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SHORT COMMUNICATIONS

Analysis of Chaotic Dynamic Regimes Using Series of Interburst Intervals

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Abstract—The problem of reconstruction of dynamic systems in the presence of noise using series of interburst intervals is solved. It is shown that the reconstruction procedure can be applied to strongly nonlinear noisy oscillatory processes. The results make it possible to generalize the method for analysis of dynamic systems with respect to recovery time to a wide variety of neuron oscillators.

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The problem of diagnostics of complex oscillatory regimes under limitations on available data occurs in several scientific and technical applications. The calculation of quantitative characteristics of chaotic dynamics (e.g., Lyapunov exponents or generalized fractal dimensions) can be implemented with the aid of the known equations of the mathematical model [1, 2]. In the presence of the time dynamics of one of dynamic variables, the problem can be solved using methods for reconstruction of dynamic systems based on experimental data [3]. In this case, the reconstruction quality and, hence, accuracy of estimations depend on the sample size, presence of noise in the detected signal, etc. [4, 5]. The reconstruction problem is difficult to solve if the data on the dynamics of the system represent series of characteristic repeated events (e.g., series of pulse generation times of neurons [6] or moments at which the phase trajectory crosses the Poincare section). Such problems emerge, in particular, when the nonlinear system that transforms signals contains a threshold unit. Theoretical study has been performed for threshold devices that can be described using the integrate-and-fire models [7]. The corresponding results make it possible to substantiate the possibility of reconstruction of dynamic systems and calculation of characteristics of chaotic oscillation regimes using input signals [8–12]. For alternative classes of threshold devices, the reconstruction quality for attractor has been tested with the aid of computer simulation [13, 14].

However, the applicability limits of the reconstruction methods have not been determined. Dynamic regimes with relatively uniform structure of attractor (e.g., Rössler model in the regime of spiral chaos) are predominantly employed as input signals in [8–13]. Such regimes do not allow full-scale tests of the applied methods. Note insufficient data on strongly nonlinear systems for which the phase trajectory contains fragments with alternation of fast and slow dynamics and the effect of noise on the accuracy of characterization of chaotic oscillations using the input signals of the threshold devices.

In this work, we solve the reconstruction problem for dynamic systems that generate chaotic bursts (trains of pulses with varying time intervals between the generation moments). A method that has been proposed in [14, 15] and improved in [16] is used for reconstruction of the chaotic attractor using series of interburst intervals. The method is based on the approximation of the instantaneous oscillation frequency. The calculation of the largest Lyapunov exponent λ_1 based on the reconstructed attractor is performed using the method of [17] with allowance for the modification proposed in [18] for the analysis of noisy series of time intervals. In accordance with the conventional approach of [17], we analyze the mean rate of the exponential divergence of trajectories. In this case, we choose the basic trajectory and determine the perturbation vector. A variation in the magnitude of the vector with time makes it possible to estimate the chaoticity. Outside the limits of the linear approximation, we perform renormalization and determine perturbation vector with smaller length that is oriented along the direction of the maximum divergence of trajectories. The analysis of the dynamics of chaotic systems based on time series must be performed with allowance for compromise of the minimization of the vector length and minimization of orientation error α, since such minimizations cannot be simultaneous. A practical method employs an interval

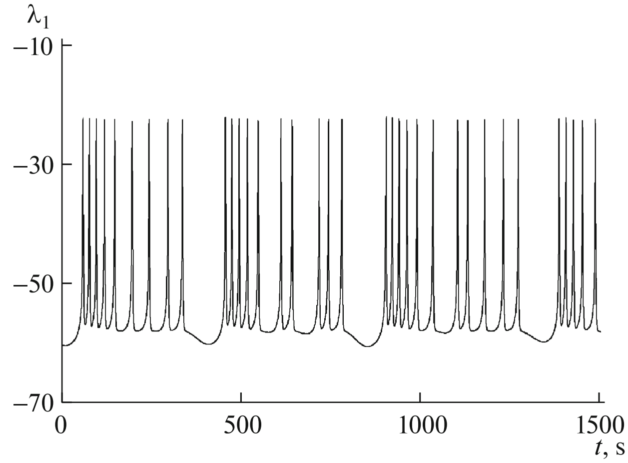

Fig. 1. Time dependence of the voltage across the membrane of the beta-cell. The intervals between the neighboring peaks of the signal are used for reconstruction of the phase portrait [16] and the subsequent calculation of the largest Lyapunov exponent.

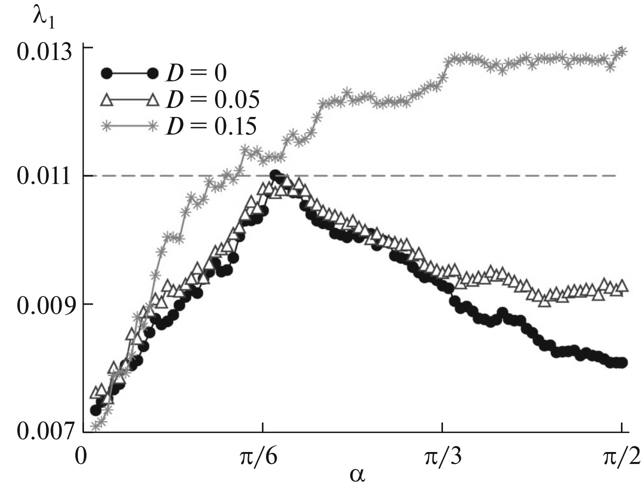

Fig. 2. Plots of the largest Lyapunov exponent vs. maximum orientation error of the perturbation vector that are calculated using the series of interburst intervals in the absence of noise and in the presence of additive noise signals with different intensities.

of possible vector lengths $[l_{\min}, l_{\max}]$ at constant orientation. The results of [18] show that the analysis of dependence $\lambda_1(\alpha)$ makes it possible to increase the calculation accuracy and determine conditions under which the presence of additive noise does not prevent correct estimation of the chaoticity of the dynamic process. Additional data on the specific features of instability of trajectories can be used with the aid of the scale-dependent Lyapunov exponents [19, 20].

A model of the pancreatic beta-cell serves as the system with several time scales that exhibits the burst regime [21]:

$$\frac{dV}{dt} = (-I_{\mathrm{Ca}} - I_{\mathrm{K}} - g_S S(V - V_{\mathrm{K}}))/\tau,$$

$$\frac{dn}{dt} = \mu(n_\infty - n)/\tau,$$

$$\frac{dS}{dt} = (S_\infty - S)/\tau_{\mathrm{S}},$$

$$I_{\mathrm{Ca}}(V) = (g_{\mathrm{Ca}} - S)/\tau_{\mathrm{S}}, \qquad (1)$$

$$I_{\mathrm{Ca}}(V) = g_{\mathrm{Ca}} m_\infty (V - V_{\mathrm{Ca}}),$$

$$I_{\mathrm{K}}(V, n) = g_{\mathrm{K}} n (V - V_{\mathrm{K}}),$$

$$x_\infty = \frac{1}{1 + \exp((V_x - V)/\theta_x)}, \quad x = m, n, S.$$

We use the following control parameters: $g_{\mathrm{Ca}} = 3.6$, $g_{\mathrm{K}} = 10.0$, $g_{\mathrm{S}} = 4.0$, $\tau = 20$ ms, $\tau_{\mathrm{S}} = 35$ s, $V_{\mathrm{Ca}} = 25$ mV, $V_{\mathrm{K}} = -75$ mV, $V_m = -20$ mV, $V_n = -16$ mV, $V_S = -40$ mV, $\theta_m = 12$ mV, $\theta_n = 5.6$ mV, $\theta_S = 10.0$ mV, and $\mu = 0.85$. Such parameters provide the generation of chaotic oscillations with a Lyapunov exponent of $\lambda_1 = 0.011$ [21]. In model (1), variable V is the voltage across the cell membrane, parameter n characterizes variations in the number of open calcium channels, and parameter S is the intracellular concentration of calcium. We analyze the time dependence of variable V (Fig. 1).

To study the possibility of diagnostics based on the series of interburst intervals, we consider deterministic dynamics and use a series consisting of 3000 time intervals between the neighboring peaks of variable $V(t)$ as the signal under study. With allowance for significant variations in time intervals between neighboring pulses in a single burst and even more significant burst-to-burst variations in such intervals, we obtain a series that is characterized by a substantial spread of time intervals and leads to nonuniformities of attractor that is reconstructed with the aid of the method of [16].

The dependence of the largest Lyapunov exponent on maximum orientation error α of the perturbation vector [17] that is calculated using a series of interburst intervals leads to a developed maximum (circles in Fig. 2) that corresponds to theoretically expected λ_1. The reason for a decrease on the left-hand side of the maximum is an increase in the length of the perturbation vector and escape from the limits of the linear approximation. On the right-hand side, parameter λ_1 is underestimated due to an increase in the components of the perturbation vector along the directions that are orthogonal to the direction of the maximum spread of trajectories.

In the presence of the additive noise in the series of the interburst intervals, we observe changes of dependence $\lambda_1(\alpha)$ at relatively large angles (triangles in Fig. 2) owing to the effect of additional (noise-induced) spread of trajectories. When the intensity of the additive noise increases, the maximum that is observed for deterministic dynamics vanishes (asterisks in Fig. 2) and the method that we use does not allow quantitative characterization of the dynamic chaos in system (1) using signals that are detected in the presence of noise. As for simpler dynamic regimes of [18], dependence $\lambda_1(\alpha)$ exhibits two important markers: maximum at relatively small angles, which makes it possible to calculate the largest exponent that is close to the expected value, and decreasing fragment of dependence $\lambda_1(\alpha)$ on the right-hand side of the maximum the size of which characterizes the effect of noise on calculated results. If the size of such a fragment decreases and the fragment vanishes at a certain intensity of the additive noise, the largest Lyapunov exponent of the noise-free dynamic regime cannot be correctly estimated.

Thus, the above analysis proves the existence of common regularities for the dependences of the largest Lyapunov exponent (calculated using a series of moments of return to the Poincare section) on the maximum orientation error of the perturbation vector in the reconstructed phase space for the regimes of deterministic chaos and noisy chaotic oscillations. This circumstance makes it possible to generalize the method for analysis of dynamic systems based on return times to a wide variety of neuron oscillators and employ the procedure in diagnostics of dynamic regime of oscillatoty systems using various experimental results.

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