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Noise-Induced Loss of Multifractality in Dynamics of Systems with Self-Sustained Oscillations

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Abstract—The phenomenon of noise-induced loss of multifractality in the dynamics of systems with self-sustained oscillations is considered. The possibility of separating regimes of deterministic and stochastic dynamics by using the method of multifractal formalism based on wavelet transform is discussed.

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The nontrivial role of noise in dynamics of nonlinear system has been studied in numerous works, which showed that fluctuations can lead to increasing regularity of oscillations, improvement of the signal to noise ratio, the appearance of dynamical regimes impossible in the absence of noise, etc. Well-known effects evidencing a nontrivial behavior of nonlinear noisy systems include, e.g., the phenomenon of stochastic resonance [1]. Under the conditions of multistability, noise induces transitions between coexisting attractors in the phase space of a dynamical system, which leads to a change in the statistical characteristics of the regimes of complex dynamics. Although the corresponding changes can be insignificant from the standpoint of characteristics such as the power spectrum or Lyapunov exponents, they can significantly modify the signal structure that is manifested by variations in the singularuty spectrum [2].

The measures of multifractality are sensitive characteristics of the regimes of dynamics that allow one to reveal changes in the statistical properties of processes that cannot be diagnosed by standard methods of digital signal processing [3–5]. In particular, this approach ensures the possibility of introducing effective diagnostic criteria applicable to the investigation of dynamics of living systems [6–8].

The present work is devoted to analysis of the phenomenon of noised-induced loss of multifractality in complex dynamics of systems with self-sustained oscillations. As an example of a model with complex behavior, let us consider a system of two coupled Rössler oscillators describing a set of equations:

$$\begin{gathered}\frac{dx_1}{dt} = -\omega_1 y_1 - z_1 + \gamma(x_2 - x_1) + I\xi(t),\\ \frac{dy_1}{dt} = \omega_1 x_1 + a y_1, \quad \frac{dz_1}{dt} = b + z_1(x_1 - c),\\ \frac{dx_2}{dt} = -\omega_2 y_2 - z_2 + \gamma(x_1 - x_2),\\ \frac{dy_2}{dt} = \omega_2 x_2 + a y_2, \quad \frac{dz_2}{dt} = b + z_2(x_2 - c)\end{gathered} \tag{1}$$

with the following values of control parameters: $\omega_1 = 1.0093$, $\omega_2 = 0.9907$, $a = 0.15$, $b = 0.2$, $c = 7.2$, and $\gamma = 0.02$. Intensity I of the normally distributed random process $\xi(t)$ was varied so as to study changes in the statistical characteristics of sequences of the return times to Poincaré section set by the equation $x_2 + y_1 = 0$. These sequences were analyzed using the method of multifractal formalism based on a wavelet transform [3]. For this analysis, we have selected sequences of return times containing 5000 counts.

At the first stage of a procedure done according to the method of [3], a sequence of return times $x(i)$ is subjected to wavelet transform as

$$W(a,k) = \frac{1}{\sqrt{a}}\sum_{i=0}^{N} x(i)\psi^*\left(\frac{i-k}{a}\right) \tag{2}$$

with basis function ψ represented by the MHAT wavelet:

$$\psi(t) = (1 - t^2)\exp\left(-\frac{x^2}{2}\right). \tag{3}$$

The parameters of scale (a) and translation (k) of wavelet function ψ were varied within broad limits. The points of singularities $x(k^*)$ on the surface of wavelet coefficients $W(a, k)$ correspond to the lines

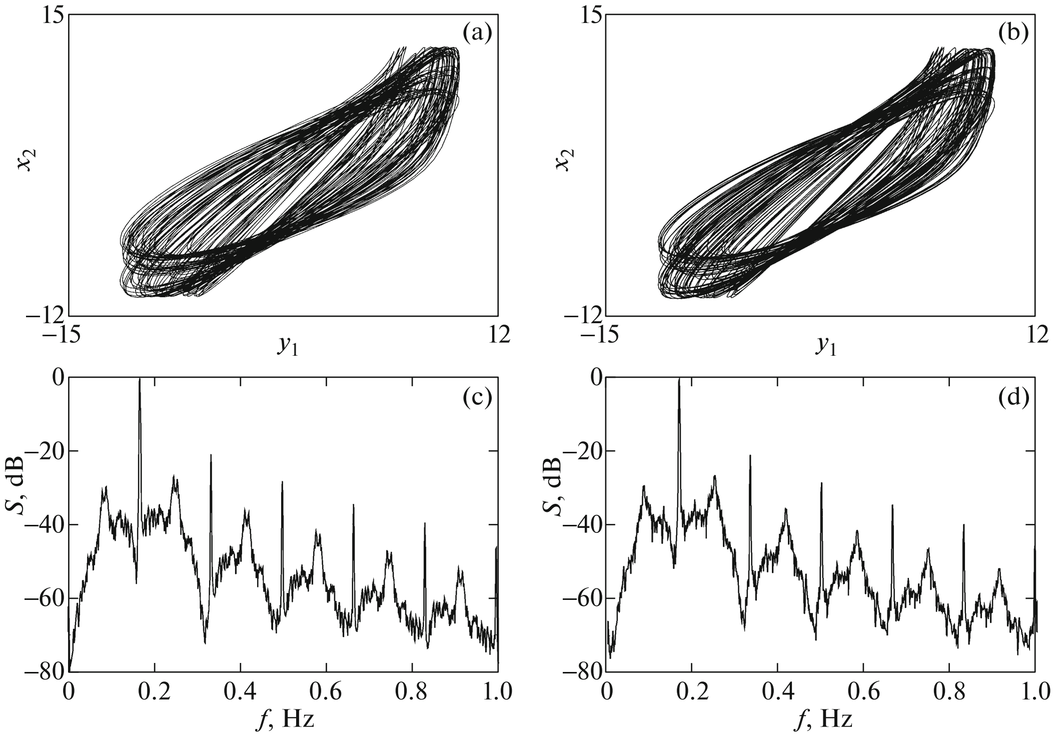

Fig. 1. (a, b) Phase portrait and (c, d) power spectra of (a, c) deterministic and (b, d) stochastic dynamics in the model system (1) of two coupled Rössler oscillators.

of local extrema, on which the wavelet transform coefficients exhibit power dependence on the scale parameter: $W(a, k^*) \sim a^h$. The power index (Hölder exponent) is a characteristic of correlation properties of the process under consideration. For a signal of complex structure, a spectrum of Hölder exponents $h(q)$ is calculated and index q is varied so as to analyze the range of small ($q < 0$) and large ($q > 0$) scaling parameters.

At the second stage of the proposed method [3], spectrum $h(q)$ is more accurately calculated using an approach based on the construction of generalized partition functions [2]. For this purpose, the lines of central extrema $W(a, k)$ (or a skeleton $L(a)$ of the wavelet transform) are separated, for which generalized partition functions

$$Z(q,a) = \sum_{l \in L(a)} |W(a,k_l(a))|^q \sim a^{\tau(q)} \quad (4)$$

also exhibit power dependence on the scale parameter. In this formula, $k_l(a)$ characterizes the position of maximum modulus of wavelet coefficients on the lth skeleton line. The spectrum of Hölder exponents is determined by differentiating exponents $\tau(q)$:

$$h(q) = \frac{d\tau(q)}{dq}. \quad (5)$$

Let us consider the application of this approach to investigation of the dynamics of model system (1). Figure 1 presents the results of integration of the model equations and calculation of the power spectra of dynamical regimes without noise ($I = 0$) and in the presence of fluctuations introduced as $I = 0.04$. According to Fig. 1, there are no visible distinctions between the deterministic ($I = 0$) and stochastic ($I > 0$) dynamics of system (1). Calculations of the largest Lyapunov exponent (which is among the most informative characteristics of complex dynamics) from the sequence of return times to the Poincaré section using a method developed by Wolf et al. [9] also showed that these regimes are almost identical in this respect ($\lambda_1 = 0.053$ vs. $\lambda_1 = 0.052$).

However, application of the multifractal formalism (capable of recognizing "fine" changes in the signal structure) revealed significant distinctions (see Fig. 2) between the spectra of Hölder exponents in the cases of deterministic and stochastic dynamics of system (1). In the former case, the dynamics is characterized by clearly pronounced multifractality (i.e., significantly different scaling characteristics for small and large fluctuations). In the latter case, the scaling properties are leveled and the process becomes more uniform in this respect. These changes can be interpreted as the noise-induced loss of multifractality. An analogous effect has been observed in the case of a stochastic resonance [10]. However, the stochastic resonance in a bistable system under an external periodic force in the presence of noise turns out to be a much simpler case

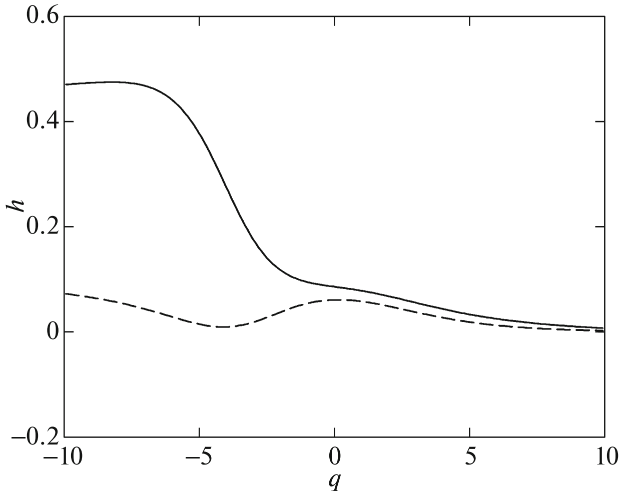

Fig. 2. Spectra of Hölder exponents in the regimes of deterministic (solid curve) and stochastic (dashed curve) dynamics in the model system

than the noise-induced loss of multifractality considered above in a model system with coexisting irregular dynamical regimes.

Some important issues need to be addressed. In contrast to the case of a stochastic resonance, a change in measure of multifractality depending on the intensity of noise does not exhibit clearly pronounced resonance character. Therefore further investigations of the laws of noise-induced loss of multifractality as dependent on the intensity of fluctuations present in the system are necessary. However, diagnostics of the corresponding variations can be performed using sample sequences of relatively short length. In particular, changes in the character of system dynamics can be reliably recognized by method [3] using sequences containing about 200–300 counts [8]. This circumstance allows the proposed approach to be applied to problems of secure data transmission with the use of chaotic carrier signals [10, 11].

In conclusion, switching between different complex dynamical regimes (Fig. 1) is not reliably recognized by methods based on standard digital processing of irregular signal, but use of the method of multifractal formalism based on a wavelet transform allows these regimes to be clearly distinguished (Fig. 1) using relatively short sample sequences.

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