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Studying Spatially Distributed Systems near the Boundary of Phase Chaotic Synchronization on Various Time Scales of Observation

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Abstract—The behavior of spatially distributed systems near the boundary of phase chaotic synchronization
1 has been studied on various time scales of observation. In this case, two types of intermittency can simultaneously exist. The results of numerical modeling are in good agreement with theoretical dependences.

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It is known that intermittent behavior is inherent in a wide range of nonlinear dynamical systems [1]. In particular, this type of behavior has been observed on the transition from periodic oscillations to chaotic regimes and near the boundaries of appearance of various types of chaotic synchronization of nonautonomous and coupled oscillators.

There exists a commonly accepted classification of the intermittent behavior, including types I–III [1],
1 1 on–off intermittency [2], eyelet intermittency [3],
1 and ring intermittency [4]. Although all these types have some common features (the presence of two
1 alternating regimes in time series), each type of intermittency has specific characteristics (dependence of the average duration of laminar phases on a control parameter, distribution of laminar phase durations at a fixed value of this parameter). The mechanisms responsible for the appearance of various types of
1 intermittency are also different.

Our recent investigations [5, 6] showed that nonlinear dynamical systems can occur in a regime in
1 which two types of intermittency exist simultaneously.
1 2 This behavior, called "intermittency of intermittencies," develops when the system admits the coexistence of two mechanisms leading to the appearance of
1 turbulent periods, each resulting in the intermittency of its own type. It should be noted that previous investigations of these regimes were performed for nonlinear systems with a small number of degrees of freedom.
1 Naturally, the question arises of whether the intermit-
2 tency of intermittencies can exist in spatially distributed systems and whether this behavior can be correctly described by a theoretical model developed for systems with a small number of degrees of freedom.

In the present work, we consider, by the example of two unidirectionally coupled Pierce diodes, the coex-
istence of two types of intermittency in spatially dis- 1
tributed systems: (i) eyelet intermittency that can be 1
observed near the boundary of phase chaotic synchro-
nization and (ii) ring intermittency that is observed in 1
a certain interval of time scales.

The main relations describing dynamics of a system of two unidirectionally coupled Pierce diodes in the framework of a hydrodynamic approximation comprise a self-consistent system of the equations of motion, equations of continuity, and Poisson's equation [7, 8]:

$$\frac{\partial \nu^{1,2}}{\partial t} = \nu^{1,2}\frac{\partial \nu^{1,2}}{\partial x} - \frac{\partial \varphi^{1,2}}{\partial x}, \tag{1}$$

$$\frac{\partial \rho^{1,2}}{\partial t} = -\frac{\partial(\rho^{1,2}\nu^{1,2})}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \varphi^{1,2}}{\partial x^2} = -\alpha_{1,2}^2(\rho^{1,2} - 1) \tag{3}$$

with the boundary conditions

$$\nu^{1,2}(0,t) = 1, \quad \rho^{1,2}(0,t) = 1, \quad \varphi^{1,2}(0,t) = 0, \tag{4}$$

where φ is the dimensionless potential of the space-charge field; ρ is the dimensionless charge density; ν is the dimensionless flux density; x is the dimensionless coordinate; t is the dimensionless time; and indices 1 and 2 refer to the driving (master) and driven (slave) beam–plasma subsystems, respectively. The sole control parameter characterizing the system dynamics is Pierce parameter α representing the undisturbed angle of electron motion at the plasma frequency. This angle is selected to be $\alpha_1 = 2.858\pi$ for the driving subsystem and $\alpha_2 = 2.860\pi$ for the driven subsystem, so as to set the detuning between the coupled Pierce diodes.

Unidirectional coupling between subsystems is achieved by varying the value of the dimensionless potential on the right-hand boundary of the driven system, while the potential on the right-hand boundary of the driving system remains unchanged:

$$\begin{cases} \varphi^1(1, t) = 0, \\ \varphi^2(1, t) = \varepsilon(\rho^2(x = 1, t) - \rho^1(x = 1, t)), \end{cases} \quad (5)$$

where ε is the coefficient of coupling between subsystems and $\rho^{1,2}(x = 1, t)$ are oscillations of the dimensionless space-charge density at the output of the corresponding subsystem. Thus, the driving subsystem occurs in a regime of self-sustained oscillations and acts upon the driven subsystem.

It should be noted that the behavior of a system of two coupled Pierce diodes has been studied on various time scales [9, 10]. According to this approach, a continuous set of signal phases is introduced via the continuous wavelet transform

$$W(s, t_0) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t) \psi^*\left(\frac{t - t_0}{s}\right) dt, \quad (6)$$

where $x(t)$ is the time series of the chaotic signal, $\psi_{s,t_0}(t)$ is the mother wavelet function, s is the time scale factor that determines the wavelet width, and the asterisk * denotes complex conjugation. The $x(t)$ signals for the Pierce diodes under consideration will represent the space-charge densities $\rho_{1,2}$ in the coupled subsystems measured at point $x = 2$ of the diode interaction space.

The mother wavelet is expediently defined via the Morlet function as

$$\psi(\eta) = (1/\sqrt[4]{\pi}) \exp(j\Omega_0\eta) \exp(-\eta^2/2) \quad (7)$$

with parameter $\Omega_0 = 2\pi$, which ensures single-valued relationship between wavelet transform time scale s and Fourier transform frequency f: $f = 1/s$. Using a complex wavelet basis set, it is possible to associate each time scale with the phase $\varphi(s, t) = \arg W(s, t)$, where $W(s, t)$ is the complex wavelet surface determined by Eq. (6).

Two coupled chaotic systems $\mathbf{x}_{1,2}(t)$ are considered to be in the regime of time scale synchronization if there is an interval of $s \in [s_1; s_2]$ in which the phase locking condition

$$|\varphi_1(s, t) - \varphi_2(s, t)| < 2\pi, \quad (8)$$

is obeyed and the wavelet spectrum energy fraction for this interval is positive:

$$E_{\text{snhr}} = \int_{s_1}^{s_2} \langle |W(s, t)|^2 \rangle ds > 0. \quad (9)$$

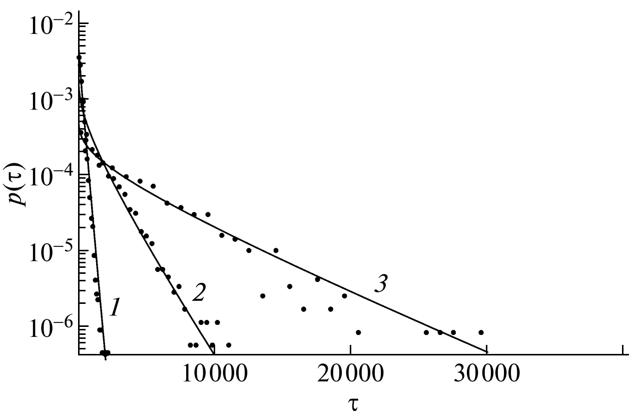

Fig. 1. (*1*)–(*3*) Distributions $p(\tau)$ of the laminar phase duration at fixed control parameters for the regime of coexistence of the eyelet and ring intermittency in two uni- 1
directionally coupled Pierce diodes (points) and corresponding analytical dependences (10) numerically calculated for these distributions (solid curves): (*1*) $\varepsilon = 0.006$, $s = 2.7255$, $T_e = 2976$, $T_r = 250$; (*2*) $\varepsilon = 0.006$, $s = 2.73$, $T_e = 2976$, $T_r = 3125$; and (*3*) $\varepsilon = 0.007$, $s = 2.7325$, $T_e = 20\,920$, $T_r = 8474$. Ordinates are plotted on a logarithmic scale.

Similarly to systems with a small number of the degrees of freedom, the spatially distributed systems also exhibit the regime of time scale synchronization for certain values of control parameters. In this case, the system has both synchronous time scales and asynchronous ones, for which conditions (8) and (9) are not satisfied. For diagnostics of the regime of time scale synchronization, the system must occur in the regime of phase chaotic synchronization, as was the case in [5, 6]. In the regime of phase synchronization at boundary time scales of observation, the system exhibits intermittent behavior. Let us consider the characteristics of this intermittency in spatially dis- 1
tributed systems.

Figure 1 shows distributions of the laminar phase duration at fixed values of control parameters, which were numerically calculated for two unidirectionally coupled Pierce diodes in a regime of simultaneous existence of the eyelet intermittency and ring inter- 1 1
mittency for three sets of coupling coefficient ε and observation time scale s. Since the mechanisms leading to intermittencies of the eyelet and ring type are 2
different, it is possible to separate the phase skips that refer to different types of intermittency and then esti- 1
mate the values of T_e and T_e (average durations of laminar behavior for intermittency of the eyelet and 1
ring types, respectively) that enter into the theoretical relation for the distribution of laminar phase

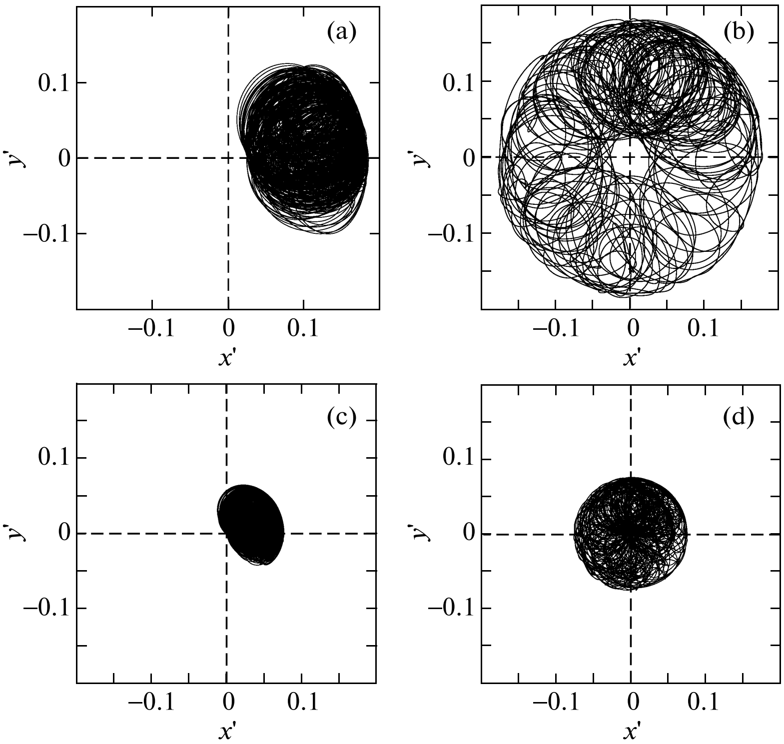

Fig. 2. Phase trajectories of the driven system on the (x', y') plane rotating about the origin in various regimes: (a) $\varepsilon = 0.02$, $s =$
1 1 4.71875 (synchronous regime); (b) $\varepsilon = 0.007$, $s = 4.71875$ (eyelet intermittency); (c) $\varepsilon = 0.02$, $s = 2.71875$ (ring intermittency);
1 and (d) $\varepsilon = 0.007$, $s = 2.71875$ (coexistence of eyelet and ring intermittency).

1 2 durations in the regime of "intermittency of intermittencies" [5]:

$$p(\tau) = \frac{\exp(-\tau/T_e)}{(T_e + T_r)}\left(1 - \frac{\tau}{T_e}\right)\Gamma\left(0, \frac{\tau}{T_r}\right) + \frac{T_e^2 + T_r^2}{T_e T_r (T_e + T_r)}\exp\left(-\frac{\tau}{T_e} - \frac{\tau}{T_r}\right) + \frac{\exp(-\tau/T_r)}{(T_e + T_r)}\left(1 - \frac{\tau}{T_r}\right)\Gamma\left(0, \frac{\tau}{T_e}\right). \quad (10)$$

As can be clearly seen from Fig. 1, the results of numerical calculations of the distributions of laminar phase durations well agree with theoretical dependence (10), which allows one to speak of the regime of
1 2 "intermittency of intermittencies" in the system under consideration.

Another conclusive piece of evidence of the exist-
1 2 ence of the regime of "intermittency of intermittencies" in the system of two unidirectionally coupled Pierce diodes is provided by analysis of the system dynamics on a rotating plane by analogy with that in [5, 6]. According to this approach, the variation of $x_{1,2} = \mathrm{Re}W_{1,2}(s, t)$ and $y_{1,2} = \mathrm{Im}W_{1,2}(s, t)$ is considered on the plane rotating about origin [4]:

$$x' = x_2\cos\varphi_1 + y_2\sin\varphi_1, \quad y' = -x_2\sin\varphi_1 + y_2\cos\varphi_1. \quad (11)$$

Figure 2 shows the behavior of two unidirectionally coupled Pierce diodes on rotating plane (11). As can be seen, it is possible to distinguish regions of coupling coefficient ε and observation time scale s in which either the eyelet intermittency (Fig. 2b) or ring inter- 1 1
mittency (Fig. 2c) is observed. In addition, there are regions of the coupling coefficient and observation time in which both these phenomena are simultaneously observed (Fig. 2d). This implies that the eyelet intermittency is interrupted by the ring intermittency 1 1
and vice versa. Thus, the behavior of the phase trajectory is also indicative of the coexistence of two types of intermittency. In this regime, the phase trajectory on 1
the (x', y') plane rotates about the origin of coordinates (which is a manifestation of eyelet intermittency) and 1
envelopes the origin (which is evidence of ring inter- 1
mittency).

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SPELL: 1. intermittency, 2. intermittencies