip occurrence during the time interval τ_ξ between two neighboring fluctuations of the control parameter A may be written in the following form:

$$P(A) = \begin{cases} 1, & A < A_*, \\ \tau_\xi/T(A), & A_* < A < A_c, \\ 0, & A > A_c. \end{cases} \quad (13)$$

As a consequence, if one considers the dynamics of system (1) during the time interval τ_ξ between the neighboring fluctuations of the external signal amplitude, the probability to observe the phase slip is defined as

$$P_1 = \int\limits_0^\infty p(A)P(A)\mathrm{d}A = \int\limits_0^{A_*} p(A)\mathrm{d}A + \int\limits_{A_*}^{A_c} \frac{\tau_\xi p(A)}{T(A)}\mathrm{d}A. \quad (14)$$

The probability of the observation of the laminar phase with the length s corresponding to k fluctuations of the A-parameter (*i.e.*, $(k-1)\tau_\xi < s < k\tau_\xi$) is defined as

$$P_2(k) = (1 - P_1)^{k-1} \cdot P_1^2, \quad (15)$$

where the probability P_1 is given by eq. (14). Indeed, a locking epoch of length s is composed of $(k-1)$ periods of τ_ξ with no phase slips. Additionally, two phase slips (the start and the end of the laminar phase) must be observed (see fig. 1(a)). The statistical independence [37] leads to the product of $(k-1)$ probabilities $(1 - P_1)$ and two probabilities P_1 (see eq. (15)).

Thus, the probability density of the laminar phase lengths obeys the law

$$\rho(s) \sim (1 - P_1)^{s/\tau_\xi}, \quad (16)$$

and, accordingly, eq. (16) may be also written as

$$\rho(s) = \kappa \exp(-\kappa s), \quad (17)$$

where

$$\kappa = -(1/\tau_\xi)\ln(1 - P_1). \quad (18)$$

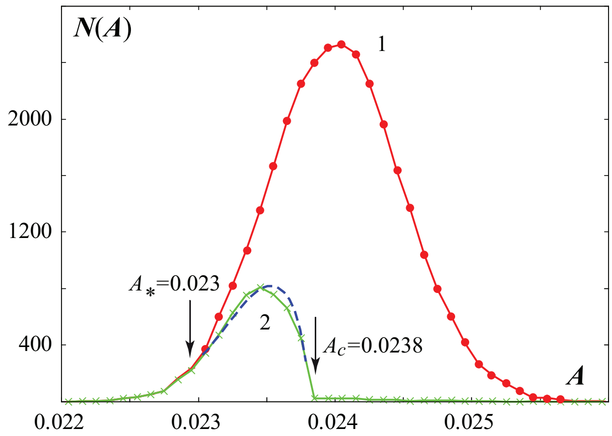

Fig. 2: (Colour on-line) The absolute frequency of observation $N(A)$ for binned values of the A-parameter values (curve 1) and the values of the amplitude of the external signal for which the phase slips have been observed (curve 2). Both distributions have been obtained for the same time series. The dependence $N_t p(A)\Delta A P(A)$ is shown by the dashed line.

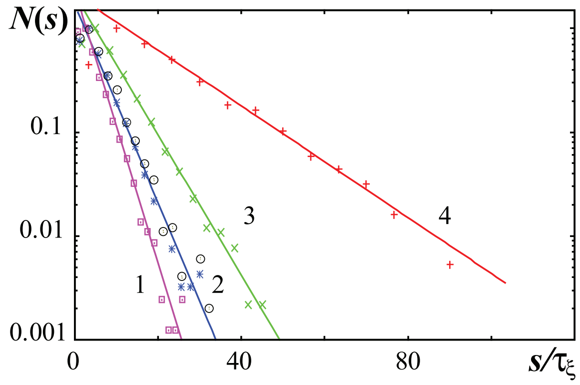

Fig. 3: (Colour on-line) Distributions of the laminar phase lengths for system (1) obtained for different values of τ_ξ. Points correspond to the data obtained numerically, the analytical law (16) is shown by the solid lines. The values of time interval τ_ξ and the probability P_1 corresponding to the distributions are the following: 1, $\tau_\xi = 1200, P_1 = 0.247$; 2, $\tau_\xi = 900, P_1 = 0.197$; 3, $\tau_\xi = 600, P_1 = 0.133$; 4, $\tau_\xi = 300, P_1 = 0.066$. Symbol "$\circ$" is used for the points corresponding to the case when the value of the A-parameter has been changed in arbitrary time intervals, with the average value being equal to 900, *i.e.*, $\langle\tau_\xi\rangle = 900$. The ordinate axis is shown in the logarithmic scale.

So, in the case of the slow fluctuations of the control parameter A the laminar phase length distribution for the system under study obeys the exponential law (17). If one normalizes the time (and, as a consequence, the lengths of the laminar phases) on the time interval τ_ξ, then the exponent κ in eq. (17) is determined only by the probability of the phase slip observation within the time interval τ_ξ between the neighboring fluctuations of the control parameter A:

$$\kappa = -\ln(1 - P_1). \tag{19}$$

Results of numerical simulations. – To verify and confirm the theoretical conclusion obtained above, we consider the results of the direct numerical simulations of system (1). The distribution of the A-parameter values obtained as a result of the numerical simulation of system (1) is shown in fig. 2 (see curve 1). Simultaneously, in the same figure the distribution of the A-parameter values for which the phase slips have been observed is illustrated by curve 2. Both distributions have been obtained for the same time series. The values A_* and A_c are shown by arrows, since they play an important role to form curve 2 of the probability density. One can see from fig. 2 that for $A \le A_*$ both distributions coincide with each other and, hence, for these values of the control parameter the phase slip occurs in 100% of the cases, *i.e.*, the turbulent phase begins in the system and the probability to observe the phase slip is close to one that agrees completely with eq. (13).

There is also a good agreement between the results of the numerical simulation and the theoretical curve for $A > A_c$. In this case there are almost no phase slips and, hence, if parameter A takes the value exceeding the synchronization threshold, the probability of the phase slip appearance is equal to zero.

Finally, in the range $A_* < A < A_c$ the amount of the phase slips decreases with the increase of A that provides the decrease of the probability P. To compare the behavior of the phase slip occurrence probability P obtained numerically and the analytical law (13) in the range $A_* < A < A_c$ the curve $N_t p(A)\Delta A P(A)$ is also shown in fig. 2 by the dashed line, where $\Delta A = 10^{-4}$ is the amplitude bin width and $N_t = 31500$ is the total number of the laminar phases. It can be seen easily from fig. 2 that eq. (13) describes very well the processes in the analyzed system.

In fig. 3 the distributions of the laminar phase lengths obtained numerically are shown for different values of the time interval τ_ξ. To compare the obtained results with the theoretical prediction the curves corresponding to analytical law (16) are also shown in fig. 3, with the value of P_1 being determined by eq. (14). Points in fig. 3 correspond to the numerical data, law (16) is shown by the solid lines for each value of τ_ξ. One can see a very good agreement between the numerical data and the theoretical law for all the considered values of the time interval τ_ξ.

Figure 3 allows to track the evolution of the laminar phase length distribution for large and small time intervals τ_ξ. When the duration of τ_ξ increases essentially the upper limit A_* of integration in eq. (14) grows and tends to A_c in accordance with (12). Therefore, for large time intervals τ_ξ the second term in eq. (14) can be neglected

$$P_1 \approx \int_0^{A_c} p(A)\,\mathrm{d}A. \tag{20}$$

As a consequence, for large values of τ_ξ the probability P_1 of the phase slip occurrence is almost constant and, therefore, the laminar phase length distributions are close to each other. This tendency becomes especially obvious if the laminar phase length distributions have been shown with the help of the normalized variables, $N(s/\tau_\xi)$. The very same situation takes place in fig. 3, where the laminar phase length distributions for $\tau_\xi = 1200$ and $\tau_\xi = 900$ turn out to be close to each other (see curves 1 and 2). At the same time, for smaller τ_ξ the corresponding distributions differ from distributions 1 and 2 obtained for $\tau_\xi = 1200$ and $\tau_\xi = 900$, respectively.

An opposite situation takes place for small values of the interval τ_ξ during which the parameter A is unchangeable. In this case with the decrease of the time interval τ_ξ the value of A_* also decreases as it follows from eq. (12), and, hence, for certain A_*-values the first integral in eq. (14) can be neglected. As a consequence, eq. (14) becomes

$$P_1 \approx \tau_\xi \int_0^{A_c} \frac{p(A)}{T(A)} \mathrm{d}A. \tag{21}$$

Therefore, for small values of τ_ξ the probability P_1 of the phase slip occurrence depends linearly on the length of the time interval between two neighboring fluctuations of the A-parameter and decreases with the decrease of τ_ξ. This means that an exponent in the exponential distribution of the laminar phase lengths obtained for the normalized time s/τ_ξ becomes less negative with the decrease of τ_ξ. As a consequence, the distribution shown in the logarithmic scale becomes flatter, which that corresponds to the appearance of the longer laminar phases. From the physical point of view it means that during the short time interval between fluctuations of the external signal amplitude A the system does not have time to react on the change of the control parameter value. Exactly the same tendency can be seen clearly in fig. 3 where the distributions shown in the logarithmic scale become flatter and flatter as the duration τ_ξ decreases (see especially dependences 3 and 4 obtained for $\tau_\xi = 600$ and $\tau_\xi = 300$, respectively).

The assumptions made above that the value of the time interval τ_ξ between the neighboring fluctuations of the parameter A is fixed and the value of the A-parameter remains unchangeable during this interval is strict enough and, as a consequence, it limits the class of systems for which the revealed regularities may be observed. At the same time, as shown by the results of our study, the obtained relations are also applicable for the case when the length of the time interval between fluctuations of the control parameter also fluctuates. In this case the use of the averaged value $\langle\tau_\xi\rangle$ instead of the parameter τ_ξ allows to get a good agreement between the theoretical predictions and the results obtained numerically.

The distribution of the laminar phase lengths in the case when time τ_ξ also fluctuates is shown in fig. 3 by circles ($\circ$), with the average value of the time interval during which A remains constant being $\langle\tau_\xi\rangle = 900$. One can see from fig. 3 that the obtained laminar phase length distribution coincides with the analogous distribution for the case $\tau_\xi =$ const $= 900$ considered above (the numerically obtained data are shown by symbols $*$) and is in good agreement with the curve corresponding to the analytical law (16).

Conclusion. – In the present work the characteristics of the intermittent behavior observed near the synchronization threshold of a Van der Pol oscillator driven by the external harmonic force have been studied in the case when the amplitude of the external signal is changing in a piecewise-constant manner with each piece having a fixed duration. The analytical law for the laminar phase lengths distribution has been deduced and the very good agreement between the obtained theoretical relation and the numerical data has been shown for the different values of the length of the time interval over which the control parameter changes its value.

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