2021 5th Scientific School Dynamics of Complex Networks and their Applications (DCNA) | 978-1-6654-4283-1/21/$31.00 ©2021 IEEE | DOI: 10.1109/DCNA53427.2021.9587298

Mixing adaptive rules in a bilayer Erdős-Rényi network

Nikita Frolov
Innopolis University
Innopolis, Russia
Immanuel Kant Baltic Federal University
Kaliningrad, Russia
n.frolov@innopolis.ru

Dibakar Ghosh
Indian Statistical Institute
Kolkata, India
diba.ghosh@gmail.com

***Abstract*—In many natural networked systems, adaptation is an essential mechanism of self-organization. Usually, network elements interact through either interdependent, high coupling at high coherence, or competitive, low coupling at high coherence, adaptive rules. The former rule supports explosive synchronization, while the latter is associated with continuous transition. Due to inherent heterogeneity of the real networks, one has to understand the transition under the mixing of such adaptive rules. Here, we address this problem from a multilayer perspective and explore the impact of multiplexing on hysteresis region associated with explosive transitions in complex networks.**

***Index Terms*—adaptive network, interdependence, competition, explosive synchronization, bilayer network**

I. Introduction

Synchronization phenomenon underlies normal and undesired dynamics in a broad range of natural and man-made systems [1]. Usually, such systems, especially biological ones, possess network organization, in which a large amount of units interact with each other in a complicated manner [2].

Among the variety of emergent behaviors exhibited by complex networks, special attention has been paid to the effect of explosive synchronization (ES) [3]. ES, being a discontinuous jump-like transition to coherence, displays properties of real networked systems, such as seizures of epileptic brain, failures of power grids, etc. [4]–[6]. From its first discovery in 2006, it has been thought for a long time that ES originates from specific microscopic correlation features between the natural frequency and the unit's degree in heterogeneous scale-free (SF) networks [7], [8] or the natural frequencies of the oscillators and their effective coupling strengths [9]–[11]. Later, Zhang et al. have generalized this conclusion on local adaptive coupling and manifested that ES requires any rule suppressing macroscopic coherence of the network [12].

In real adaptive networks, elements do not always share the same rules of adaptation due to the inherent heterogeneity [13], [14]. Thus, investigation of mixed adaptive scenarios – interdependence and competition – is necessary for understanding the dynamics of realistic networked systems. Recently, Dai et al. have addressed this problem in the case of monolayer network [15]. They have demonstrated that a certain fraction of competitively-coupled units switches the network to continuous transition.

Recent literature sources indicate that multiplexing may crucially influence network's dynamics, specifically to induce or suppress coherent states [16]–[18] and to support ES [19]–[22]. In this brief report, we explore how the multiplex architecture impacts the boundaries of ES if interdependent and competitive units coexist within such network. We investigate this problem using an extensive numerical simulation of a bilayer Erdős-Rényi graph.

II. Numerical model

We analyze a multiplex ($L = 2$ layers) network with $N = 1000$ Kuramoto phase oscillators in each layer and compare it with a monolayer case ($L = 1$). A bilayer network's dynamics is described by the following system of differential equations:

$$\begin{aligned}\dot{\theta}_{i,1} &= \omega_{i,1} + \lambda \mathcal{D}_i^{2\to 1} \sum_{j=1}^{N} \mathcal{A}_{ij,1} \sin(\theta_{j,1} - \theta_{i,1}),\\ \dot{\theta}_{i,2} &= \omega_{i,2} + \lambda \mathcal{D}_i^{1\to 2} \sum_{j=1}^{N} \mathcal{A}_{ij,2} \sin(\theta_{j,2} - \theta_{i,2}),\end{aligned} \tag{1}$$

where subscripts 1 and 2 correspond to layer-1 and its replica, layer-2, respectively. In a bilayer case, $\mathcal{D}_i^{2\to 1}$ and $\mathcal{D}_i^{1\to 2}$ define the rule of interlayer adaptation, so that it is controlled by the local coherence of the *replica* unit from the opposite layer. For an interdependent fraction f we define $\mathcal{D}_i^{2\to 1} = r_{i,2}$ and $\mathcal{D}_i^{1\to 2} = r_{i,1}$, while for remaining (competitive) fraction $(1-f)$ this rule is governed by $\mathcal{D}_i^{2\to 1} = 1 - r_{i,2}$ and $\mathcal{D}_i^{1\to 2} = 1 - r_{i,1}$. The adjacency matrices $\mathcal{A}_{i,j,\{1,2\}}$ define Erdős-Rényi graphs with $\langle k_{i,\{1,2\}}\rangle = 12$.

To evaluate phase coherence within network layers we use averaged global order parameter:

$$R_{1,2} = \frac{1}{N(t_{max} - t_{tr})} \sum_{t=t_{tr}}^{t_{max}} \left| \sum_{j=1}^{N} e^{\mathrm{i}\theta_{j,\{1,2\}}(t)} \right|, \tag{2}$$

This work has been supported by the Russian Foundation for Basic Research (Grant No. 19-52-45026), the Department of Science and Technology, Government of India (Project no. INT/RUS/RFBR/360), and the Council on grants of the President of Russian Federation (Grant No. NSh-2594.2020.2).

where $t_{max} = 1.5 \times 10^6$ iterations and $t_{tr} = 1.2 \times 10^6$ iterations are maximal simulation time and duration of the transient period presented in the number of iterations.

III. Results

We witness the expanding of hysteresis area on the parameter plane (f, λ) in the case of a bilayer network ($L = 2$) opposed to a monolayer model ($L = 1$) Fig. 1a. Specifically, one can see that the boundaries of both forward (solid lines) and backward transition (dashed lines) are shifted to higher values of the coupling strength λ in bilayer model. Despite that, the width of the hysteresis area does not experience a considerable growth due to to multiplexing for relatively small-sized competitive fractions ($f > 0.8$ in Fig. 1b). However, multiplexing makes it possible to achieve ES for larger size of competitive populations compared with a single-layer problem.

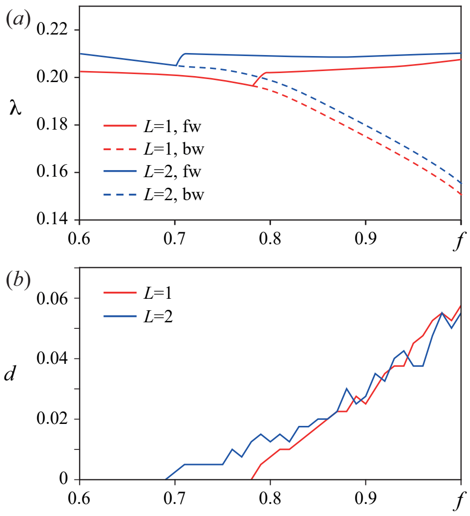

Fig. 1. (a) Hysteresis areas on the parameter plane (f, λ) in the case of monolayer ($L = 1$, red), and bilayer ($L = 2$, blue) networks. Here, solid and dashed lines indicate approximate boundaries of forward and backward transitions. (b) Corresponding hysteresis width d versus f for mono- (red line) and bilayer (blue line) networks.

IV. Conclusions

In this brief report, we evidence that the multiplexing of complex network with mixed adaptive rules expands the area of hysteresis, a key feature of ES. Moreover, we achieve the shifting of ES thresholds to stronger couplings and facilitated ES for a larger size of competitive population in the multiplex bilayer network.

References

[1] A. Pikovsky, M. Rosenblum, and J. Kurths, Synchronization: a universal concept in nonlinear sciences. Cambridge University Press, 2003.

[2] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, "Synchronization in complex networks," Physics Reports, vol. 469, no. 3, pp. 93–153, 2008.

[3] S. Boccaletti, et al. "Explosive transitions in complex networks' structure and dynamics: Percolation and synchronization," Physics Reports, vol. 660, pp. 1–94, 2016.

[4] Zh. Wang, et al. "A small change in neuronal network topology can induce explosive synchronization transition and activity propagation in the entire network," Scientific Reports, vol. 7, no. 1, pp. 1–10, 2017.

[5] N. Frolov, and A. Hramov, "Extreme synchronization events in a Kuramoto model: The interplay between resource constraints and explosive transitions," Chaos: An Interdisciplinary Journal of Nonlinear Science, vol. 31, no. 6, p. 063103, 2021.

[6] S. V. Buldyrev, et al. "Catastrophic cascade of failures in interdependent networks," Nature, vol. 464, no. 7291, pp. 1025–1028, 2010.

[7] J. Gómez-Gardenes, et al. "Explosive synchronization transitions in scale-free networks," Physical Review Letters, vol. 106, no. 12, p. 128701, 2011.

[8] W. Liu, et al. "Effects of frequency-degree correlation on synchronization transition in scale-free networks," EPL (Europhysics Letters), vol. 101, no. 3, p. 38002, 2013.

[9] X. Zhang, et al. "Explosive synchronization in a general complex network," Physical Review E, vol. 88, no. 1, p. 010802, 2013.

[10] I. Leyva, et al. "Explosive synchronization in weighted complex networks," Physical Review E, vol. 88, no. 4, p. 042808, 2013.

[11] I. Leyva, et al. "Explosive transitions to synchronization in networks of phase oscillators," Scientific Reports, vol. 3, no. 1, pp. 1–5, 2013.

[12] X. Zhang, et al. "Explosive synchronization in adaptive and multilayer networks," Physical Review Letters, vol. 114, no. 3, p 038701, 2015.

[13] S. Majhi, S. N. Chowdhury, and D. Ghosh, "Perspective on attractive-repulsive interactions in dynamical networks: Progress and future," EPL (Europhysics Letters), vol. 132, no. 2, p. 20001, 2020.

[14] N. Frolov, et al. "Chimera-like behavior in a heterogeneous Kuramoto model: The interplay between attractive and repulsive coupling," Chaos: An Interdisciplinary Journal of Nonlinear Science, vol. 30, no. 8, p. 081102, 2020.

[15] X. Dai, et al. "Explosive synchronization in populations of cooperative and competitive oscillators," Chaos, Solitons & Fractals, vol. 132, p. 109589, 2020.

[16] V. A. Maksimenko, et al. "Excitation and suppression of chimera states by multiplexing," Physical Review E, vol. 94, no. 5, 052205, 2016.

[17] N. S. Frolov, et al. "Macroscopic chimeralike behavior in a multiplex network," Physical Review E, vol. 98, no. 2, p. 022320, 2018.

[18] E. Pitsik, et al. "Inter-layer competition in adaptive multiplex network," New Journal of Physics, vol. 20, no. 7, p. 075004, 2018.

[19] S. Jalan, A. Kumar, and I. Leyva, "Explosive synchronization in frequency displaced multiplex networks," Chaos: An Interdisciplinary Journal of Nonlinear Science, vol. 29, no. 4, p. 041102, 2019.

[20] P. Khanra, et al. "Explosive synchronization in phase-frustrated multiplex networks," Physical Review E, vol. 98, no. 5, p. 052315, 2018.

[21] S. Jalan, et al. "Inhibition-induced explosive synchronization in multiplex networks," Physical Review E, vol. 99, no. 6, p. 062305, 2019.

[22] A. D. Kachhvah, et al. "Interlayer Hebbian plasticity induces first-order transition in multiplex networks," New Journal of Physics, vol. 22, no. 12, p. 122001, 2020.