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Binary generalized synchronization

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ABSTRACT

In this paper we report for the first time on the binary generalized synchronization, when for the certain values of the coupling strength two unidirectionally coupled dynamical systems generating the aperiodic binary sequences are in the generalized synchronization regime. The presence of the binary generalized synchronization has been revealed with the help of both the auxiliary system approach and the largest conditional Lyapunov exponent calculation. The mechanism resulting in the binary generalized synchronization has been explained. The finding discussed in this paper gives a strong potential for new applications under many relevant circumstances.

1. Introduction

Chaotic synchronization is known to be one of the fundamental phenomena, widely studied recently since it has both the theoretical and applied significance [1,2]. The study of the chaotic synchronization phenomenon is the central branch of the dynamical chaos theory dictated by its major fundamental significance accompanied with the broad spectrum of the practical applications including (but not limited to) the biological [3–5], physiological [6–9], chemical [10,11], controlling chaos [12,13] and information transmission [14–22] tasks.

There are the different types of chaotic synchronization revealed and studied recently such as phase synchronization [23,24], generalized synchronization [25,26], lag synchronization [27,28], complete synchronization [29,30], time scale synchronization [31,32], anti-synchronization[33], passive synchronization [19], adaptive synchronization [34,35], projective synchronization [36–38], generalized projective synchronization [39–41], modified generalized projective synchronization [21], etc. Among the different types of the synchronous behavior of chaotic systems the generalized synchronization (GS) [25,26,42] stands out due to its interesting features [43–46] and possible applications [18,47,48]. This kind of synchronous behavior means the state vectors of the interacting chaotic systems being in the generalized synchronization regime are related with each other. It has been observed in many systems both numerically [43,49–51] and experimentally [46,52,53].

The significant progress in the generalized synchronization studies has been achieved recently. In parallel with the revealing the mechanism being responsible for the generalized synchronization regime arising in the unidirectionally coupled chaotic oscillators [45,54,55], the concept of the generalized synchronization phenomenon has been extended to the mutually coupled systems and networks [56], as well as the relationship between the interacting systems has been clarified [57,58]. At the same time, generalized synchronization has been observed hitherto only for the *analog* systems (both the flows and maps), whereas phase synchronization and complete synchronization are known to be

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found recently in the *binary* systems whose signals contain only bits “0” and “1” (see Ref. [59] and Ref. [60], respectively).

Therefore, there is a fundamental problem, whether the type of synchronization with the properties of GS can exist for the systems generating the aperiodic binary signals. This problem is important from the point of view of both the generalized synchronization theory development and the generality of the chaotic synchronization phenomenon. In other words, the problem is the following: whether for the certain values of the coupling strength ε two coupled dynamical systems $\mathbf{x}(t)$ and $\mathbf{y}(t)$ generating the aperiodic signals $x(t)$ and $y(t)$ (where $x(t)$ and $y(t)$ take only the values “0” or “1”, t is time which may be both continuous and discrete) can be in the generalized synchronization regime.

In this work we report for the first time on the generalized synchronization between two unidirectionally coupled binary systems. Such type of synchronization we call as *binary generalized synchronization* (BGS).

2. Binary generalized synchronization

Let $\mathbf{x}$ and $\mathbf{y}$ are the drive and response systems (coupled unidirectionally with the coupling strength ε) generating the aperiodic binary signals $x = h(\mathbf{x})$ and $y = g(\mathbf{y})$, where h and g are some functions, with x and y taking only the values “0” or “1”. The dynamics of the systems $\mathbf{x}$ and $\mathbf{y}$ are governed by the evolution operators $\mathbf{H}[\cdot]$ and $\mathbf{G}[\cdot]$

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{H}[\mathbf{x}(t)]\\ \dot{\mathbf{y}}(t) &= \mathbf{G}[\mathbf{y}(t), \varepsilon x(t)]\end{aligned} \tag{1}$$

in the case of the flow systems and by

$$\begin{aligned}\mathbf{x}_{n+1} &= \mathbf{H}[\mathbf{x}_n]\\ \mathbf{y}_{n+1} &= \mathbf{G}[\mathbf{y}_n, \varepsilon x_n]\end{aligned} \tag{2}$$

in the case of the discrete maps.

The binary generalized synchronization takes place, when the condition

$$y = F[x] \tag{3}$$

is satisfied, where $F[\cdot]$ is the functional. The functional $F[\cdot]$ means the state of the response system signal y depends not only on the state of the drive signal x (whose value can be “0” or “1”) at the fixed moment of the continuous or discrete time, but on the pre-history of the evolution of the drive system (see works [57,58] where this point is considered in detail).

To detect the generalized synchronization regime in the unidirectionally coupled systems the different techniques have been proposed, e.g., the nearest neighbor method [25] or the conditional Lyapunov exponent calculation [26]. Among these techniques the auxiliary system approach proposed for the unidirectionally coupled chaotic oscillators may be generally considered as the most easy, clear and powerful tool to study the generalized synchronization regime in chaotic systems. Starting from the seminal paper of *Abarbanel* et al. [42], the auxiliary system approach has become de-facto the standard of generalized synchronization studies. Although the auxiliary system approach is not applicable for the mutual type of coupling [61] it is the very effective tool to detect the GS regime in unidirectionally coupled chaotic systems. The auxiliary system approach has been used in the plenty of theoretical and experimental works (see, e.g., [52,54,62,63]).

In our work we use two approaches mentioned above to detect the BGS regime, namely the auxiliary system approach and the largest conditional Lyapunov exponent calculation. The core idea of the auxiliary method approach consists in the parallel consideration of the dynamics of the response system $\mathbf{y}$ and the auxiliary system $\mathbf{z}$, whose dynamics is governed by

$$\dot{\mathbf{z}}(t) = \mathbf{G}[\mathbf{z}(t), \varepsilon x(t)] \qquad \text{or} \qquad \mathbf{z}_{n+1} = \mathbf{G}[\mathbf{z}_n, \varepsilon x_n]. \tag{4}$$

The auxiliary system must be completely identical to the response system $\mathbf{y}$, but it starts with the other initial conditions, i.e., $\mathbf{u}(t_0) \neq \mathbf{y}(t_0)$. If the generalized synchronization regime takes place, the signals of the response system, $y = g(\mathbf{y})$, and the auxiliary system, $z = g(\mathbf{z})$, become identical after the transient, since $y = F[x]$ and, simultaneously, $z = F[x]$. Obviously, in this case the condition $y = z$ should be fulfilled. On the contrary, in the case of the absence of generalized synchronization, the dynamics of the response $\mathbf{y}$ and auxiliary $\mathbf{z}$ systems are unrelated, $y \neq z$. Therefore, to detect the BGS regime, one has to compare the time series of the response and auxiliary systems with each other. To compare these time series, one can consider *the error*

$$E = y - z, \tag{5}$$

between values of the response and auxiliary system signals.

The presence of the generalized synchronization may be detected also with the help of the largest conditional Lyapunov exponent calculation. If the dimensions of the drive and response systems are equal to N, the *Lyapunov exponent spectrum* of the interacting systems is $\Lambda_1 \geq \Lambda_2 \geq \cdots \geq \Lambda_{2N}$. Since the drive system dynamics is independent on the behavior of the response system, this spectrum may be divided into two parts: exponents of the drive system $\Lambda_1^d \geq \cdots \geq \Lambda_N^d$ and conditional Lyapunov exponents [30,64] $\Lambda_1^r \geq \cdots \geq \Lambda_N^r$. The generalized synchronization regime takes place if and only if $\Lambda = \Lambda_1^r < 0$ (see [26] for detail).

3. Binary generalized synchronization in discrete systems

We have observed *the binary generalized synchronization* in two unidirectionally coupled systems whose equations read as

$$\begin{aligned}x_{n+1} &= H(\eta_{n+1}), \quad \eta_{n+1} = f(\eta_n, \lambda_d),\\ y_{n+1} &= H(\zeta_{n+1}), \quad \zeta_{n+1} = f(\zeta_n, \lambda_r) + \varepsilon\zeta_n^2 x_n,\end{aligned} \tag{6}$$

where x_n, y_n are the binary sequences under study, η_n and ζ_n are supposed to be the interior (hidden) variables whose dynamics is governed by the evolution operator

$$f(\xi, \lambda) = 1 - \lambda\xi^2, \tag{7}$$

λ_d and λ_r are the control parameters of the drive and response systems, respectively, ε is the coupling strength

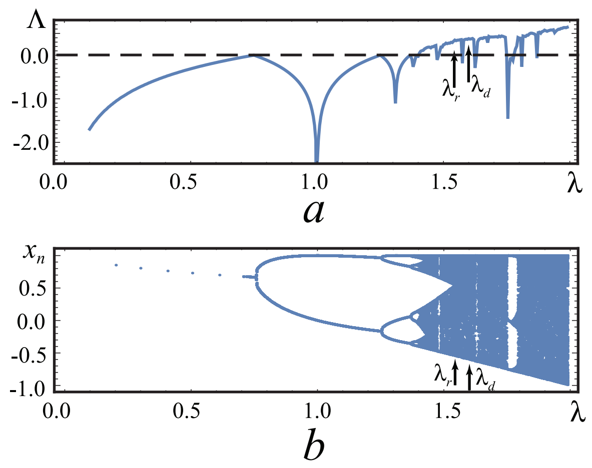

Fig. 1. (a) The bifurcation diagram of the evolution map (7) and (b) the dependence of its Lyapunov exponent, Λ, on the λ-parameter. The values of $\lambda_d = 1.6$ and $\lambda_r = 1.54$ corresponding to the drive and response systems, respectively, are shown with the help of arrows.

and

$$H(\xi) = \begin{cases} 0, & x < 0, \\ 1, & x \geq 0, \end{cases} \tag{8}$$

is the Heaviside function. We have chosen the values of the control parameters as $\lambda_d = 1.6$ and $\lambda_r = 1.54$ for both the drive and response systems in the absence of coupling (i.e., for $\varepsilon = 0$, when the dynamics of the hidden variables η_n and ζ_n are governed by logistic map (7)) to exhibit the chaotic behavior (see Fig. 1).

To reveal the BGS regime in (6) we have used the auxiliary system approach described above. In parallel with the response system we have considered the auxiliary system z_n (see Eq. (4))

$$z_{n+1} = H(\varsigma_{n+1}), \quad \varsigma_{n+1} = f(\varsigma_n, \lambda_r) + \varepsilon \varsigma_n^2 x_n, \tag{9}$$

which is completely identical to the response one, but starts with the different initial condition in comparison with the response system, i.e., $\varsigma_0 \neq \zeta_0$.

The behavior of two coupled systems (6) generating the binary signals is illustrated in Figs. 2 and3 for two different values of the coupling strength ε. Fig. 2 corresponds to the relatively weak coupling $\varepsilon = 0.2$ between the systems when the generalized synchronization is not observed. One can see that the binary sequences generated by the drive, response and auxiliary systems differ from each other (Fig. 2a–c). To make sure that for $\varepsilon = 0.2$ the interacting systems are in the asynchronous regime, in Fig. 2d the difference between the states of the response and auxiliary systems, $y_n - z_n$, is given.

With the increase of the coupling strength ε, the interacting systems undergo into the generalized synchronization regime (Fig. 3). Whereas the behavior of the drive system remains unchanged (due to the unidirectional type of coupling), after the short transient (approx. 100 units of the discrete time) the dynamics of the response and auxiliary systems becomes identical (Fig. 3b–d) that is the evidence of the generalized synchronization of the binary drive and response systems (6).

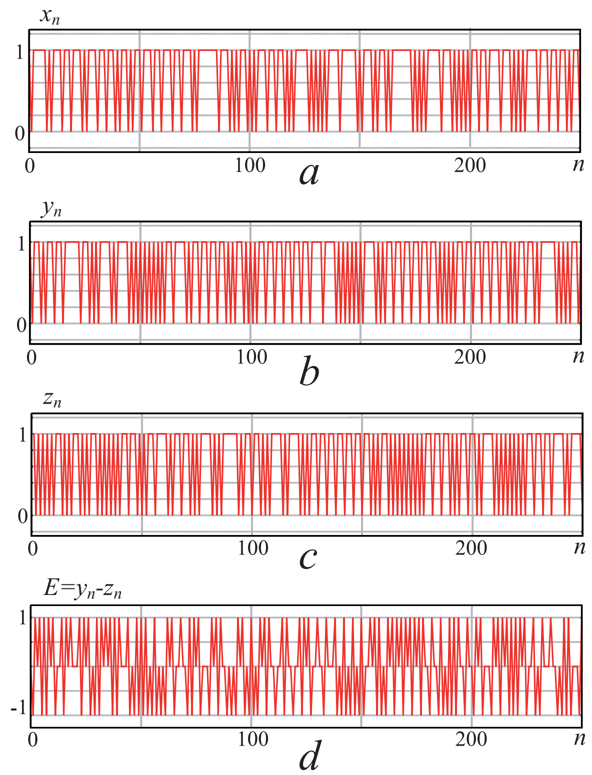

Fig. 2. Asynchronous dynamics of two unidirectionally coupled binary systems, $\varepsilon = 0.2$. Time series shown in Figure correspond to (a) the drive system, (b) the response system, (c) the auxiliary system and (d) the difference E between the time series of the response and auxiliary systems.

To validate the presence of the generalized synchronization regime, in parallel with the auxiliary system approach, we have also used the calculation of the conditional Lyapunov exponent (CLE) of the response system [26]. Although for the binary signals there is no possibility to get the Lyapunov exponent value (because the output variable takes only two possible values "0" and "1" and one can not find the small variable variation which is necessary for the LE calculation), we can calculate CLE of the system (6) as

$$\Lambda = \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln |2(\varepsilon x_i - \lambda_r)\zeta_i|, \tag{10}$$

since for the model system we can suppose that the hidden variable ζ is known.

The dependencies of the conditional Lyapunov exponent Λ of the response system on the coupling strength ε for three different values of the control parameter λ_r are shown in Fig. 4. One can see that for the considered above value $\lambda_r = 1.54$ the conditional Lyapunov exponent value decreases with the increase of the coupling strength and becomes sufficiently negative above $\varepsilon_{BGS} \approx 0.35$ (the solid line 1 in Fig. 4) that can be considered as the binary generalized synchronization onset. With the growth of the control parameter λ_d (curves 2 and 3 in Fig. 4), the dynamics of the response system becomes more complex (see the corresponding values of CLE for $\varepsilon = 0$) and, as a consequence, the coupling strength

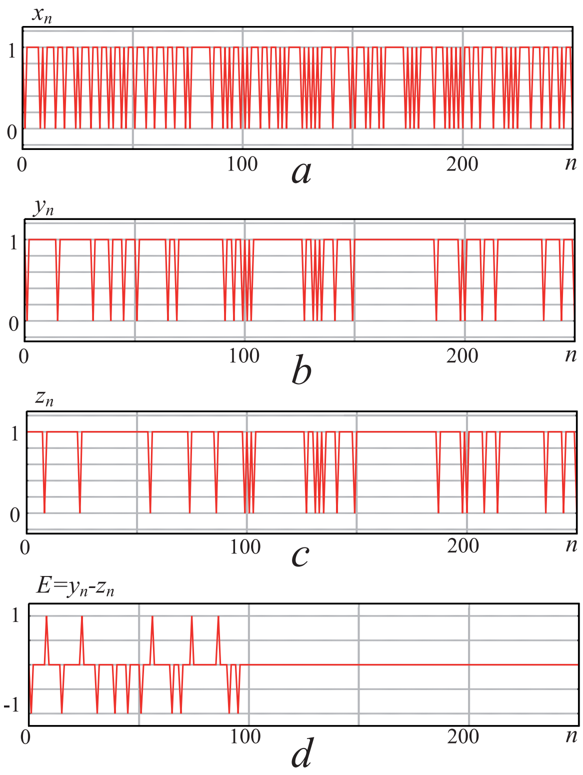

Fig. 3. Binary generalized synchronization regime, $\varepsilon = 0.54$. Time series shown in Figure correspond to (a) the drive system, (b) the response system, (c) the auxiliary system and (d) the difference E between the time series of the response and auxiliary systems.

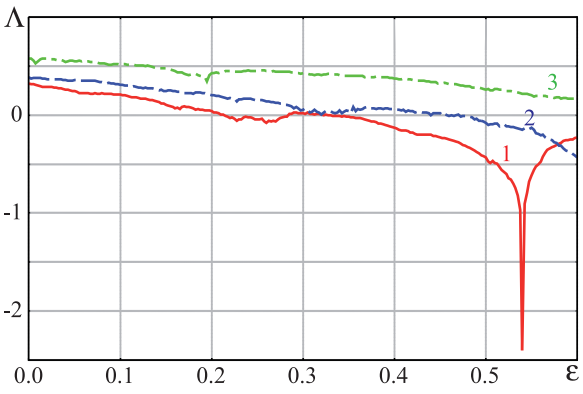

Fig. 4. The dependencies of the conditional Lyapunov exponent Λ of the response system on the coupling strength ε for three different values of the control parameter λ_r: $\lambda_r = 1.54$—solid line 1, $\lambda_r = 1.62$—dashed line 2, $\lambda_r = 1.95$—dotted line 3.

must be greater to get the binary generalized synchronization regime. So, for $\lambda_r = 1.62$ the onset of BGS is at $\varepsilon_{BGS2} \approx 0.48$ (the dashed line 2 in Fig. 4), whereas for $\lambda_r = 1.95$ the binary generalized synchronization is not observed at all in the considered range of the coupling strength values.

The consideration of the conditional Lyapunov exponent allows also to understand the relationship between the synchronous dynamics (from the point of view of the generalized synchronization regime) of the hidden and binary variables. It is obviously, that the generalized synchronization of the hidden variables (i.e., η_n and ζ_n) implies definitely the generalized synchronization in terms of the binary variables, x_n and y_n. At the same time, the reverse relationship is not so obvious, but, fortunately, CLE allows one to solve this problem. Indeed, the increase/decrease of the difference between the hidden variables of the response and auxiliary systems, $\delta = \zeta - \varsigma$, is determined completely by the sign of CLE. When CLE is positive (the generalized synchronization regime is not observed), the difference δ between the values of the hidden variables of the response and auxiliary system increases, and, since the values of the hidden variables are bounded ($-\zeta_m < \zeta < \zeta_m$, $-\zeta_m < \varsigma < \zeta_m$, $0 < \zeta_m \le 1$), δ-variable exhibits the chaotic behavior, with its value located within the range $(-2\zeta_m, 2\zeta_m)$. Obviously, when $-2\zeta_m < \delta < -\zeta_m$ and $\zeta_m < \delta < 2\zeta_m$ the hidden variables ζ and ς are characterized by the different signs, and, as a consequence, the variables y and z corresponding to the response and auxiliary systems, respectively, are also different. So, the binary variables are synchronized (in terms of the generalized synchronization regime) if and only if the hidden variables are in the generalized synchronization regime.

It should be noted that from the engineering point of view the binary systems demonstrate several non-ideal behaviors such as jitter, time rise and time fall affecting the binary pulses (see, e.g., [65,66]). In numerical experiments on the BGS such processes can be simulated by the addition of noise in the systems under study or by the control parameter mismatch [67]. At the same time, as well as the GS of unidirectionally coupled chaotic systems (see, e.g., [46]), the BGS regime possess a great enough stability to the external perturbations. In such case the stability of the synchronous regime is defined by the stability of the GS regime established between the hidden variables of interacting systems just in the same way as for the GS phenomenon [46]. At the same time, since the type of coupling between interacting systems considered in [46] is different in comparison with the system under study, the quantitative values of the boundaries of GS and BGS regime stability are also differed.

To illustrate the stability of BGS in logistic maps (6) to external noise we have analyzed the dependencies of the boundary values of the synchronous regime onset on the noise intensity. We have assumed that both the response and auxiliary systems are subjected to the additional noise signal $D\xi$ where ξ is the stochastic process which probability density is distributed uniformly on the interval [0; 1], D defines the intensity of noise.

Fig. 5 illustrates the dependencies of the threshold of the BGS regime onset on the noise intensity for three different values of the control parameter λ_r and fixed values of other control parameters. On the horizontal axis the signal to noise ratios (SNR, [dB]) corresponding to these noise intensities are also indicated[1]. It is clearly seen that in all considered cases the boundary value of the BGS regime does not practically depend on the noise intensity $D \in [0, D_c]$ where D_c (shown by arrows in Fig. 5) depends on the choice of the control

[1] The SNR value has been calculated as $\mathrm{SNR} = 10\lg \frac{P_{sign}}{P_{noise}}$, where P_{sign} is a power of chaotic signal, P_{noise} is a power of noise affected the chaotic system [68].

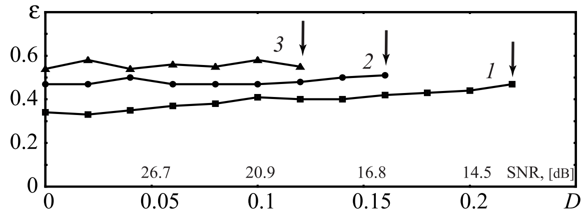

Fig. 5. Dependencies of the threshold value of the BGS regime onset in logistic maps (6) on the noise intensity (the SNR-values corresponding to the noise intensities are also shown) for different values of the control parameters: $1 - \lambda_r = 1.54$, $2 - \lambda_r = 1.62$, $3 - \lambda_r = 1.7$. Critical values of the noise intensity D_c, up to which the GS regime in system (6) is observed, are marked by arrows.

parameters in the same way as it has been discussed in [46]. The further increase of the noise intensity $D > D_c$ results in the runaway of the representation point to infinity that is caused by the properties of the logistic map itself (see [46] for details). At the same time, for the selected values of the control parameters $D_c \in [0.12; 0.22]$ (SNR $\in [19.07; 13.95]$ dB, respectively). In other words, the BGS regime in the system under study possesses high but limited stability to external noise. It should be noted that if the response and auxiliary systems would be subjected to different noise, the stability of the synchronous regime would show the same properties as for the systems being in the phase synchronization regime (see, e.g. [18,69]).

4. Binary generalized synchronization in flow systems

To prove the generality of the revealed phenomenon we have also shown that the binary generalized synchronization may be also observed for the systems generating the continuous binary signals, where the length of bits is variable (contrary to the discrete systems such as (6) generating the discrete binary signals). For this purpose we have considered the flow binary system based on Rössler oscillator being the standard object of the nonlinear dynamics. The irregular drive binary signal $m(t)$ is supposed to be generated by Rössler system

$$\begin{aligned} \dot{x}_d &= -\omega_d y_d - z_d, \\ \dot{y}_d &= \omega_d x_d + a y_d, \\ \dot{z}_d &= p + z_d(x_d - c) \end{aligned} \tag{11}$$

as

$$m(t) = H(x_{th} - x_d(t)), \tag{12}$$

where $H(x)$ is the Heaviside function (8), $x_{th} = 5.0$ is the threshold value governing the switching between bits “0” and “1”, the control parameters of Eq. (11) have been set to $a = 0.15$, $p = 0.2$, $c = 10.0$, $\omega_d = 0.93$. The response binary signal $n(t)$, in turn, is generated by the response flow system

$$\begin{aligned} \dot{x}_r &= -\omega_r y_r - z_r - \varepsilon m x_r, \\ \dot{y}_r &= \omega_r x_r + a y_r, \\ \dot{z}_r &= p + z_r(x_r - c) \end{aligned} \tag{13}$$

as

$$n(t) = H(z_r(t) - z_{th}), \tag{14}$$

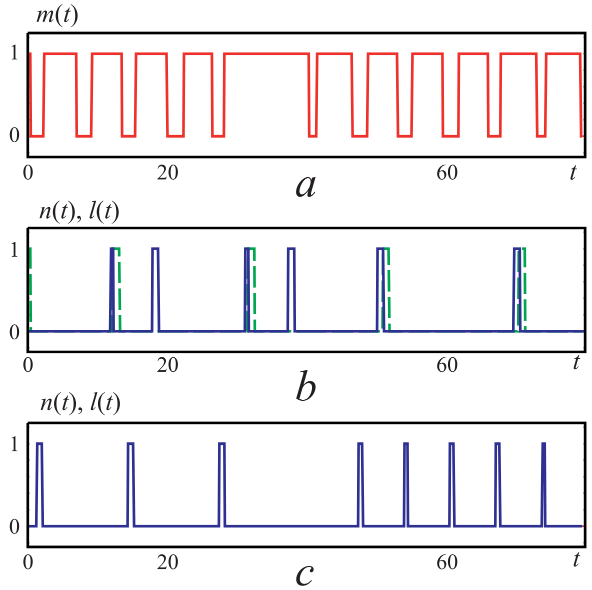

Fig. 6. (a) the drive binary signal generated by systems (11) and (12), (b) the asynchronous binary signals of the response and auxiliary systems (13) and (14), $\varepsilon = 0.05$, (c) the binary generalized synchronization regime, $\varepsilon = 0.13$, the binary signals of the response and auxiliary systems coincide with each other.

where $\omega_r = 0.95$, the threshold value is set to $z_{th} = 3.0$, the parameter of coupling ε governs the response system dynamics. The values of the control parameters $a, p, c, \omega_{d,r}$ provide the chaotic dynamics of the considered Rössler oscillators without coupling (i.e., for $\varepsilon = 0$) exactly just as in the case of the discrete systems considered above in Section 3. To detect the BGS regime in parallel with the response system we have also consider the auxiliary system whose equations coincide with Eqs. (13) and (14), but the initial conditions are different.

The behavior of two coupled systems (11)–(14) generating the binary signals are illustrated by Fig. 6 for two different values of the coupling strength ε. The drive binary signal $m(t)$ is shown in Fig. 6a. The switchings between states “0” and “1” is realized practically immediately (in our numerical simulations their length is equal to one time step, $h = 10^{-3}$) and, theoretically, the length of time intervals corresponding to these switchings should be considered as zero. Since the coupling between the systems is unidirectional, the dynamics of the drive system does not depend on the coupling strength and is the same for all values of parameter ε. The binary sequences $n(t)$ and $l(t)$ generated by the response and auxiliary systems, respectively, are given in Fig. 6b ($\varepsilon = 0.05$) and Fig. 6c ($\varepsilon = 0.13$). One can see that for the small value of the coupling strength $\varepsilon = 0.05$ the binary signals generated by the response and auxiliary systems do not coincide with each other (Fig. 6b) that is the evidence of the asynchronous dynamics. With the increase of the coupling strength ($\varepsilon = 0.13$), the binary generalized synchronization takes place that is proven both by the identical binary signals of the response and auxiliary systems (Fig. 6c) and the behavior of the largest conditional Lyapunov exponent calculated for the response system (Fig. 7).

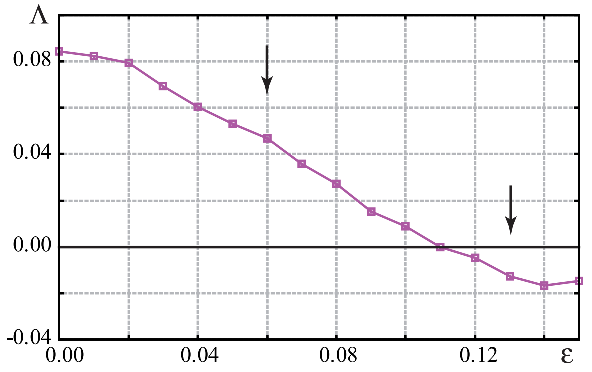

Fig. 7. The dependence of the largest conditional Lyapunov exponent Λ of the response Rössler system on the coupling strength ε. The coupling strength values corresponding to Fig. 6 are shown by arrows.

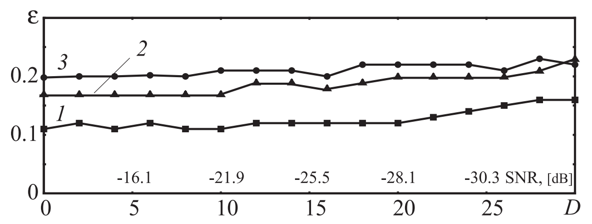

Fig. 8. Dependencies of the threshold value of the BGS regime onset in Rössler systems (11)–(14) on the noise intensity (the SNR-values corresponding to the noise intensities are also shown) for different values of the control parameters: *1* – $\omega_r = 0.95$, *2* – $\omega_r = 0.99$, *3* – $\omega_r = 0.89$.

The mechanism resulting in the binary generalized synchronization can be explained within the framework of the theory proposed earlier in [54]. There are known two main mechanisms being responsible for the GS regime, namely, (i) arising of the additional dissipation caused by the coupling, which suppresses the proper chaotic dynamics of the response system and (ii) the destruction of the proper chaotic dynamics of the response system by the drive signal with the large amplitude and moving the phase trajectory of the response oscillator into the regions of the phase space with the strong dissipation. As we have found out, arising of the binary generalized synchronization in the considered systems is explained by the first mechanism. Indeed, the coupling term changes the property of both the discrete (6) and flow systems (11)–(14) in such a way that the dissipation in the response oscillator increases (that leads to the negativeness of the largest conditional LE) resulting in arising of the generalized synchronization regime.

It should be noted that as in the case of the discrete maps considered above, the BGS regime in flow systems also possesses a high enough stability to external noise. In particular, if the response and auxiliary Rössler systems are additionally subjected to the noise influence (the stochastic term $D\xi$ should be added to the first equation of the system (13) and its replica) the boundary value of the synchronous regime onset would be practically the same as in the noiseless case. In Fig. 8 the dependencies of the boundary value of the BGS regime onset in system (11)–(14) on the noise intensity for different values of the control parameters have been shown. It is clearly seen that in all considered cases the threshold of the BGS regime onset does not change dramatically with the noise intensity increasing, and even in the case when the power of noise exceeds the power of signal considerably the boundary of BGS would be practically the same as in the case of the absence of noise.

5. Conclusion

In conclusion, in this paper we have reported for the first time on the binary generalized synchronization, when for certain values of the coupling strength two unidirectionally coupled dynamical systems generating aperiodic binary sequences are in the generalized synchronization regime. Along with the previously revealed possibility of the binary systems to demonstrate the complete and phase synchronization regimes [59,60], the results of our paper concerning generalized synchronization, suggest the possibility of the development of the theory of chaotic synchronization of binary systems. Due to the high enough stability of the BGS to external perturbation we believe that the finding discussed in this paper gives a strong potential for new applications under many relevant circumstances including the chaotic communication field.

Acknowledgment

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