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Daniil V. Kirsanov, "Emergence of macroscopic chimera states in multilayer multiplex network," Proc. SPIE 11067, Saratov Fall Meeting 2018: Computations and Data Analysis: from Nanoscale Tools to Brain Functions, 110670R (3 June 2019); doi: 10.1117/12.2527692

Event: International Symposium on Optics and Biophotonics VI: Saratov Fall Meeting 2018, 2018, Saratov, Russian Federation

Emergence of macroscopic chimera states in multilayer multiplex network

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ABSTRACT

In this paper we study the dynamics of a multi-layer network composed of identical layers of non-locally coupled Kuramoto-Sakaguchi phase oscillators. Throughout the intensive numerical study we consider three-layer multiplex network and reveal conditions for a specific form of multi-layer network behavior, the macroscopic chimera-like state. It represents an excitation of different spatiotemporal patterns in initially identical layers of multiplex network under their interaction. Also, we show that transition to such macroscopic chimera patterns can be achieved not only variation of phase shift, but according to introduction of heterogeneity of network elements.

Keywords: complex network, chimera state, multiplex netrwork, multistability, nonlocal coupling

1. INTRODUCTION

The chimera state phenomenon was first observed in 2002 by Kuramoto and Battogtokh.[1] The modern term *Chimera state* was introduced in 2004 by Abrams and Strogatz who theoretically proved this phenomenon[2] and described it as a specific behavior of a nonlinear oscillator ensemble where the network has simultaneously existing coherent and incoherent groups of dynamical units. Since that time the phenomenon of chimera behavior is of great interest in modern science. For the last decade a great number of theoretical and experimental works were published on the topic (see Review paper on chimera states by Panaggio and Abrams[3]) in which chimera patterns have been observed in various systems, e.g. laser networks,[4,5] neural ensembles,[6–9] coupled chemical,[10–12] mechanical[13–17] and electronic oscillators,[18–20] etc. Although numerous attempts were made on the way to understanding the nature of chimera states and its possible applications in science and technology but a lot is yet to be done and many issues are still not studied.

The usual way of studying chimera states properties is through the analysis of complex networks, whose nodes contain single nonlinear oscillators coupled according to different link topologies. Another approach which we consider rather interesting is to analyze similar effects in networks, whose nodes themselves are complex subnetworks.[21] Thus we can get the network topology that could be a relevant model to describe many real-life systems, which demonstrate complex organization and hierarchical structure, e.g. transportation networks,[22] population networks,[23] social networks,[24] functional network of brain cortex,[25,26] etc. In this case, the interaction between subnetworks reflects macroscopic properties of the whole network, while the processes of self-organization and structure formation taking place inside each subnetwork are seen as microscopic properties of the whole network.[27] Thus, the issue related to macro-level pattern formation analysis essentially follows from the nature of such networks and has been poorly studied so far. The chimera state have already been studied in a triangular network,[28] which is the simplest network topology that demonstrates chain-like and ring-like properties and contains three subnetworks. There, it has been shown that such system demonstrates two stable chimera attractors, which are associated with the coexistence of coherent and incoherent groups and are born through a saddle-node bifurcation, along with all-coherent and all-incoherent group behavior.

Another important notion in network science related to the topology, where subnetworks are interconnected and interact with each other, is multilayer multiplex networks.[23,29–31] This approach to complex network construction presents subnetworks as isolated layers, where each individual node takes places in all layers simultaneously. In this network topology we observe properties of many real-life and complexly organized systems, for

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Saratov Fall Meeting 2018: Computations and Data Analysis: from Nanoscale Tools to Brain Functions,
edited by Dmitry E. Postnov, Proc. of SPIE Vol. 11067, 110670R · © 2019 SPIE
CCC code: 0277-786X/19/$18 · doi: 10.1117/12.2527692

example, neural network of brain cortex, where each layer represents network dynamics corresponding to different brain rhythms.[32,33] Recent studies of chimera pattern interactions formed at different levels of a two-layer multiplex network[34,35] have shown that depending on the value of inter-layer coupling, chimera patterns could be either excited or suppressed. In these works authors mostly analyzed micro-level network dynamics taking into account only chimera patterns properties individually on each layer of the multiplex network.

The discovered effects raise new important questions. One of them is the problem of possibility of observing a chimera state on the macroscopic level of the multilayer multiplex network. In other words, we need to find out if multiplex topology allows for a macroscopic chimera or chimera-like state in a similar way as observed by Martens in the case of a triangular network. Finding such regimes of multiplex network behavior characterized by a macroscopic level symmetry breaking, when the subnetworks located at different layers split into different spatio-temporal patterns, would widen the view on phenomena of chimera state, which has been originally discovered for a single group of interacting nonlinear oscillators. In this context we call this phenomenon *macroscopic chimera*.

This work aims at studying regimes of macroscopic chimera-like behavior under the interaction between identical layers in a three-layer network coupled by multiplexing. In particular, we have found long-living macroscopic chimera attractors, which appear in the formation of different spatial patterns on different network layers. Finding and examining the macroscopic chimera regimes is relevant in the context of deeper understanding of complex systems described in the framework of multiplex models. Along with the finding of such chimera-like regimes in multiplex networks, we infer the relations between microscopic and macroscopic self-organization processes.

2. MATHEMATICAL MODEL

The model under study is the multi-layer multiplex network which consists of $L = 3$ layers each having $N = 100$ nodes. Each every node in its tern performs two types of coupling — intra-layer (solid lines) and inter-layer (dashed lines), as shown in Fig. 1, (a). Identical Kuramoto-Sakaguchi (KS) equations, which are paradigmatic models allowing for chimera patterns, describe the dynamics of those nodes:

$$\frac{d\phi_i^l}{dt} = \omega_0 - \frac{\lambda_1}{2R_c} \sum_{r=i-R_c}^{r=i+R_c} \sin\left(\phi_i^l - \phi_r^l + \alpha_1\right) + \frac{\lambda_2}{2} \sum_{k \neq l} \sin\left(\phi_i^l - \phi_i^k + \alpha_2\right) , \tag{1}$$

where ϕ_i^j is the phase of the i-th KS oscillator on the j-th layer, ω_0 is the natural frequency, λ_1 and λ_2 are the strengths of intra- and inter-coupling, respectively, R_c is the radius of the non-local intra-layer coupling, α_1 and α_2 are the coupling phase-lag corresponding to intra- and inter-layer coupling, respectively. Here, the subscripts denote the number of the KS oscillator and superscripts denote the number of the layer. One should note, that $\phi_{-i}^j = \phi_{N_0-i}^j$. The second term on the right side in Eq. (1) stands for a non-local *intra-layer* coupling and the third one determines a multiplex all-to-all *inter-layer* coupling. Without loss of generality we set $\omega_0 = 0$ throughout the study.

Initially phase distributions are shaped as a cosine wave, which is slightly shifted on each layer with respect to the others, which allows for a microscopic chimera pattern formation:

$$\phi_i^l(0) = -\pi \cos\left(2\pi \frac{i-l}{N}\right). \tag{2}$$

In order to reduce the number of network control parameters, we set fixed coupling radius $R_c = 35$ and fixed coupling strength $\lambda_1 = 0.085$, which determine the intra-layer coupling properties. In this case, the coupling phase-lag α_1 alone controls the microscopic dynamics of each single layer. Having all that in mind let us now return to the entire three identical layer network coupled by multiplexing. One should note that term "identical" regarding our multiplex network means that all its layers are described via identical mathematical equations (1) with identical control parameters but slightly mismatched initial phase distributions (2). Further in our study the macroscopic states of the network are denoted with three capital letters, which reflects the type of microscopic state on each layer of the network. For instance, *SSS* marks such network state, where all layers perform a synchronized behavior on the microscopic level.

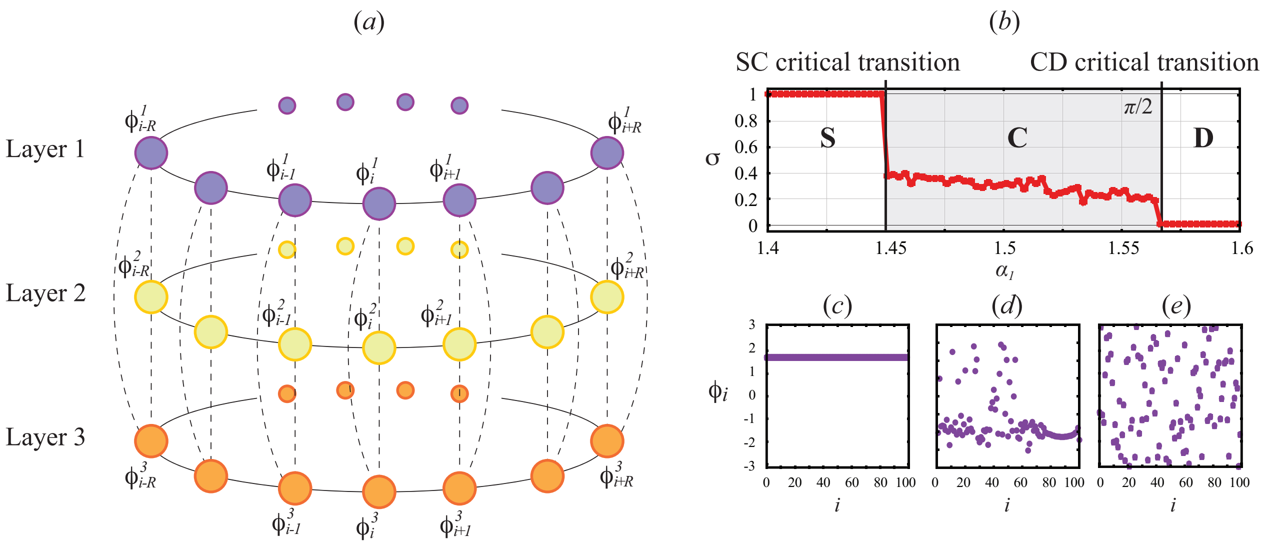

Figure 1. (a) Schematic illustration of a three-layer multiplex network of Kuramoto-Sakaguchi oscillators with a non-local *intra-layer* coupling and a global *inter-layer* coupling (Eq. (1)). (b) Dependence of coherence measure σ on controlling parameter α_1 for a single network layer with the following parameters: λ_1=0.085, $R_c = 35$. Bold vertical lines indicate single layer critical transitions: "synchronized–chimera" transition at $\alpha_1 = 1.45$ and "chimera–desynchronized" transition at $\alpha_1 = \pi/2$. Examples of different single-layer states corresponding to different regimes: (c) synchronized as "S" ($\alpha_1 = 1.42$); (e) chimera as "C" ($\alpha_1 = 1.52$); (d) desynchronized as "D" ($\alpha_1 = 1.59$).

3. MACROSCOPIC CHIMERA STATE

In the begining of this section we should note that the 4-th order Runge-Kutta method was used to analyse the considered three-layer multiplex network and solve the system of ordinary differential equations (1) numerically. We have used this method with chosen integration time step of $dt = 0.01$.

In order to estimate the type of the microscopic state, in other words, to measure a coherence level in each layer of our multiplex network we use the *CM* (coherence measure) parameter which was first introduced by Frolov et al.[36] This method based on the recurrence approach[37] provides the estimation of the relative size of coherent subpopulation within a single layer of the network. Thus, $\sigma = 0$ corresponds to a totally desynchronized behavior within the layer, $\sigma = 1$ corresponds to a totally synchronized layer dynamics and $0 < \sigma < 1$ indicates the formation of partially synchronous or chimera patterns as shown in Fig. 1, (b)–(d). Let us denote the microscopic states associated with synchronized, chimera and desynchronized layer dynamics by single capital letters S, C and D respectively. Indeed, one can see in Fig. 1,(b), that single-layer dynamics undergoes two critical transitions under variation of phase-lag α_1. Increasing of α_1 leads to the transition from a synchronous layer dynamics S (Fig. 1,(c)) to desychronization of the layer nodes D (Fig. 1,(e)) through the birth of a partially synchronized chimera state C (Fig. 1,(d)). Namely, the transition from S to C is observed at $\alpha_1 = 1.45$, and the transition from C to D corresponds to $\alpha_1 = \pi/2$. In this particular case, the coherent subpopulation of the network layer includes 22 nodes and $\sigma = 0.22$. Thus, we can conclude that calculation of *CM* provides a good estimation of the relative coherent group size of the network layer.

The first case that we analyze is one of weak inter-layer coupling. In this situation, one can expect that the weak coupling between the layers, which exhibit close dynamical regimes, will contribute to either synchronization or desynchronization of interacting network layers without changing the dynamical properties within each layer. However, such regimes are rather trivial and understandable. Our study is aimed at finding the conditions, which allow for a symmetry breaking on the macroscopic level of the considered three-layer network, i.e. the situation when two layers of the multiplex network demonstrate similar dynamics, whereas the third one performs the other type of behavior. In this sense we call such regimes macroscopic chimera-like regimes. We suppose that such chimera-like regimes can be observed in the neighborhood of microscopic critical transitions shown in Fig. 1,(b).

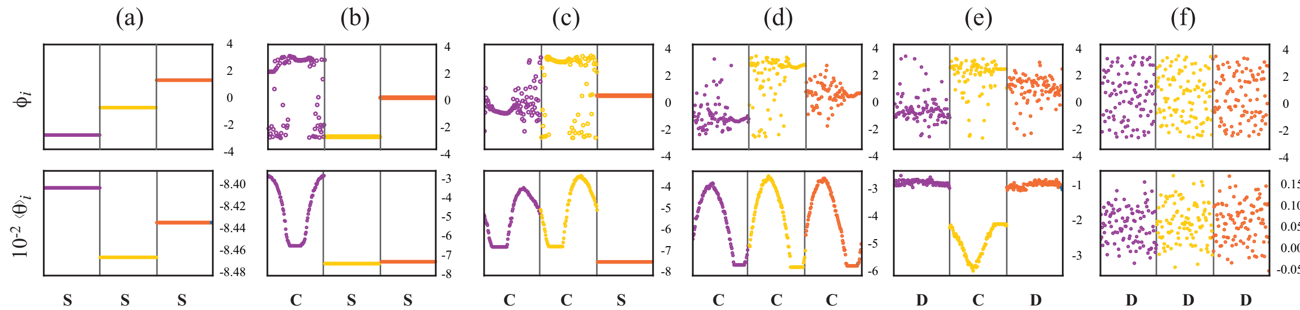

Figure 2. The snapshots of phase ϕ_i and mean phase velocity $\langle\theta\rangle_i$ of multilayer network are taken at 7000 time units after introduction of inter-layer coupling are presented at weak inter-layer coupling $\lambda_2 = 0.005$ in case of low value of inter-layer phase-lag $\alpha_2 = 0.5$ under increase of intra-layer phase-lag α_1: (a) $\alpha_1 = 1.43$; (b) $\alpha_1 = 1.445$; (c) $\alpha_1 = 1.452$; (d) $\alpha_1 = 1.5$; (e) $\alpha_1 = 1.555$; (f) $\alpha_1 = 1.57$.

Fig. 2 illustrates the transitions between different types of macroscopic behavior of the considered multiplex network under the variation of α_1 in the case of weak inter-layer coupling $\lambda_2 = 0.005$ and a small value of inter-layer phase-lag $\alpha_2 = 0.5$. We can see, that network macroscopic states, in which similar microscopic states at all the layers are established, are represented in a wide range of α_1. In particular, the SSS-state with $\sigma_1 = \sigma_2 = \sigma_3 = 1$ lies in the range $\alpha_1 \leq 1.44$, the DDD-state in the range $\alpha_1 \geq 1.56$ and the CCC-state corresponds to $\alpha_1 \in [1.4575; 1.5575]$. These ranges are characterized by the presence of long-living (at least, a few thousands of time units) macroscopic states. At the same time, the excitation of macroscopic chimera-like states in two rather narrow areas can be noticed: (i) at the boundary between SSS and CCC ($\alpha_1 \in [1.4425; 1.455]$) and (ii) at the boundary between CCC and DDD ($\alpha_1 \in [1.5525; 1.5575]$). Notably, in the first region chimera-like states consist of coexisting chimeric and synchronized layers (CSS and CCS states), whereas the second one is composed of one chimera layer and two desynchronized layers (CDD-state). Surprisingly, CCD states in considered multiplex system have not been observed – instead, those states, initiated after switching the inter-layer coupling typically collapsed into either CDD or DDD states.

4. HETEROGENOUS NETWORK

Let us consider the influence of heterogeneity of network elements on their collective behavior and formation of macroscopic chimera pattern in studied multiplex network. With this goal in mind, we introduce heterogeneity in phase oscillators frequency. Instead of previously considered case $\omega_i = \omega_0 = 0$ for all $i \in [1, N]$, here we assume that KS oscillators frequencies are randomly distributed in accordance with Lorentz distribution:

$$g(\omega) = \frac{1}{\pi\gamma}\frac{\gamma^2}{(\omega-\omega_0)^2+\gamma^2}, \tag{3}$$

where γ is a Lorentz distribution parameter, which controls network heterogeneity, and $\gamma = 0$ corresponds to homogenous network.

Based on the method of estimating the volume of the stability basin,[38] the region of the inhomogeneity parameter γ was identified, which is characterized by the existence of several stable macroscopic states of a multilayer network. In particular, we found that the coexisting states are characterized by various microstates of the network, i.e. intralayer patterns. As seen from Figure 3 introduction of heterogeneity of natural frequencies distribution induces multistability in considered network. In addition to absolutely stable SSS state in case of homogenous network, small heterogeneity creates multiple stable attractors corresponding to CSS, CCS and CCC state. Increasing of heterogeneity level enlarges basin size of CCC state and leads to its absolute stability. Thus, we observe the route from absolutely stable SSS to absolutely stable CCC through the multistability, which is followed be emergence of intermediate states CSS and CCS. The discovered dynamic feature of a complex mult-layer network under the conditions of heterogeneity of its elements introduces an understanding of the processes of self-organization of real network structures, which are characterized by the formation of different spatial-temporal patterns on different layers.

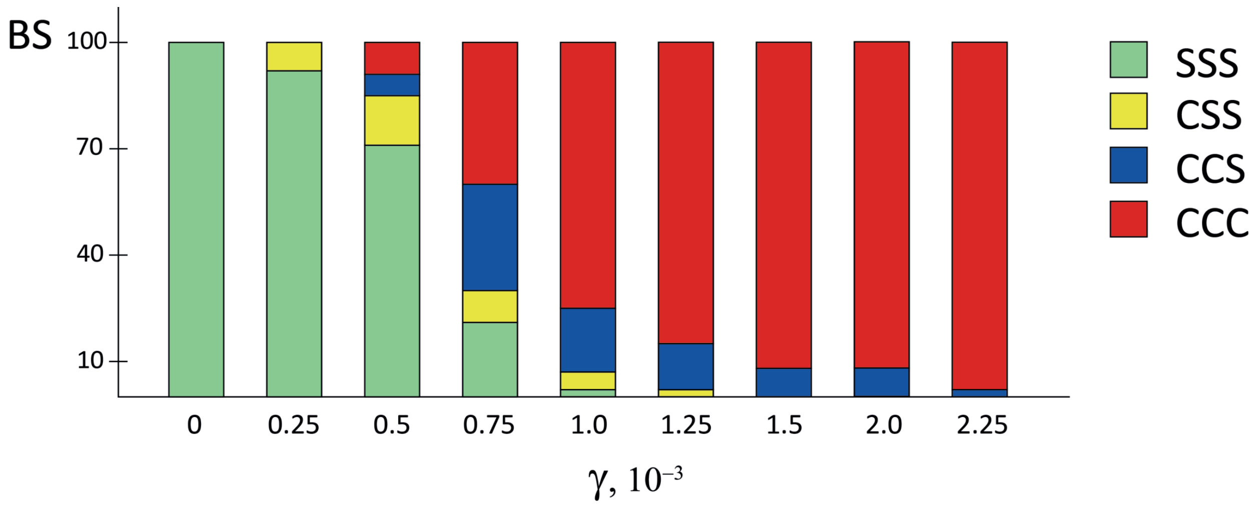

Figure 3. Dependence of Basin Stability (BS) on heterogeneity parameter γ.

5. CONCLUSION

In this paper we have studied a new phenomenon, the macroscopic chimera-like state, which emerges in a multi-layer multiplex networks. In particular, we have focused on consideration of three-layer multiplex network, where each layer is composed of identical Kuramoto-Sakaguchi phase oscillators with non-local coupling. We have uncovered that this phenomenon consists in a split of the layers with initially close dynamics into subgroups, where the group of two layers performs one type of dynamics, whereas the rest exhibit the other type, after the introduction of inter-layer coupling. Based on the provided numerical analysis we reveal conditions, which allow for macroscopic chimera state emergence. We have also shown the role of network heterogeneity in formation of macroscopic chimera patterns – they can be achieved due to the multistability developed under the heterogeneity of network elements.

The conducted research opens a number of important issues. For instance, how does the heterogeneity of the multiplex network nodes affect the macroscopic state of the network and how does it influence the formation of macroscopic chimera? In this sense, macroscopic chimera could be a relevant model for the description of multistable visual image perception performed by human brain neural network and heterogeneity of the network elements could be associated with brain cognitive noise[39, 40] or decision-making uncertainty.[41]

6. ACKNOWLEDGMENTS

This work has been supported by the Russian Science Foundation (Grant No. 17-72-30003).

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