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Riemannian geometry of eye-movement covariance matrices during a prolonged working memory task

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Abstract Eye movements reflect cognitive processing during working memory performance. However, it remains unclear how prolonged task execution and increasing fatigue affect the integrated organization of oculomotor behavior under different memory demands. In this study, we analyzed covariance patterns of trial-wise eye-movement features during a prolonged Sternberg task using a Riemannian geometry framework. To quantify the differentiation between oculomotor states associated with different levels of task demand, we introduced the Oculomotor Differentiation Index (ODI). The results showed that oculomotor states corresponding to different working memory demands were clearly differentiated during the earlier stages of the experiment, but this differentiation progressively weakened and was no longer evident at the final stage. In addition, higher subjective fatigue was associated with lower differentiation between demand-related oculomotor states. These findings suggest that increasing fatigue is associated with a weakening of task-specific oculomotor tuning, so that eye-movement behavior becomes less selectively organized with respect to cognitive demand. The study also shows that a covariance-based manifold approach can capture integrated changes in oculomotor state organization that are not evident from isolated eye-movement measures alone.

1 Introduction

Working memory performance relies on the coordinated operation of multiple cognitive and sensorimotor processes, whose interactions give rise to structured and adaptive behavior. From this perspective, eye movements can be viewed not simply as isolated behavioral markers, but as an observable expression of the integrated organization of a cognitive system under task demands. Recent studies show that oculomotor behavior during working memory tasks reflects not only overt visual sampling, but also the allocation of internal attention to maintained information [1, 2]. In particular, fine-grained gaze biases and fixational dynamics have been shown to track shifts of attention within working memory, supporting the view that eye movements are closely linked to the internal organization of cognitive processing rather than merely to external stimulus inspection [1]. In this sense, oculomotor behavior may be viewed within a self-organization framework, in which coordinated task-related patterns emerge and reorganize through the interaction of multiple cognitive and sensorimotor components. This makes the oculomotor system a useful model for studying how cognitive demand and fatigue reshape the organization of coordinated behavior over time.

Accumulating evidence shows that working memory load influences several oculomotor parameters [2–5]. For instance, during free viewing with an updating-based load, increased load leads to longer fixations and shorter scan paths, yet does not reduce the spatial area covered by gaze. This pattern suggests that load primarily modulates the timing of visual exploration rather than its spatial breadth [4]. In addition, other studies have found that gaze metrics can reveal internal working memory processes, including covert attention shifts and rehearsal [2]. Taken together, these results indicate that working memory load produces a multicomponent oculomotor signature, not a single universal marker.

At the same time, prolonged cognitive performance introduces an additional source of oculomotor variation [6]. Recent fatigue studies show that sustained task execution can alter blink dynamics, pupil-related measures, and saccade-related variables, while the magnitude and even direction of these effects depend on task context and the

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balance between engagement, effort, and exhaustion [3, 6]. Importantly, recent work on mental fatigue prediction from eye tracking has emphasized that fatigue accumulates gradually and is better captured by multiple eye-based features than by a single indicator. This is particularly relevant for prolonged working memory tasks, in which current cognitive load and accumulated fatigue are likely to act simultaneously on the oculomotor system[6].

Despite the multidimensional nature of these effects, many studies still analyze eye-movement measures in isolation. However, recent findings suggest that working memory load does not affect all oculomotor components uniformly: some aspects of gaze dynamics change systematically, whereas others remain relatively stable [4, 7]. Fatigue-related changes are also increasingly described using combinations of eye-based features rather than a single marker [8–10]. These observations suggest that the relevant signal may lie not only in isolated feature changes, but also in the joint organization of oculomotor behavior. Consistent with this view, multivariate analyses of eye movements can discriminate task and cognitive state even when aggregate eye-movement measures overlap substantially across conditions [11, 12].

A natural way to represent such joint organization is through covariance matrices constructed from multiple oculomotor features. Covariance matrices encode the second-order structure of a multivariate system, but they are not ordinary Euclidean objects: they belong to the space of symmetric positive definite (SPD) matrices. In adjacent fields, especially neuroimaging and covariance-based neural signal analysis, this property has motivated the use of Riemannian geometry, which provides mathematically appropriate distances, averages, and trajectories for SPD-valued data [13, 14]. Recent work has shown that functional connectivity matrices can be analyzed more faithfully when their SPD geometry is respected, and dynamic Riemannian state-space models have been proposed for time series of SPD matrices, outperforming Euclidean alternatives in modeling and classification tasks[15, 16].

In contrast, the covariance structure of multiple oculomotor features has received relatively little attention in eye-movement research. However, covariance-based approaches offer a fundamentally different perspective. Rather than describing eye-movement behavior as a set of isolated parameter changes, they characterize it in terms of the coordinated variation of multiple components. Because these components are inherently interdependent, their covariance structure may capture aspects of cognitive-state-related organization that remain hidden in univariate summaries. In this sense, a Riemannian treatment of oculomotor covariance matrices provides a useful framework for studying how cognitive demand and fatigue reshape the multivariate organization of gaze behavior over time.

2 Methods

2.1 Experimental procedure

The experiment was based on a prolonged Sternberg working memory task. Each trial consisted of several successive stages. First, a fixation cross was presented at the center of the screen for 1.5-−2.5 s to orient attention. This was followed by the presentation of a memory set for 1.5-−2.5 s, during which participants were instructed to memorize the presented letters. The memory set consisted of seven symbol positions, some of which were occupied by uppercase Cyrillic letters, whereas the remaining positions were filled with asterisks. After the encoding stage, a blank screen was shown for 3–7 s, corresponding to the retention stage. Next, a single lowercase probe letter was presented, and the participant was required to determine whether it had been included in the preceding memory set. The response interval lasted 4 s and included 2 s of probe presentation followed by 2 s of a black screen.

Task difficulty was manipulated by varying the number of letters in the memory set. Trials containing 2–4 letters were assigned to the low-difficulty condition, whereas trials containing 5–7 letters were assigned to the high-difficulty condition.

The main part of the experiment consisted of four consecutive blocks. Each block included 72 trials presented in randomized order. Before and after each block, participants rated their current level of subjective fatigue using a visual analog scale, where the slider position indicated fatigue from low to high. Eye movements were recorded continuously throughout the task. The total duration of the experiment was approximately 70 min.

Fifteen healthy volunteers aged 18 to 22 years (5 females and 10 males) with normal or corrected-to-normal visual acuity participated in the experiment. All participants provided written informed consent before participation and were informed about the experimental procedure. The study was conducted in accordance with the Declaration of Helsinki.

2.2 Eye-movement metrics

Eye movements were recorded using an EyeLink 1000 Plus eye tracker (SR Research, Canada) at a sampling rate of 1000 Hz. Oculomotor behavior was analyzed separately for two task stages: encoding, during which the letter array was visually presented, and retention, during which participants maintained the encoded information in working memory. Four types of oculomotor measures were extracted to characterize complementary aspects of gaze behavior: fixation timing, saccade dynamics, spatial coverage of task-relevant regions, and repeated inspection of previously viewed locations. Mean fixation duration ($\overline{FD}$) and mean peak saccade velocity ($\overline{PSV}$) were quantified separately for the encoding and retention stages, whereas letter coverage (LC) and gaze overlap (GO) were computed for the encoding stage only.

For each trial and task stage, $\overline{FD}$ was computed as the mean duration of all detected fixations, whereas $\overline{PSV}$ was computed as the mean of the peak velocities of all detected saccades.

Spatial gaze measures were quantified for the encoding stage only, as this stage directly reflects active visual sampling of the presented stimulus array. Both LC and GO were computed on a trial-by-trial basis using a circular attentional window with a diameter corresponding to approximately 2° of visual angle. To account for differences in the spatial extent of the relevant stimulus elements, both measures were normalized by the total area occupied by the target letters.

Specifically, LC was defined as the proportion of the total area of the target letters that was covered by the attentional window at least once during a trial, thus reflecting the overall extent of visual inspection of task-relevant stimulus regions. In turn, GO was defined as the proportion of the total area of the target letters that was covered by the attentional window more than once within the same trial, thereby quantifying repeated inspection of previously viewed target locations.

2.3 Manifold-based analysis of oculomotor state geometry

Individual eye-movement measures provide only partial descriptions of task-related behavior. Because oculomotor behavior emerges as a coordinated response involving multiple interdependent features, the relevant structure lies not only in separate variables, but also in their joint organization. Covariance matrices provide a natural representation of such integrated organization by capturing the pattern of relationships among eye-movement features. On this basis, each experimental condition was described by a covariance matrix constructed from trial-wise eye-movement features, and these covariance representations were compared using Riemannian distances.

Let $\mathbf{x}_{ib\tau t} \in \mathbb{R}^p$ denote the vector of p oculomotor features for participant i, block b, stimulus condition τ, and trial t, where stimulus conditions corresponded to the number of letters in the array, $\tau \in \{2, 3, 4, 5, 6, 7\}$. The feature vector comprised six trial-wise measures: mean fixation duration during encoding, mean fixation duration during retention, mean peak saccade velocity during encoding, mean peak saccade velocity during retention, letter coverage, and gaze overlap.

For each participant i and feature j, standardization was performed across all available observations:

$$\tilde{x}_{im}^{(j)} = \frac{x_{im}^{(j)} - \mu_i^{(j)}}{\sigma_i^{(j)}}. \tag{1}$$

where m indexes all observations available for participant i, and $\mu_i^{(j)}$ and $\sigma_i^{(j)}$ denote the mean and standard deviation of feature j across all observations available for participant i, respectively.

For each combination $(i,\, b,\, \tau)$, the trial-level data matrix was defined as

$$\mathbf{X}_{ib\tau} = \begin{bmatrix} \tilde{\mathbf{x}}_{ib\tau 1}^{\top} \\ \tilde{\mathbf{x}}_{ib\tau 2}^{\top} \\ \vdots \\ \tilde{\mathbf{x}}_{ib\tau n_{ib\tau}}^{\top} \end{bmatrix} \in \mathbb{R}^{n_{ib\tau} \times p}. \tag{2}$$

where $\tilde{\mathbf{x}}_{ib\tau t}$ denotes the standardized feature vector for trial t, and $n_{ib\tau}$ is the number of trials available for participant i, block b, and stimulus condition τ.

Because the number of trials per condition was limited, covariance estimation was regularized to improve stability [17, 18]. For each $(i,\, b,\, \tau)$ and bootstrap replicate $r = 1, \ldots, R$, with $R = 1000$, rows of $\mathbf{X}_{ib\tau}$ were resampled with replacement, and the covariance matrix was estimated using the Oracle Approximating Shrinkage estimator:

$$\mathbf{C}_{ib\tau}^{(r)} = (1 - \hat{\rho}_{ib\tau}^{(r)})\, \mathbf{S}_{ib\tau}^{(r)} + \hat{\rho}_{ib\tau}^{(r)} \frac{\mathrm{tr}(\mathbf{S}_{ib\tau}^{(r)})}{p} \mathbf{I}, \tag{3}$$

where $\mathbf{S}_{ib\tau}^{(r)}$ is the sample covariance matrix of the r-th bootstrap sample, $\hat{\rho}_{ib\tau}^{(r)} \in [0,\, 1]$ is the shrinkage coefficient estimated analytically under the Oracle Approximating Shrinkage criterion, $\mathrm{tr}(\cdot)$ denotes the matrix trace, p is the number of oculomotor features, and $\mathbf{I}$ is the $p \times p$ identity matrix.

To obtain a robust condition-specific covariance representation, the bootstrap ensemble was summarized by its geometric median on the manifold $\mathcal{S}_{++}^{p}$ of $p \times p$ symmetric positive definite matrices. The geometric median was preferred over the Fréchet mean to reduce the influence of occasional outlying bootstrap samples on the aggregated covariance estimate:

$$\hat{\mathbf{C}}_{ib\tau} = \arg\min_{\mathbf{M}\in\mathcal{S}_{++}^{p}} \sum_{r=1}^{R} d_{\mathrm{AIRM}}\Big(\mathbf{M},\, \mathbf{C}_{ib\tau}^{(r)}\Big). \tag{4}$$

Pairwise distances between covariance matrices were computed using the Affine-Invariant Riemannian Metric:

$$d_{\mathrm{AIRM}}(\mathbf{A},\, \mathbf{B}) = \Big\|\log\Big(\mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\Big)\Big\|_F, \qquad \mathbf{A},\, \mathbf{B} \in \mathcal{S}_{++}^{p}. \tag{5}$$

where $\|\cdot\|_F$ denotes the Frobenius norm.

Task difficulty was defined by stimulus set size:

$$\mathcal{G}_L = \{2,\, 3,\, 4\}, \qquad \mathcal{G}_H = \{5,\, 6,\, 7\}. \tag{6}$$

For each participant i and block b with all six matrices $\hat{\mathbf{C}}_{ib\tau}$ available, the dissimilarity matrix $\mathbf{W}_{ib} \in \mathbb{R}^{6\times 6}$ was defined as

$$W_{ib}(\tau_1,\, \tau_2) = d_{\mathrm{AIRM}}\Big(\hat{\mathbf{C}}_{ib\tau_1},\, \hat{\mathbf{C}}_{ib\tau_2}\Big), \qquad \tau_1 \neq \tau_2. \tag{7}$$

If lower- and higher-demand conditions correspond to distinct oculomotor organizations, covariance matrices should be more similar within the same demand range and more dissimilar across demand ranges. To summarize this relation within each block, we contrasted between-group and within-group dissimilarity across the six stimulus conditions. The sets of within-group and between-group condition pairs were defined as:

$$\mathcal{P}_{\mathrm{w}} = \{(\tau_1,\, \tau_2) : \tau_1 < \tau_2,\;\; \ell(\tau_1) = \ell(\tau_2)\}, \tag{8}$$

$$\mathcal{P}_{\mathrm{b}} = \{(\tau_1,\, \tau_2) : \tau_1 < \tau_2,\;\; \ell(\tau_1) \neq \ell(\tau_2)\}, \tag{9}$$

where

$$\ell(\tau) = \begin{cases} L, & \tau \in \mathcal{G}_L, \\ H, & \tau \in \mathcal{G}_H. \end{cases} \tag{10}$$

The Oculomotor Differentiation Index (ODI) was defined as

$$\mathrm{ODI}_{ib} = \frac{1}{|\mathcal{P}_{\mathrm{b}}|} \sum_{(\tau_1,\, \tau_2)\in\mathcal{P}_{\mathrm{b}}} W_{ib}(\tau_1,\, \tau_2) - \frac{1}{|\mathcal{P}_{\mathrm{w}}|} \sum_{(\tau_1,\, \tau_2)\in\mathcal{P}_{\mathrm{w}}} W_{ib}(\tau_1,\, \tau_2). \tag{11}$$

Positive values of ODI_{ib} indicate that covariance structures associated with lower- and higher-demand conditions are more dissimilar to each other than covariance structures observed within the same demand level.

2.4 Statistical analysis

Trial-level oculomotor features were standardized within each participant before analysis. The effects of block and task difficulty on individual oculomotor measures were assessed using two-way repeated-measures ANOVA with block and task difficulty as within-subject factors, followed by post hoc comparisons where appropriate. Block-related changes in the ODI were assessed using the Friedman test, followed by pairwise Wilcoxon signed-rank tests with Benjamini–Hochberg false discovery rate correction. One-sample Wilcoxon signed-rank tests were additionally used to determine whether ODI values differed from zero within each block. The association between ODI and subjective fatigue was evaluated using repeated-measures correlation. Statistical significance was set at $p < 0.05$.

3 Results

The results are presented in two stages. First, we report the effects of block and task difficulty on individual oculomotor features. We then examine the dynamics of the integrated manifold-based index and its association with subjective fatigue.

To evaluate the effects of experimental progression and task difficulty on oculomotor behavior, a two-way repeated-measures ANOVA was applied to all extracted eye-movement features (Table 1). Significant main effects of block were observed for mean fixation duration during both encoding and retention, and for mean peak saccade velocity during retention. Significant main effects of task difficulty were found for letter coverage, gaze overlap, and mean peak saccade velocity during encoding. No significant block × difficulty interactions were detected for any feature, indicating that the trajectory of block-related changes was comparable across difficulty conditions.

The features showing significant main effects of block are illustrated in Fig. 1. Mean fixation duration decreased across blocks during both encoding and retention. During retention, $\overline{FD}$ was significantly lower in blocks 2, 3, and 4 relative to block 1. During encoding, significant differences were observed between blocks 1 and 3, 1 and 4, 2 and 3, and 2 and 4, indicating an overall decrease over the course of the experiment. Mean peak saccade velocity during retention showed a distinct pattern: values in block 4 were significantly higher than in blocks 2 and 3, suggesting a shift in oculomotor dynamics at the final stage of the task rather than a gradual trend.

The difficulty-related effects are presented in Fig. 2. Both letter coverage and gaze overlap were significantly higher under low-difficulty conditions, indicating broader and more repeated visual exploration of the stimulus array under lower memory load. Mean peak saccade velocity during encoding showed the opposite pattern, being significantly higher under high-difficulty conditions, consistent with faster oculomotor shifts under higher task demands.

Whereas the analyses above addressed individual oculomotor features in isolation, the ODI was used to quantify the overall differentiation between low- and high-difficulty oculomotor states at the level of their joint covariance structure. The block-related dynamics of the ODI are shown in Fig. 3. A Friedman test revealed a significant effect of block ($\chi^2(3) = 10.28$, $p = 0.016$), indicating that the degree to which the oculomotor system differentiated between difficulty levels varied across the experiment. ODI values were largest in blocks 1 and 3, somewhat reduced in block 2, and lowest in block 4, suggesting that the multivariate differentiation between oculomotor states associated with low- and high-difficulty conditions diminished markedly by the end of the experiment.

Table 1 Results of the two-way repeated-measures ANOVA for oculomotor features, with block and task difficulty as within-subject factors

Feature	Effect	df	F	p	η^2_G
GO	Block	3, 42	0.45	0.720	0.014
	Block × difficulty	3, 42	0.38	0.766	0.010
	Difficulty	1, 14	261.38	< 0.001***	0.758
LC	Block	3, 42	0.33	0.803	0.015
	Block × difficulty	3, 42	0.41	0.747	0.007
	Difficulty	1, 14	1254.24	< 0.001***	0.898
$\overline{FD}$ (encoding)	Block	3, 42	9.74	< 0.001***	0.277
	Block × difficulty	3, 42	1.62	0.198	0.018
	Difficulty	1, 14	1.41	0.255	0.029
$\overline{FD}$ (retention)	Block	3, 42	14.94	< 0.001***	0.421
	Block × difficulty	3, 42	0.50	0.683	0.005
	Difficulty	1, 14	0.47	0.504	0.006
$\overline{PSV}$ (encoding)	Block	3, 42	1.85	0.153	0.067
	Block × difficulty	3, 42	1.03	0.388	0.019
	Difficulty	1, 14	7.55	0.016*	0.093
$\overline{PSV}$ (retention)	Block	3, 42	5.01	0.005**	0.190
	Block × difficulty	3, 42	1.48	0.235	0.024
	Difficulty	1, 14	1.67	0.217	0.014

Asterisks denote significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

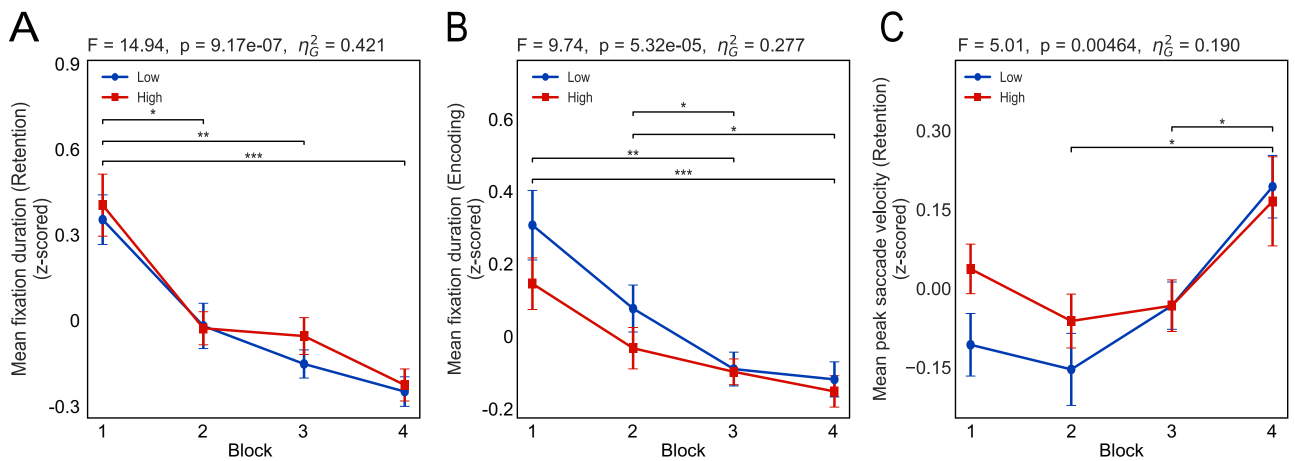


Fig. 1 Block-related changes in oculomotor features with significant main effects of block: (A) mean fixation duration during retention, (B) mean fixation duration during encoding, and (C) mean peak saccade velocity during retention. Blue and red lines correspond to low- and high-difficulty conditions, respectively. Points represent group means, and error bars indicate SEM. Horizontal brackets mark significant post hoc pairwise comparisons between blocks. Asterisks denote statistically significant effects in post hoc comparisons: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

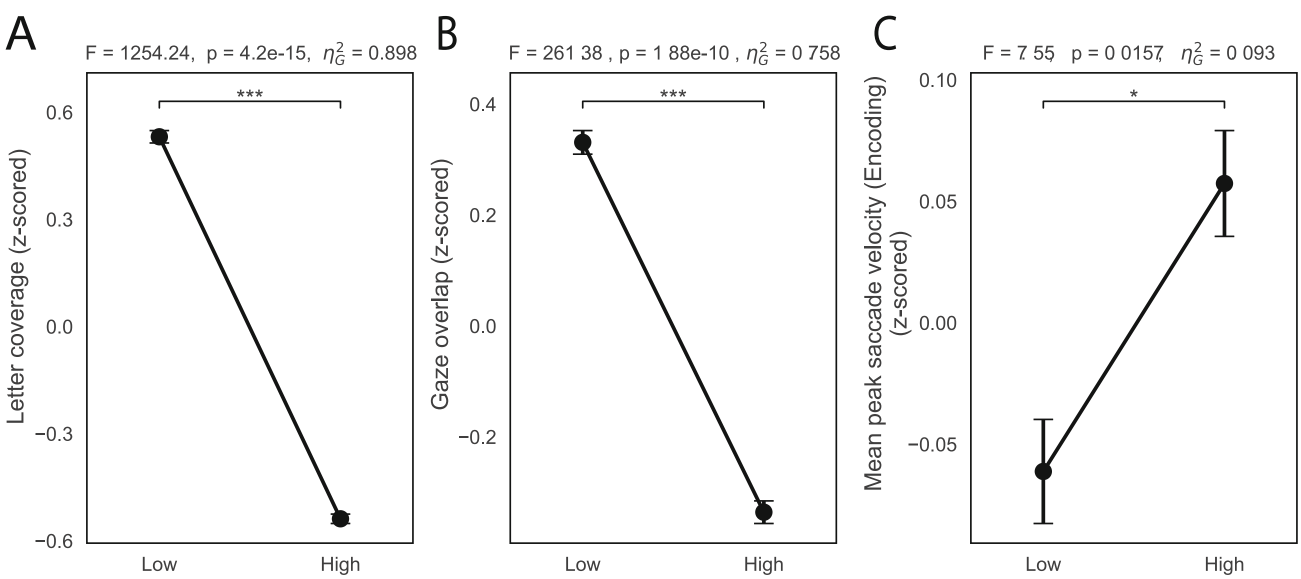
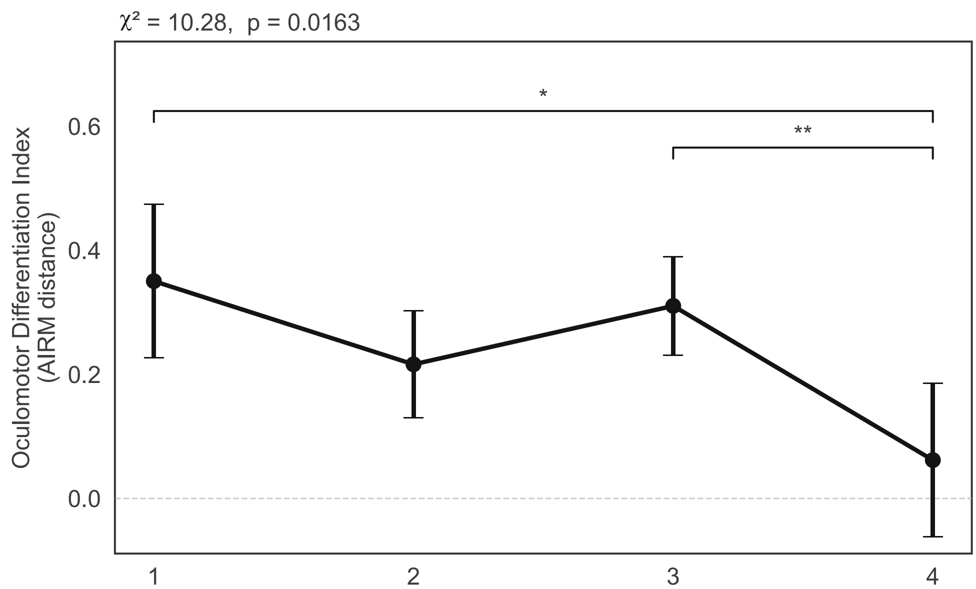


Fig. 2 Effects of task difficulty on oculomotor features with significant main effects of difficulty: (A) letter coverage, (B) gaze overlap, and (C) mean peak saccade velocity during encoding. Points represent group means for low- and high-difficulty conditions, and error bars indicate SEM. Horizontal brackets mark significant pairwise comparisons between difficulty levels. Asterisks denote statistically significant effects: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

To further characterise the source of this effect, we conducted within-block and between-block comparisons. One-sample Wilcoxon signed-rank tests confirmed that ODI values were significantly greater than zero in block 1 ($p = 0.004$), block 2 ($p = 0.026$), and block 3 ($p < 0.001$), indicating that across these blocks the dissimilarity between covariance structures from different difficulty levels reliably exceeded the dissimilarity observed within difficulty groups. In block 4, however, the ODI did not differ significantly from zero, indicating that by the final block the covariance structure of oculomotor features no longer showed reliable differentiation between task difficulty levels.

Post hoc pairwise comparisons between blocks confirmed that ODI in block 4 was significantly lower than in blocks 1 and 3. No significant differences were observed among blocks 1, 2, and 3, indicating that the reduction in oculomotor differentiation was not gradual but occurred specifically at the transition to the final block.

Fig. 3 Block-related dynamics of the ODI. Points represent group means, error bars indicate SEM. Asterisks above points indicate significant deviation from zero, and brackets indicate pairwise block comparisons. Asterisks denote statistically significant effects in post hoc comparisons: * $p < 0.05$, ** $p < 0.01$



Taken together, these findings demonstrate that the multivariate differentiation of oculomotor states with respect to task difficulty was reliably maintained during the first three blocks of the experiment, but was substantially reduced in the final block.

Given the marked decline in ODI observed in the later part of the experiment, we examined whether this reduction was related to subjective fatigue ratings. A repeated-measures correlation revealed a significant negative association between ODI and fatigue ($r_{\mathrm{rm}} = -0.32$, $p = 0.03$), indicating that participants reporting higher fatigue showed weaker differentiation between oculomotor states corresponding to low- and high-difficulty conditions. This result suggests that the reduced differentiation between low- and high-difficulty oculomotor states in the final block may be related, at least in part, to the influence of accumulated fatigue on adaptive oculomotor control.

4 Discussion

The present findings show that working memory demand and prolonged task execution shape oculomotor behavior during a prolonged Sternberg task. Task demand affected several individual eye-movement measures, whereas prolonged performance was associated with weaker differentiation between demand-related oculomotor states. In addition, higher subjective fatigue was associated with lower values of the ODI. Together, these findings indicate that cognitive demand and fatigue are reflected not only in separate oculomotor features, but also in the overall organization of their covariance structure.

At the level of individual features, the present results are consistent with previous studies showing that increasing working memory load changes the way visual information is sampled. In our data, letter coverage and gaze overlap were higher in the low-difficulty condition, whereas mean peak saccade velocity during encoding was higher in the high-difficulty condition. This pattern suggests that higher working memory demand is associated with a more selective mode of visual sampling, in which gaze is distributed less broadly and repeated inspection is reduced. Such an interpretation agrees with reports that greater cognitive demand is accompanied by more constrained exploration and by a stronger concentration of gaze on task-relevant information [4, 19]. The increase in peak saccade velocity during encoding under higher difficulty is also compatible with previous findings, although results for this metric vary across paradigms. Recent reviews suggest that saccade velocity may increase or decrease depending on the balance between arousal, engagement, and fatigue [7]. In the present data, higher saccade velocity during encoding may reflect a more mobilized and temporally compressed mode of information acquisition under greater mnemonic demand.

Importantly, none of the individual oculomotor features showed a significant block × difficulty interaction. This indicates that the block-dependent change in differentiation between low- and high-difficulty conditions was not expressed at the level of single measures, but emerged only at the multivariate level captured by the ODI. Accordingly, a central finding of the study was that the differentiation between oculomotor states associated with different task demands weakened over the course of the experiment and was no longer evident at the final stage. This pattern suggests that prolonged task execution reduces the specificity of oculomotor organization with respect to task difficulty. In other words, with increasing fatigue, eye-movement behavior becomes less distinctly tuned to differences in working memory demand, leading to a convergence of demand-related oculomotor states. Rather than indicating changes in isolated eye-movement measures alone, this result points to a reorganization at the level of their joint multivariate structure.

One possible interpretation of this pattern is that prolonged task execution leads to a shift toward more similar oculomotor strategies across conditions. In the earlier stages of the experiment, eye-movement behavior remained more specifically organized with respect to task difficulty, whereas later this specificity became weaker, suggesting a more uniform mode of visual and oculomotor coordination. Such a shift may reflect a reduction in the flexibility with which the oculomotor system adapts to different levels of working memory demand under increasing fatigue. This interpretation is broadly consistent with the fatigue literature, which suggests that mental fatigue is better captured by coordinated changes across multiple eye-based features than by any single oculomotor marker [6]. It may also be discussed in relation to compensatory-control accounts, according to which sustained task performance can be accompanied by changes in resource allocation and behavioral regulation even when overt performance remains relatively preserved [3, 20, 21]. At the same time, the present design does not fully disentangle accumulated fatigue from other factors associated with block progression. However, the within-participant association between ODI and subjective fatigue suggests that fatigue contributes to the observed reorganization beyond time-on-task effects alone.

Taken together, these results indicate that the effects of working memory demand and prolonged task execution are expressed not only in separate oculomotor variables, but also in the multivariate organization of their relationships. The reduction in differentiation between demand-related oculomotor states suggests that increasing fatigue is associated with a weakening of task-specific oculomotor organization. More broadly, these findings support the view that eye-movement behavior during working memory performance is better understood as an integrated functional state. The present study therefore highlights the value of covariance-based Riemannian analysis for investigating cognitive-state-dependent changes in oculomotor organization.

4.1 Limitations

The main limitation of this study is the relatively small sample size. Another limitation concerns the grouping of task difficulty. In the present work, the division into lower-demand (2–4 letters) and higher-demand (5–7 letters) conditions was fixed and applied uniformly to all participants, rather than being adjusted to individual differences in working memory capacity or subjective task difficulty. This could have influenced the results, given interindividual variability in the experienced demands of the task.

An additional methodological limitation is related to the covariance-based nature of the proposed approach. Reliable estimation of covariance structure generally requires a sufficient number of trials within each condition. In the present design, each block contained 72 trials distributed across six stimulus conditions, yielding on average 12 trials per condition per block. For a 6×6 covariance matrix this is close to the minimum practical sample size, which inevitably increases the variance of individual covariance estimates. Shrinkage estimation, bootstrap resampling, and geometric-median aggregation were used to partially compensate for this limitation, but caution is still warranted when interpreting covariance-based differences under such a limited number of observations per cell.

Finally, in the present study the multivariate organization of oculomotor behavior was characterized primarily through distances between covariance matrices and their summary index. This provided a compact and interpretable description of condition-related differentiation, but it does not exhaust all possible ways of describing the structure of relations among conditions. Future work may therefore extend the present framework by considering more detailed relational or network-based descriptions.

5 Conclusions

The results of this study indicate that the effects of cognitive demand and fatigue on eye movements are expressed not only through changes in separate oculomotor features, but also through changes in the overall organization of oculomotor coordination. The progressive weakening of differentiation between demand-related oculomotor states suggests that fatigue reduces the specificity with which the oculomotor system adapts to task demands. This supports the view that eye-movement behavior during working memory performance should be understood as a multicomponent state. Methodologically, the findings show that covariance-based Riemannian analysis can provide information about cognitive-state-dependent oculomotor organization beyond that available from isolated measures alone.

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Data Availability The data presented in this study are available on request from the corresponding author.

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