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Noise-Induced Binary Synchronization in Nonlinear Systems

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Abstract—The phenomenon of noise-induced binary synchronization has been discovered in two independent dynamical systems generating aperiodic binary signals under the action of a common noise source. The presence of a synchronous regime was confirmed by the calculation of Lyapunov exponents for the two systems. The mechanism of development of the noise-induced binary synchronization regime has been found. A relation of the observed regime to binary generalized synchronization is established.

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Chaotic synchronization is among the fundamental concepts of the modern natural sciences [1]. The interest in this phenomenon is also related to the possibility of detecting and/or using various types of asynchronous behavior in a wide circle of various dynamic systems, including telecommunications and biological, physiological, social, and many other systems [2–5].

At present, several types of chaotic synchronous behavior are distinguished among which the most important and extensively studied are the regimes of total synchronization (TS) [6], lag synchronization [7], phase synchronization (PS) [8], generalized synchronization (GS) [9], and noise-induced synchronization (NS) [10]. It is important to note that all these types of chaotic synchronization have been mostly found and studied in analog systems. However, it is also possible theoretically to observe these regimes in binary (e.g., digital communication) systems where interacting subsystems generate aperiodic sequences of "zero" and "unity" bits. In the past two decades, researchers in this field have revealed and described TS, PS, and NS regimes in binary systems, which are called the "binary total" [11], "binary phase" [12], and "binary generalized synchronization" (BGS) [13]. Taking into account the common nature of GS and NS regimes in analog systems [14], it can also be expected that NS exists in binary systems and, according to the above classification, must be called the binary noise-induced synchronization (BNS).

The present work was aimed at studying the possibility that the NS regime exists in binary systems, revealing the mechanisms of its appearance, and establishing its relationship to the BGS regime.

First, let us introduce the concept of BNS. For this purpose, consider two independent systems $\mathbf{x}$ and $\mathbf{y}$ with continuous or discrete time and identical control parameters, which occur under the action of a common source of noise $\varepsilon f(\xi)$. Here, factor ε characterizes the noise signal intensity, f is a linear or nonlinear function, and ξ is a stochastic process. Assume that the systems generate aperiodic binary time series of $x = h[\mathbf{x}]$ and $y = h[\mathbf{y}]$ such that $x \neq y$ in the absence of noise for $\mathbf{x}_0 \neq \mathbf{y}_0$. Here, $\mathbf{x}_0$ and $\mathbf{y}_0$ are the vectors of initial conditions and h is some function that takes only two possible values: 0 or 1. Accordingly, variables x and y also take only the values of zero and unity. By analogy with the traditional NS regime [10, 14], the BNS will imply a regime in which binary time series x and y generated by the two systems become fully identical ($x = y$) after a transient process, provided that the noise intensity exceeds a certain critical level ε_c.

For BNS regime diagnostics, it is possible to use either direct comparison of x and y signals or calculation of the synchronization error, similarly to the case of analog signals [10]. In addition, the BNS diagnostics can be based on calculation of the spectrum of conditional Lyapunov exponents for one of the two ($\mathbf{x}$ or $\mathbf{y}$) systems occurring under the action of noise [14]. A criterion for the presence of synchronization in this case is a negative value of the highest conditional Lyapunov exponent.

The BNS phenomenon was found in a system of two uncoupled logistic maps

$$\begin{aligned} X_{n+1} &= g(X_n, \lambda) + \varepsilon(g(\xi_n, \lambda) - g(X_n, \lambda)), \\ Y_{n+1} &= g(Y_n, \lambda) + \varepsilon(g(\xi_n, \lambda) - g(Y_n, \lambda)), \end{aligned} \qquad (1)$$

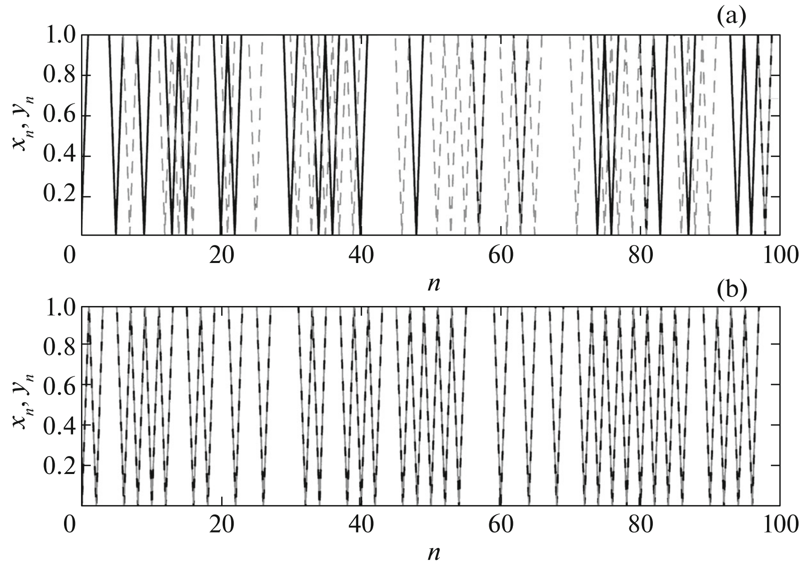

Fig. 1. Aperiodic binary time series obtained by numerically solving Eqs. (1) and (2) for two values of the external noise amplitude: (a) $\varepsilon = 0$ (asynchronous regime); (b) $\varepsilon = 0.2$ (BNS regime).

where X and Y are variables characterizing the states of interacting systems, $g(x, \lambda) = 1-\lambda x^2$, $\lambda = 1.6$ is the control parameter, ξ_n is the stochastic Gaussian process with mean value $\mu_0 = 0.5$ and variance $\sigma = 0.12$, and ε is the parameter of noise intensity. The external noise action upon logistic maps (1) was introduced by analogy with the case of traditional NS [14]. Binary time series were obtained from initial analog signals X_n and Y_n of logistic maps (1) by application of the Heaviside function $H(\zeta)$ so that

$$\begin{aligned} x_n &= H(X_n), \\ y_n &= H(Y_n). \end{aligned} \tag{2}$$

In the absence of noise ($\varepsilon = 0$), the two aperiodic time series x_n and y_n turn out to be different despite identical values of control parameter λ in the interacting subsystems (Fig. 1a). As the noise intensity increases, the two signals become close to each other and, when the synchronization error E approaches zero (Fig. 2a), a BNS regime appears in the system. The synchronization error for binary systems with discrete time was calculated using the following formula:

$$E = \frac{1}{N - N_0} \sum_{n=N_0}^{N} |x_n - y_n|,$$

where N is the number of iterations and N_0 is that corresponding to the transient process. In this case, binary time series x_n and y_n after termination of the transient process exactly coincide (Fig. 1b).

The presence of BNS in the system studied is also confirmed by the dependence of the highest conditional Lyapunov exponent Λ on intensity ε of the noise action (Fig. 2b). As can be seen from Fig. 2b, an increase in ε leads to the shift of Λ to the region of negative values, which is indicative of the onset of a BNS regime. It should be emphasized that the BNS regime onset identified in this way almost exactly coincides with the threshold determined by calculations of the synchronization error (cf. Figs 2a and 2b), which indicative of the possibility of using both methods for NBS diagnostics in the case under consideration.

Let us briefly discuss mechanisms responsible for development of the BNS regime, which turn out to be analogous to the mechanisms of NS in analog systems (see [14]). Since Eqs. (1) contain additional terms $-\varepsilon g(X, \lambda)$ responsible for growing dissipation in the system with increasing amplitude of external noise, it is evident that, similarly to the case of BGS, the development of a synchronous regime in the present case is determined primarily by suppression of the intrinsic chaotic dynamics under the action of external noise. Increased dissipation in systems occurring under the action of external noise leads to a shift of the highest conditional Lyapunov exponent to the negative region, which is a criterion for the appearance of both GS and NS regimes.

In concluding, analysis of the behavior of a model system with discrete time in this work allowed us for the first time to establish the possibility of BNS regimes. It is shown that the diagnostics of this regime can be based both on direct comparison of the states of subsystems occurring under common noise action and on calculations of the highest conditional Lyapunov exponent for one of the interacting systems. It is established that, similarly to the case of BGS, the development of a synchronous regime in the system studied is determined primarily by suppression of the intrinsic chaotic dynamics under the external noise action.

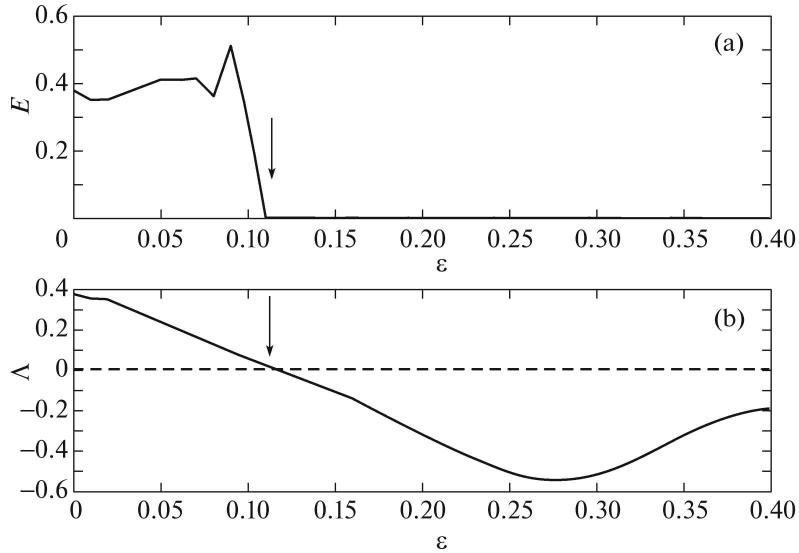

Fig. 2. Plots of (a) synchronization error E and (b) highest conditional Lyapunov exponent Λ vs. amplitude ε of the external noise action. The onset of BNS is indicated by the arrow in both plots.

It should be noted that a phenomenon analogous to the BNS can be observed if the noise is replaced by a harmonic signal at a frequency varying within broad limits. This substitution leads to quantitative changes in behavior of the system, but does not qualitatively modify the phenomenon under consideration.

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