

Intermittency in Forecasting Stochastic System Behavior Using Reservoir Computing

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Abstract—We study intermittent behavior of reservoir computing (RC) when predicting stochastic systems. We demonstrate that near threshold parameter values, RC exhibits accurate prediction most of the time, but with intermittent intervals where prediction fails completely. This behavior corresponds to on-off intermittency, a well-known phenomenon in nonlinear dynamics. Using a stochastic FitzHugh-Nagumo neuron as our test system, we analyze the statistical properties of this intermittent prediction behavior and develop a technique to estimate the “effective noise” amplitude that characterizes prediction quality. Our findings reveal important limitations in RC-based prediction of stochastic systems and provide insights for improving prediction reliability in practical applications.

Index Terms—reservoir computing, stochastic system, intermittency, prediction, noise, FitzHugh-Nagumo system.

I. INTRODUCTION

Intermittency represents a fundamental universal nonlinear phenomenon found in many different dynamical systems [1]. In stochastic systems, noise can induce intermittent behavior that follows predictable patterns despite its random nature. A prominent example is on-off intermittency [2], [3], which emerges near supercritical bifurcations under noise influence and manifests consistently across diverse systems including scale-free oscillator networks [4], human motion dynamics [5], and neuronal ensembles in epileptic brains [6].

Various machine learning methods have been used for forecasting and classification of nonlinear systems behavior where intermittency effect can be observed [7], [8]. One of popular methods is reservoir computing (RC), which effectively predicts nonlinear dynamics [9], including stochastic systems [10]–[12]. RC was also successfully used for reconstruction of hidden variables [13], [14] and time series classification tasks [15].

Recent theoretical work suggested that RC prediction performance is maximized in a regime of generalized synchronization with the system being predicted [16]. This insight led us to hypothesize that near the boundary of accurate prediction, intermittent behavior might emerge, potentially following the patterns of on-off intermittency observed in systems near the boundry of generalized synchronization.

This work was supported by Russian Science Foundation (Grant No. 23-71-30010).

In this paper, we verify this hypothesis by examining RC-based prediction of a stochastic FitzHugh-Nagumo neuron across varying noise parameters. We demonstrate that intermittent prediction behavior occurs when the noise parameters of the system and RC model are slightly detuned, and we characterize this behavior using established frameworks from nonlinear dynamics. Our work reveals fundamental limitations in RC-based prediction of stochastic systems that must be considered in practical applications.

II. METHODS

A. Stochastic FitzHugh-Nagumo Neuron

We consider the stochastic FitzHugh-Nagumo neuron as our test system, described by:

$$\begin{aligned}\dot{\eta}_1 &= \eta_1 - \eta_1^3/3 - \eta_2 + \beta, \\ \dot{\eta}_2 &= 0.08(\eta_1 - 0.8\eta_2 + 0.7) + D_n\xi(t),\end{aligned}\quad (1)$$

where η_1 and η_2 denote the excitatory and recovery variables. The term $D_n\xi(t)$ represents zero-mean white Gaussian noise with amplitude D_n . We operate in the excitable regime with $\beta = 0.3$ (below the Hopf bifurcation point $\beta_c \approx 0.322$). For numerical solution, we use the Euler-Maruyama method with integration step $\Delta t = 0.1$.

B. Reservoir Computing Model

Our RC model consists of input, reservoir (hidden), and output layers. The input vector is $\mathbf{I}(t) = (\eta_1(t), \eta_2(t), D_r\xi(t))^T$, where D_r is the noise amplitude fed to the RC. The reservoir state evolves according to:

$$\mathbf{R}(t) = \tanh(\mathbf{Q}\mathbf{R}(t - \Delta t) + \mathbf{V}\mathbf{I}(t)), \quad (2)$$

where $\mathbf{Q}$ is the reservoir adjacency matrix characterized by average node degree $\langle k \rangle$ and spectral radius λ , and $\mathbf{V}$ is the input matrix. The output vector $\mathbf{G}(t + \Delta t) = (g_1(t + \Delta t), g_2(t + \Delta t))^T$ predicts the next state of the system variables.

During training, we minimize the L_2 -error between true and predicted values using Tikhonov regularization. In prediction mode, the RC operates autonomously by feeding its output back to the input while continuing to receive the noise signal.

C. Intermittency Analysis

We evaluate prediction quality using the root-mean-square deviation:

$$\delta(t) = \sqrt{(\eta_1(t) - g_1(t))^2 + (\eta_2(t) - g_2(t))^2}. \quad (3)$$

Accurate prediction is defined when $\bar{\delta}_T < \delta_0$ (where $\bar{\delta}_T$ is the time-averaged deviation and $\delta_0 = 0.1$ is a threshold), while lack of prediction occurs when $\bar{\delta}_T > \delta_0$. Intermittent behavior manifests as alternating intervals of accurate prediction (laminar phases) and prediction failure (turbulent phases).

III. RESULTS

A. Hyperparameter Optimization

We optimized RC hyperparameters ($\langle k \rangle$ and λ) through grid search across 209 configurations. The best-performing RC had $\langle k \rangle = 20$ and $\lambda = 1.9$, trained with noise amplitude $D_0 = 0.2$ corresponding to the coherence resonance mode of the stochastic neuron.

B. Intermittent Prediction Behavior

When the noise amplitude fed to the RC (D_r) differs slightly from that of the neuron (D_n), intermittent prediction behavior emerges. In the first example $D_n = 0.2$ and $D_r = 0.32$. During laminar phases, prediction is accurate ($\delta(t) \approx 0$), while during turbulent phases, prediction completely fails with large deviations.

C. Statistical Characteristics of Intermittency

The distribution of laminar phase durations L follows a power law $N(L) \sim L^\alpha$ with exponent $\alpha = -1.53$, characteristic of on-off intermittency. Additionally, the mean laminar phase duration $\langle L \rangle$ depends on the parameter mismatch $\Delta D = |D_n - D_r|$ as $\langle L \rangle \sim |\Delta D|^\rho$.

When varying the neuron's noise parameter (D_n) while keeping $D_r = D_0$ fixed, we observe $\rho = -1.01$, precisely matching the theoretical prediction for on-off intermittency. However, when varying the RC's noise parameter (D_r) while keeping $D_n = D_0$ fixed, we find $\rho = -1.21$, indicating a deviation from the "ideal" on-off intermittency.

D. Effective Noise Model

To explain the deviation from ideal on-off intermittency when varying D_r , we propose the concept of "effective noise" with amplitude D^* that characterizes RC self-errors. This effective noise increases exponentially with $|\Delta D_r| = |D_r - D_0|$:

$$D^*(\Delta D_r) = D_0^* \exp(+k\Delta D_r) \quad (4)$$

where $D_0^* \approx 6 \times 10^{-3}$ and $k = 13.69$. By incorporating this effective noise into the parameter mismatch ($\Delta D^* = |D_n - D_r| + D^*$), we recover the ideal power law with $\rho = -1.01$, confirming that the observed behavior is indeed on-off intermittency.

IV. CONCLUSIONS

We have identified and characterized a novel intermittent behavior in RC-based prediction of stochastic systems. This behavior, corresponding to on-off intermittency, manifests as alternating intervals of accurate prediction and prediction failure when the noise parameters of the system and RC model are slightly detuned. To explain the deviation from target on-off intermittency angle of $\rho = -1$, we have proposed the concept of effective noise and determined a method of its amplitude evaluation. Including effective noise in calculations lead to the correction of on-off intermittency angle.

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