Mean phase coherence modified for piecewise constant phase difference data

Vladimir Khorev
Department of Dynamic Modeling and Biomedical Engineering
Saratov State University
Saratov, Russia
Neuroscience and Cognitive Technology Lab, Center for Technologies in Robotics and Mechatronics Components
Innopolis University
Innopolis, Russia
0000-0001-6613-8940

***Abstract* — We propose a modification of the mean phase coherence method based on the statistics arrange of phase difference data. The modification aimed towards better analysis of data of coupled oscillators under influence of strong noise inducing phase skips.**

Keywords—mean phase coherence, connection, coupling, modification.

I. INTRODUCTION

The task of determining the degree of interaction between dynamical systems is an object of interest in various fields of science[1–3]. One of the most simple and fast methods was proposed in [4]. Nevertheless, it also has limits of applicability, for the expansion of which a solution is proposed in this work. To modify the mean phase coherence index method, it is required to analyze the change in synchronization properties of interacting systems in the presence of noise. To analyze the behavior of the phase difference signal in the presence of noise, consider an example of particle motion in the presence of a potential barrier: if the noise is strong enough, or if it is Gaussian, then the particle occasionally jumps over the potential barrier and fairly quickly comes to the next state of equilibrium $\varphi_0 \pm \varepsilon$. Physically, this means that the phase point makes an extra part of the turn, as compared with the external force, along the limiting cycle. These relatively fast changes in phase difference are called phase skips [5].

In practice, there are often situations where "overshoot" is observed on a number of phase differences. In presence of weak and limited noise, the phase fluctuates around a constant value, i.e. overshoot is not observed. In presence of strong and unlimited noise, phase skips are usually observed. However, the probabilities of skipping to the right and left are now different: naturally, the particle more often jumps down than up. Therefore, although most of the time the particle fluctuates around a state of equilibrium, on average it slips down the potential. Therefore, the phase dynamics is substantially non-uniform (at least with low noise): long synchronization intervals alternate with phase skips

We now discuss the relationship between the phases of self-oscillations and strength. It was noted that the phase difference $\Delta\varphi$ can be arbitrarily large due to phase diffusion (unless the noise is limited and weak). Thus, for noise systems, generally speaking, one cannot speak of phase capture, since the phase difference is not limited. On the other hand, the particle is more often at the minimum of the potential, and therefore certain values of $\Delta\varphi$ are observed with greater probability. If we consider the phase difference on the $[0,2\pi]$ interval and construct the phase distribution density, we will see that it is not uniform, but has a pronounced maximum.

II. DATA AND METHODS

Phases are calculated by rows of initial signals. Then, the probability density in the range from 0 to 2π is calculated from the series of the wrapped phase difference. Next, the obtained values of the probability density distributions are sorted. This range of values is approximated by an exponential function in the form:

$$Z_I = a \cdot \exp(bx_i) \tag{1}$$

where x_i are the initial values of the function, a, b are the approximation coefficients determined using the least squares method, in which the Levenberg-Marquardt algorithm was used for optimization. In the case when the synchronization level is high, the coefficient b should also take high values in the limiting cases for a series with a constant phase difference (limiting case) with a length of 100 characteristic periods a value of 7.2 was obtained. For the case of no synchronization, the coefficient b will take values close to 0.

In this modification, it we propose to calculate the distribution of phase differences and divide the distribution of phase differences into L bins. According to statistical studies, the optimal number of cells is determined by the formula:

$$L = |e^{(0.626+0.4\ln(N-1))}| \tag{2}$$

where L is the number of cells in the plot of the density of distribution of the function, N is the number of elements of the series [6]. Next is the sum of the distributions, which exceed the threshold value k, which is found by the following formula:

$$k = \varepsilon N/L \tag{3}$$

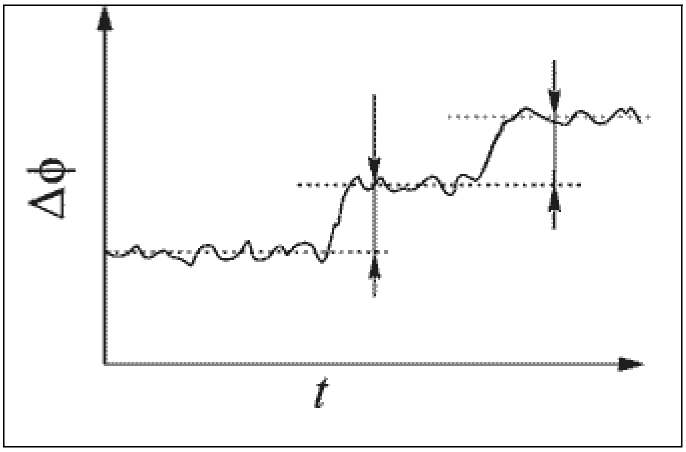

Fig. 1 An exemplary dependence of the phase difference from the time in case of switching to the synchronizatio and presence of noise when shifts occur irregularly.

It is easy to notice that in the case of constantly and monotonously increasing phase difference or if the phase difference is a random value similar to white noise, the height of all bins will tend to a constant value of N/L and for real data it is reasonably to add scaling factor ε depending on the signal/noise ratio.

III. Results

We can take this value as a threshold, but due to short time series and complex properties of the dynamics of real systems requires a normalization factor. To check the performance of the modified method of calculating the mean phase coherence index using a threshold value, a system of equations describing coupled phase oscillators with coupling coefficients were taken [7].

Fig. 2 shows a graph of the phase synchronization index versus the coupling parameter of two coupled oscillators. With high noise coefficients and large lengths of series, the phase synchronization indices of the proposed method show more accurate indications for two oscillators that are not fully synchronized with each other.

From the above, it can be concluded that the method of calculating the phase synchronization index using the approximating function, is more sensitive to the coupling strength, but requires further study for application in real systems.

In the course of the work, the possibilities of estimating phase synchronization were studied, taking into account the delay in coupling between systems, estimated using the phase coherence index using the model and real data. Namely, it was found out that in some cases it is possible to recover the delay in the connection between the two systems. It was determined that taking into account the delay in case of an increase in dynamic noise allows us to obtain a higher level of the mean phase coherence.

Thus, in this work we analyzed the possibility of modifying the method of calculating the phase synchronization between two oscillatory systems using model time series for the case of a piecewise constant phase difference and proposed methods for such a modification. The performance check of the proposed modifications was implemented on model data and demonstrated the potential applicability for the analysis of time series obtained from real systems [8-11].

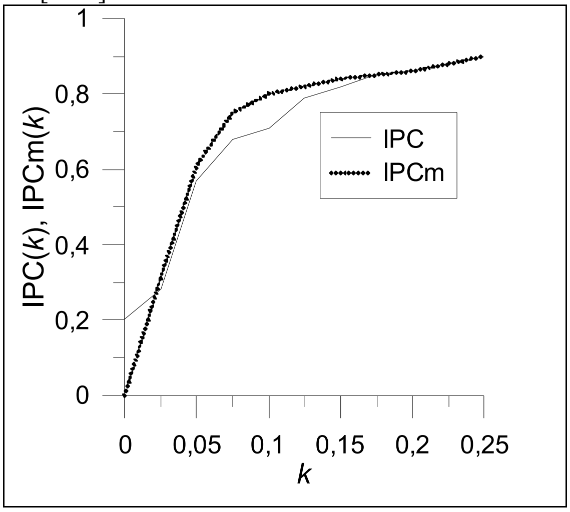

Fig. 2 The dependence of the mean phase coherence index (IPC) and modified phase coherence index (IPCm) on the coupling parameter for a number of phase difference of coupled oscillators in presence of noise.

Acknowledgment

This work has been supported by the Center for Technologies in Robotics and Mechatronics Components (Innopolis University).

References

[1] L. Cimponeriu, M. Rosenblum, A. Pikovsky, Estimation of delay in coupling from time series. Phys. Rev. E 2004. V.70 046213

[2] A. S. Karavaev, A. R. Kiselev, A. E. Runnova, et al. Synchronization of infra-slow oscillations of brain potentials with respiration CHAOS, 2018; vol. 28, 081102.

[3] O. I. Moskalenko, A. D. Koloskova, M. O. Zhuravlev, A. A. Koronovskii, A. E. Hramov, Intermittent phase synchronization in human epileptic brain, Proc. SPIE, 2017; 1006316-1.

[4] F. Mormann, K. Lehnertz, P. David, C. E. Elger. Mean phase coherence as a measure for phase synchronization and its application to the EEG of epilepsy patients. Physica D, 2000; vol. 144, pp. 358–369.

[5] A. S. Pikovsky, G. V. Osipov, M. G. Rosenblum, M. Zaks, J. Kurths, Phys. Rev. Lett., 1977; vol 79(1), pp. 47.

[6] R. Otnes and L. Enochson, Digital Time Series Analysis, New York: Wiley, 1972; pp. 112.

[7] D. Cumin, C. P. Unsworth, Generalising the Kuromoto model for the study of neuronal synchronisation in the brain, Physica D, 2007; vol. 226, pp. 181–196.

[8] M. M. Danziger, O. I. Moskalenko, S. A. Kurkin, X. Zhang, S. Havlin, S. Boccaletti, "Explosive synchronization coexists with classical synchronization in the Kuramoto model," Chaos: An Interdisciplinary Journal of Nonlinear Science, V. 26, no. 6, p.065307, 2016.

[9] S. A. Kurkin, A. E. Hramov, A. A. Koronovskii, "Nonlinear dynamics and chaotization of virtual cathode oscillations in annular electron beam in uniform magnetic field," Plasma Phys. Rep, V. 35, no. 8, pp.628-642, 2009.

[10] S. A. Kurkin, A. A. Koronovskii, A. E. Hramov, "Formation and dynamics of a virtual cathode in a tubular electron beam placed in a magnetic field," Technical Physics, V. 54, no. 10, p. 1520, 2009.

[11] P. Chholak, G. Niso, V.A. Maksimenko, S.A. Kurkin, N.S. Frolov, E.N. Pitsik, A.E. Hramov, A.N. Pisarchik, "Visual and kinesthetic modes affect motor imagery classification in untrained subjects," Scientific reports, V. 9, no. 1, p. 9838, 2019.