2021 5th Scientific School Dynamics of Complex Networks and their Applications (DCNA) | 978-1-6654-4283-1/21/$31.00 ©2021 IEEE | DOI: 10.1109/DCNA53427.2021.9587095

Coherence Resonance in Complex Networks

Alexander N. Pisarchik
Center for Biomedical Technology
Universidad Politécnica de Madrid
Madrid, Spain
Innopolis University
Innopolis, Russia
ORCID: 0000-0003-2471-2507

***Abstract*—The paper is devoted to the analysis of coherence resonance phenomena in complex networks of coupled nonlinear oscillators. We explain how coherence and anticoherence resonances can be revealed and experimentally measured in stochastic and deterministic systems, and provide some evidence in various networks. Particular attention is paid to a study of the influence of coherent resonance on cognitive activity. According to our research, intrinsic brain noise, which affects neural activity at the microscopic level, has a positive effect at the macroscopic level. Namely, it coordinates responses of different brain areas and forces their interaction to efficiently process sensory information. We find that brain noise can be altered as a result of attention and cognitive training to optimize the efficiency of information processing. The experimental and theoretical studies provide substantial evidence for beneficial effects of internal brain noise on cognitive performance. Coherence resonance in the brain response to a cognitive task not only increases neural activity in certain brain areas, but also provides pathways for neural communication between distant regions. Thus, the study of cognitive resonance allows us to find optimal parameters for better system performance and efficient control of complex network dynamics.**

***Index Terms*—complex network, coherence resonance, noise, chaos, coupled oscillators, brain**

I. Introduction

The emergence of order in stochastic and chaotic networks has been a challenging problem for scientists from various fields of science. In some cases, the order resonates with the level of noise or chaos. This phenomenon known as *coherence resonance* was first discovered in a stochastic system where the regularity (or coherence) maximizes at a certain level of noise [1]. Then, a similar resonance behavior was observed in noiseless chaotic systems [3]–[6] and called *deterministic coherence resonance*. It should be noted that the well-known phenomenon of *stochastic resonance* [7] is a particular case of coherence resonance in periodically driven systems.

Coherence resonance was extensively studied in networks of coupled neural oscillators, such as Rulkov maps [8], FitzHugh–Nagumo [9], Morris–Lecar [10], and Hodgkin–Huxley [11] models, and evidenced experimentally in distributed cortical neural network during sensory information processing [11].

On the other hand, an inverse resonant behavior, referred to as *anticoherence resonance*, was also discovered, i.e., the system order (or coherence) minimizes with respect to the noise intensity [12], [13].

II. Characterization of Coherence Resonance

There are several measures to quantify coherent resonance [2], [5]. These are

- normalized autocorrelation function,
- correlation time,
- normalized standard deviation of peak amplitude (amplitude coherence),
- normalized standard deviation of inter-peak interval (time coherence),
- dominant spectral component (spectral coherence),
- signal-to-noise ratio (SNR),
- similarity function,
- entropy,
- connectivity (topological coherence).

To characterize deterministic coherence resonance, in addition to the above measure, the following values can be used

- signal-to-chaos ratio (SCR),
- Lyapunov exponents.

The dependences of all these measures versus noise or chaos amplitude display a resonant behavior in the case of coherence or anticoherence resonances.

Now we will show how these characteristics are defined.

A. Normalized Autocorrelation Function

The normalized autocorrelation function is defined as

$$C(\tau) = \frac{\langle \tilde{x}(t)\tilde{x}(t+\tau)\rangle}{\langle \tilde{x}^2\rangle}, \tag{1}$$

where x is the measurable system variable, $\langle x\rangle$ is its time average, $\tilde{x} = x - \langle x\rangle$, and τ is the time lag. The higher $C(\tau)$ means stronger coherence.

B. Correlation Time

The characteristic correlation time is defined as

$$\tau_c = \int_{t_0}^{t_{max}} C^2(t)dt, \tag{2}$$

where t_0 is the duration of transients and t_{max} is the duration of time series. The larger τ_c, the better the coherence.

This work was supported by the Russian Science Foundation (Grant No. 19-12-00050).

C. Normalized Standard Deviation of Peak Amplitude (Amplitude Coherence)

The standard deviation of the peak amplitude A_p normalized to the mean amplitude $\langle A_p \rangle$ can also serve as a measure of the coherence resonance. For a network of oscillators, this value is defined as

$$R_a = \frac{\sqrt{\langle \bar{A}_p^2 \rangle - \langle \bar{A}_p \rangle^2}}{\langle \bar{A}_p \rangle}, \tag{3}$$

where the over-line indicates the average over nodes. The amplitude coherence resonance occurs at the minimum of R_a with respect to the noise or chaos amplitude.

D. Normalized Standard Deviation of Interpeak Interval (Time Coherence)

Similarly, time coherence of a network of oscillators is defined as the standard deviation of the interpeak interval t_p normalized to the mean interpeak interval $\langle t_p \rangle$ as follows

$$R_p = \frac{\sqrt{\langle \bar{t_p^2} \rangle - \langle \bar{t_p} \rangle^2}}{\langle \bar{t_p} \rangle}. \tag{4}$$

When R_p reaches a minimum value with respect to the noise or chaos amplitude, we deal with coherence resonance.

E. Dominant Spectral Component (Spectral Coherence)

In a network of N coupled oscillators, the sum spectral power S_{f_d} at dominant frequency f_d is higher when the network synchronizes. Therefore, S_{f_d} can serve as a measure of the network coherence:

$$S_{f_d} = \sum_{i \in N} S_i. \tag{5}$$

F. Signal-to-Noise Ratio

The signal-to-noise-ratio (SNR) is the most common measure of the system coherence. Usually, SNR is calculated to characterize stochastic resonance in systems with external periodic signal. When the external modulation is absent, SNR can be measured from the power spectrum as the ratio of the spectral component S_P at the dominant frequency f_d to the background spectral value S_N at the same frequency, as shown in Fig. 1. In the semilog scale of the power spectrum (in dBm), this value can be measured as the difference

$$\text{SNR(dBm)} = S_P - S_N. \tag{6}$$

G. Similarity Function

Coherence of system of coupled oscillators is related to their synchronization [14], that can be characterized by similarity function. The similarity function Z_{ij} of motions x_i and x_j of oscillators i and j can be derived from

$$Z_{ij}^2(\tau) = \frac{\langle [x_j(t) - x_i(t+\tau)]^2 \rangle}{\sqrt{\langle x_j(t)^2 \rangle \langle x_i(t)^2 \rangle}}, \tag{7}$$

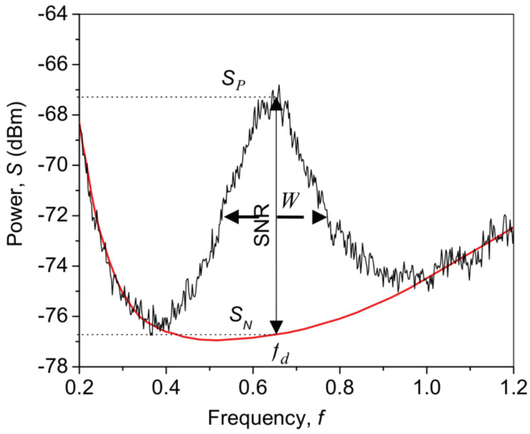

Fig. 1. Definition of signal-to-noise ratio (SNR) and signal-to-chaos ratio (SCR) from the power spectrum.

where τ is the time lag between the state vectors of the interacting oscillators. The similarity function of a network of N coupled oscillators can be calculated as

$$Z(\tau) = \sum_{i \in N} \sum_{j \in N} Z_{ij}(\tau), \; i \neq j. \tag{8}$$

The value of the minimum of the similarity function $\delta = \min_\tau Z(\tau)$ is related to lag synchronization. When the dependence of δ on a control parameter displays extrema, this means that the system exhibits coherence or anticoherence resonance. The lower the minimum of the similarity function, the higher the coherence.

H. Signal-to-Chaos Ratio

To characterize deterministic coherence resonance, signal-to-chaos ratio (SCR) can be used. Namely, this is the ratio of the spectral component S_P at the dominant frequency f_d to the full-width at half-height W defined as

$$\text{SCR} = \frac{S_P}{W}. \tag{9}$$

The full-width at half-height is measured with respect to the background spectral value S_N (see Fig. 1).

I. Lyapunov Exponents

The Lyapunov exponents characterize the dependence of the system dynamics on a small change in initial conditions. In chaotic systems, the largest Lyapunov exponent is positive. However, other exponents can be either negative or zero. In the case of deterministic coherence resonance, some of the exponents take maximum or minimum values with respect to a control parameter. As an example, in Fig. 2 we plot the largest Lyapunov exponent of the system of three unidirectionally ring-coupled Rössler oscillators in the parameter space of mismatch Δ between their fundamental frequencies and coupling strength σ [6]. One can see that the largest Lyapunov exponent takes a minimum value (up to zero for $\Delta \approx 0.25$) with respect to the coupling strength σ when the frequency

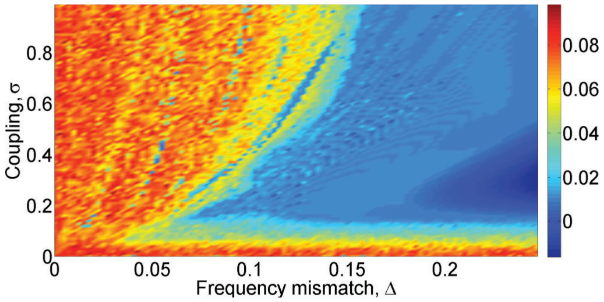

Fig. 2. Largest Lyapunov exponent (right color panel) of three ring-coupled Rössler oscillators in the (Δ, σ)-parameter space.

mismatch Δ is fixed. This means that the chaotic oscillators behave periodically for a certain coupling strength ($\sigma \approx 0.3$) and thus their dynamics is more coherent.

J. Entropy

Since entropy is a measure of order, it can evidently be used to quantify coherence. There are different type of entropy. One of them, Shannon entropy of complex network is defined as [15]

$$H(\mathbf{q}) = -\sum_{k=1}^{N} q(k) \log(q(k)), \tag{10}$$

where $\mathbf{q} = (q(1), \ldots, q(i), \ldots, q(N)$ and $q(k)$ is the remaining degree given by

$$q(k) = \frac{(k+1)P_{k+1}}{\langle k \rangle} \tag{11}$$

with P_k being the degree distribution and $\langle k \rangle$ is the average degree. However, the above definition of the Shannon entropy does not account for the network topology. Therefore, the improved definition of *topological* Shannon entropy for the case of a simple undirected network is derived as [16]

$$H = -\sum_{i<j}^{N} p_{ij} \log p_{ij}, \tag{12}$$

where p_{ij} is the probability to have a link between nodes i and j.

K. Connectivity

When we deal with a complex network, it is important to estimate its topological coherence determined by the number of links which connect the network nodes. In this context, we assume that the network topology is more coherent when a larger number of nodes interact. This is especially appealing when we deal with neural networks, for instance, the human brain. The brain connectivity is crucial while processing and analyzing information. In this case, the number of links connecting distinct brain areas and their strengths can be revealed by recording neurophysiological brain activity, e.g., by electroencephalography (EEG) of magnetoencephalography (MEG).

III. Examples of Coherence Resonances in Complex Networks

Coherence and anticoherence resonances were found in various networks of coupled oscillators. Consider now some examples.

A. Coherence and Anticoherence Resonances in a Star Network

Now, we will demonstrate the existence of deterministic coherence and anticoherence resonances in a small network of unidirectional coupled oscillators in a star configuration in the presence of a small mismatch between natural frequencies of the coupled oscillators. The resonances are quantified in amplitude and time by normalized standard deviations of the peak amplitude and inter-peak interval with respect to the coupling strength and frequency mismatch.

In particular, we consider the star network of 11 chaotic Rössler oscillators with a small mismatch between their natural frequencies. The normalized standard deviations of the peak amplitude and inter-peak interval of one of the oscillators are shown in Fig. 3 as a function of the coupling coefficient σ.

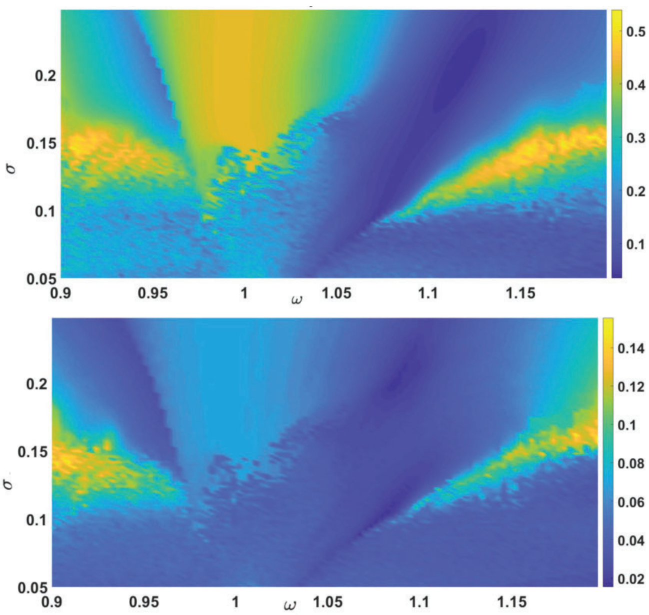

Fig. 3. Normalized standard deviation (right color bar) of peak amplitude (upper panel) inter-peak interval (lower panel) of one of the variables of one of the oscillators in a star network of chaotic Rössler oscillators in the parameter space of the oscillator natural frequency ω and common coupling strength σ.

The alternation of different colors in both upper and lower panels in Fig. 3 displays a clear evidence of the existence of amplitude and time coherence and anticoherence resonances in the network.

B. Coherence Resonance in Cortical Network

Another evidence of coherence resonance was recently found in the distributed cortical network of the human brain [11]. The resonance was experimentally revealed in the brain connectivity during sensory information processing by EEG.

Mona Lisa images with different contrast were present to subjects and simultaneously EEG was recorded. The contrast level was interpreted as the noise level. After the analysis of the brain activity, the connectivity between distinct brain areas was recovered in alpha (8–12 Hz) and beta (15–30 Hz) frequency ranges. The resulted brain connectivity structure is shown in Fig. 4.

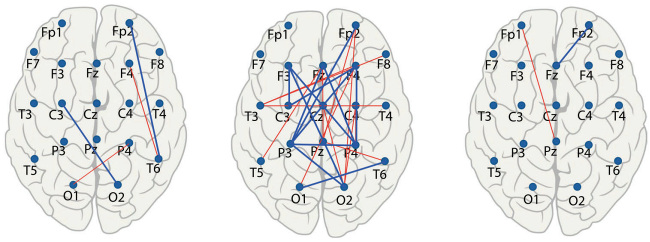

Fig. 4. Brain connectivity in alpha (red links) and beta (blue links) bands for different image contrasts (left) low contrast, (middle) middle constrast, and (right) high contrast. The link strengths were estimated via wavelet bicoherence.

The increasing number of links and their strength confirm that efficient neural connectivity in the frontoparietal cortical network is achieved through coherence resonance. We have found that internal brain noise affects neural activity at a microscopic level that leads to a positive effect at a macroscopic level. In most cases, as was shown in recent MEG experiments [17], brain noise helps to process sensory information more efficiently. Specifically, the brain noise level is determined by the size of the active neural network and can change as a result of cognitive load and training in solving a specific cognitive task. Hence, increased brain noise may indicate more efficient information processing and improved cognitive performance. This discovery bridges the gap between the neural noise paradigm and global workspace theory.

IV. Conclusion

In this paper we have shown how coherence can be characterized in complex network, and present some examples of coherence and anticoherence resonances in small and large complex network, in particular, in a star network of Rössler oscillators and in the distributed cortical neural network. The quantitative description of the coherence phenomena allows better understanding of the noise influence on the system dynamics and efficient control by adjusting parameters and noise intensity.

Although significant advance has been achieved in the study of network coherence, e.g., neural communication through coherence resonance, some coherence measures still require further investigation, in particular, network entropy and structural coherence resonance. The study of stochastic resonance arises important questions for future research: Is the endogenous brain noise constant or changes over time? and if the latter is true, what are the mechanisms underlying the noise change?

Acknowledgments

This work was supported by the Russian Science Foundation (Grant No. 19-12-00050). The author thanks A. Hramov, A. Andreev, V. Maksimenko, N. Frolov, and R. Jaimes-Reátegui for collaboration and fruitful discussions.

References

[1] H. Gang, T. Ditzinger, C.-Z. Ning, and H. Haken, “Stochastic resonance without external periodic force,” Phys. Rev. Lett., vol. 71, pp. 807–810, 1993.

[2] A. S. Pikovsky and J. Kurths, “Coherence resonance in a noise-driven excitable system,” Phys. Rev. Lett., vol. 78, pp. 775–778, 1997.

[3] A. N. Pisarchik and R. Corbalán, “Stochastic resonance in a chaotic laser,” Phys. Rev. E, vol. 58, pp. R2697–R2700, 1998.

[4] D. Postnov, S. K. Han, T. G. Yim, and O. Sosnovtseva, “Experimental observation of coherence resonance in cascaded excitable systems,” Phys. Rev. E, vol. 59, pp. R3791–R3794, 1999.

[5] A. N. Pisarchik, R. Jaimes-Reátegui, “Deterministic coherence resonance in coupled chaotic oscillators with frequency mismatch,” Phys. Rev. E, vol. 92, pp. 141–150, 2015.

[6] M. García-Vellisca, A. N. Pisarchik, R. Jaimes-Reátegui, “Experimental evidence of deterministic coherence resonance in coupled chaotic systems with frequency mismatch,” Phys. Rev. E, vol. 94, art. no. 012218, 2016.

[7] R. Benzi, A. Sutera, and A. Vulpian, “The mechanism of stochastic resonance,” J. Phys. A: Math. Gen., vol. 14, pp. L453–L457, 1981.

[8] A. V. Andreev, V. V. Makarov, A. E. Runnova, A. N. Pisarchik, and A. E. Hramov, “Coherence resonance in stimulated neuronal network,” Chaos Soliton. Fract., vol. 106, pp. 80–85, 2018.

[9] R. Toral, C. R. Mirasso, and J. D. Gunton, “System size coherence resonance in coupled Fitzhugh–Nagumo models,” Europhys. Lett., vol. 61, pp. 162–167, 2003.

[10] Q. Wang, M. Perc, Z. Duan, and G. Chen, “Synchronization transitions on scale-free neuronal networks due to finite information transmission delays,” Phys. Rev. E, vol. 80, art. no. 026206, 2009.

[11] A. N. Pisarchik, V. A. Maksimenko, A. V. Andreev, N. S. Frolov, V. V. Makarov, M. O. Zhuravlev, A. E. Runnova, and A. E. Hramov, “Coherent resonance in the distributed cortical network during sensory information processing,” Sci. Rep., vol. 9, art. no. 18325, 2019.

[12] A. M. Lacasta, F. Sagués, and J. M. Sancho, “Coherence and anti-coherence resonance tuned by noise,” Phys. Rev. E, vol. 66, art. no. 045105(R), 2002.

[13] P. Zhu and D. C. Mei, “Anti-coherence and coherence resonance induced by nonlinear time delay in autonomous stochastic system,” Eur. Phys. J. B, vol. 87, art. no. 109, 2014.

[14] S. Boccaletti, A. N. Pisarchik, C. I. del Genio, and A. Amann, Synchronization: From Coupled Systems to Complex Networks, Cambridge Univ. Press: Cambridge, 2018.

[15] R. V. Solé and S. J. Valverde, “Information theory of complex networks: on evolution and architectural constraints, Lect. Notes Phys., vol. 650, pp. 189–207, 2004.

[16] K. Anand and G. Bianconi, “Entropy measures for networks: Toward an information theory of complex topologies”, Phys. Rev. E, vol. 80, art. no. 045102(R), 2009.

[17] A. N. Pisarchik, P. Chholak, and A. E. Hramov, “Brain noise estimation from MEG response to flickering visual stimulation”, Chaos Soliton. Fract. X, vol. 1, art. no. 100005, 2019.